Analysis of Queueing System and Impact of Digital Payments in Supermarket

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Abstract — *This paper deals with digital payments and cash payments in supermarket. Initially we take two counters for comparison of digital and cash payments. The first counters for digital payments and the second for cash payments and calculate billing times from both counters. Our aim is to reducing the customers waiting time by increasing the number of servers according to the conditions, both digital and cash payments. The analysis of various parameters of the queueing system, calculate utilization factor, service rate, arrival rate, calculate idle bill payment counter, customer satisfaction rates, and waiting time. After analyzing the parameters of the parameters of queueing system model, it is observed that digital payments save time.*

Keywords— *digital payment, cash payment, utilization factor, percentage of idle bill counter, waiting time, M/M/C model.*

I. INTRODUCTION

Queueing is a part of present day life that we experience at every step in our daily activities like to get tickets at railway station or to pay bill in super market. In supermarket system customers waiting to get service from server are represented by queue and also called waiting line. It is also use in the field of communication network, computer system and operation machine plants and so on. The queueing system is related to the three important factors, customer, queues and service. Customer come for the service system for taking service, after finding service customer leaves the system. Queueing theory is the Mathematical study of waiting lines or queue. The theory enables Mathematical analysis of several related processes, including arriving at the queue, waiting in the queue and being served by the servers at the front of the queue. Long waiting times are one of the most common issues at supermarkets. Shorter waiting times can increase service experience, customer loyalty and thereby potentially increases market share. The perception of waiting time is important; customers with fewer items can perceive a longer waiting time than customers with more items even if the customers waiting time was the same. Most of the part of this paper focuses on the single server queueing models with multi-phase requirements and digital payment system.

The whole queueing system based on the following terms:

The Input Process: The input defines the pattern in which the customers arrive for taking service. Since the customers arrive in a random manner, therefore their arrival pattern can be considered in terms of probabilities.

Queue (Waiting line): The customer arrive and enter in the queueing system, if the server is busy or the number of customers are more than the service capacity then queue will be formed. The word “size of queue” defines the maximum number of customers which are waiting for service. The size of queue can be finite or infinite. It only depends on the system, how many customers can stay in the system? If it is for finite capacity then queue length will be finite and for infinite queue, we can take an example of online system i.e. online reservation, online shopping, online booking for cinema tickets etc.

Service Discipline (Queue Discipline): Service discipline give some rules in which the members in the queue are selected for the service, one is first come first serve (FCFS) under this rule the customers are served in manner of their arrivals i.e. any customer join the queue first then he/she will be served first. Other rules are as follows Last come first served (LCFS) under this rule the customer arrive in the last that served first, service in random order (SIRO) in this rule the customer are served in random order which is independent of their arrival patterns. In many cases customers are served on some priority such that customers are served before others without observe of their order of arrivals.

Service Mechanism (Service Pattern): Service mechanism introduce to service time and the service facilities. Service time is the interval from starting to the completion of service. Service facility depends on many factors.

- **(i)** Single queue and single server
- **(ii)** Several queue several servers
- **(iii)** Single queue several servers
- **(iv)** Several queue one server

Customer's Behavior in Queue: In the queueing system, customers behave differently given are following:

- **(i)** **Balking:** In this, the waiting line is too long and customers having enough amount of time therefore he/she does not join the queue and do not take any service facility.
- **(ii)** **Reneging:** Under this behavior, customer joins the queue but after spending much amount of time he/she lose his/her patience and leave the queue.
- **(iii)** **Jockeying:** Under this behavior, the service system should have multiple numbers of queues, customer joins one queue but in case of small length of another queue, he/she joins another queue i.e. customer may jump from one queue to another queue.
- **(iv)** **Collusion:** It is different from another behaviors, in this group of customers may collaborate and one of them may join the queue and find service for example at the cinema ticket window one person may join the queue and buy tickets for the group.
- **(v)** **Priorities:** In some cases, some customers have priority therefore they are served before other customers i.e. service facility is independent of their arrivals.

States of System: The main aspect of queueing system also involves the state of the system. The states of the system define only in the term time. There are two types of states:

- **(i)** **Transient State:** The system is said to be in transient state when the operating factor such as input, output, average, queue length etc are dependent of time.
- **(ii)** **Steady State:** The system said to be steady state when the operating factors are not affected by the amount of time i.e. they are independent of time.

II. LITERATURE REVIEW

Sivathanu [2] studied on the actual usage (AU) of digital payment systems by the consumers during the period of demonetization in India. He worked on the unified theory of acceptance and use of technology (UTAUT 2) and innovation resistance theory using the partial least squares (PLS)-structural equation modeling (SEM) technique. The results suggest that the behavioral intention (BI) to use and innovation resistance (IR) affect the usage of digital payment systems. The relation between BI to use digital payment systems and the AU of digital payment systems is moderated by the stickiness to cash payments. **Rengarajan et al [11]** analyze the perception of customer on the digital payments and give the comparative study between the private sector and public sector banks regard to the usage and awareness of digital payments. They designed the well-structured questionnaire for the customers is to study their demographic factors, relationship with banks, awareness, and usage and then they comparing the two sectors to get the valuable inputs. Finally they find out the various problems about the area of usage of digital payment system and to create consciousness about the importance of electronic services. **Roy & Sinha [10]** determined the factors influencing consumer's adoption on the light of Technology Acceptance Model. Survey based questionnaires are designed and Factor Analysis is used to find reliable and consistent factors. Proposed model illustrates the level of fulfillment of each acceptance factors and therefore predicts its adoption and indicates areas of improvement. **Shastrakar & Pokley [5]** studied that how to minimize the waiting time of customer at bill paying counter of supermarket by using the queueing theory model, measures different parameters and analyzed that parameters of Queueing model it has been observed that the new suggested model gives better result than the existing model. **Arita & Schadschneider** [4] deal with situations in which queues are formed with the different structures supermarket checkout or ticket counter. They worked on the behavior of crowds which may be described by the queuing dynamics. They generate the exclusive queuing process in which the queue in on microscope level. This process as same as the totally symmetric exclusion process (TASEP) of varying length gives the exact result. **Jhala & Bhathawala [8]** worked on the efficiency of the queueing models in terms of utilization, length of the queue waiting length and they found the solution by increasing the number of queues so that the total cost is optimized. **Thirupathi et al [7]** worked on the case study research tries to find the impact of digital payment apps and its impact after demonetization. **Peterson [10]** discussed the on-going debate and country-level projects directed at greater financial inclusion via digital finance in developing and emerging economies. **Igwe et al [1]** determined a efficient queue management in Makurdi (Nigeria) as case study by using "M/M/1" model. The paper has consequently made suggestions to help mitigate the prevailing queueing problems in Makurdi town, which will be relevant to many cities in developing countries having similar challenges. **Shukla et al. [12]** discussed how to reduce bounce rate of Indian customers in online shopping.

III. METHODOLOGY

Here we consider the M/M/C: N/FCFS queueing model with arrival rate follows the Poisson distribution and service rate follows the exponential distribution, N represents the maximum number of population in the queueing system and C represents the number of server. The research method used in this work is a quantitative research approach. We have observed the situation of supermarket from Thursday to Sunday and collected the data of total arrival and total departure with respect to each server and per hour. Here we analysis the various parameter of the system number of customer in the system, number of system in the queue, average waiting time, system capacity. The model developed was used to test the queueing system against the number of servers and customers arrival rate of the establishment.

1. **For the cash payments, we have the following observations:**

CASH PAYMENT

Table – 1.1 DAY - 1 (THURSDAY)

	Counter – 1		Counter – 2		Counter – 1	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	17	11	18	11	16	10
5:00- 6:00	20	15	22	12	20	15
6:00- 7:00	22	15	20	14	21	16
7:00- 8:00	18	10	19	10	20	18
8:00- 9:00	15	12	16	9	16	12
Total	**92**	**63**	**95**	**56**	**93**	**71**

Table – 1.2 DAY - 2 (FRIDAY)

	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	20	15	22	14	15	8
5:00- 6:00	25	20	27	16	26	15
6:00- 7:00	30	15	31	21	28	17
7:00- 8:00	20	14	23	15	24	12
8:00- 9:00	25	12	24	14	26	13
Total	**120**	**76**	**127**	**80**	**119**	**65**

Table – 1.3 DAY - 3 (SATURDAY)						
	Counter-1		**Counter-2**		**Counter-3**	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	25	13	27	13	28	13
5:00- 6:00	29	19	30	18	29	13
6:00- 7:00	30	17	33	20	31	14
7:00- 8:00	32	15	29	19	28	15
8:00- 9:00	33	15	28	14	29	12
Total	**149**	**79**	**147**	**84**	**145**	**67**

Table – 1.4 DAY - 4 (SUNDAY)						
	Counter-1		**Counter-2**		**Counter-3**	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	30	15	29	12	28	12
5:00- 6:00	31	17	28	14	26	19
6:00- 7:00	32	19	30	13	31	20
7:00- 8:00	29	14	33	17	32	19
8:00- 9:00	32	17	29	16	31	18
Total	**154**	**82**	**149**	**72**	**148**	**88**

Table – 1.5						
	Counter-1		**Counter-2**		**Counter-3**	
	Arrival	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
Day 1	92	63	95	56	93	71
	18.4	12.6	19	11.2	18.6	14.2
Day 2	120	76	127	80	119	65
	24	15.2	25.4	16	23.8	13
Day 3	149	79	147	84	145	67
	29.8	15.8	29.4	16.8	29	13.4
Day 4	154	82	149	72	148	88
	30.8	16.4	29.8	14.4	29.6	17.6

M	1	2	3	4	5	6	7	8	9
P_0	-0.74	0.069519	0.157622	0.172183	0.174899	0.17541	0.175502	0.175518	0.175584

Total Arrival rate for server 1 = 25.75

Total Arrival rate for server 2 = 25.90

Total Arrival rate for server 3 = 25.25

Total service rate for server 1 = 14.95

Total service rate for server 2 = 14.60

Total service rate for server 3 = 14.55

Number of Servers $(M) = 6$

Average Utilization Factor $(\rho) = \frac{\lambda}{\mu} = \frac{25.63}{14.7} = 1.74$

Probability of Zero Units in the System $(P_0) = \sum_{n=0}^{M-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^M}{M!\left(1 - \frac{\lambda}{M\mu}\right)} = 0.1756$

Average Number of Customers in the Queue $(L_q) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)(\mu M - \lambda)^2} = 0.00937$

Average Waiting Time for an Arrival $(w_a) = \frac{1}{M\mu - \lambda} = 0.0159$ hr

Average Waiting Time of Customer Wait in the Queue $(w_q) = \frac{L_q}{\lambda} = 0.003655$

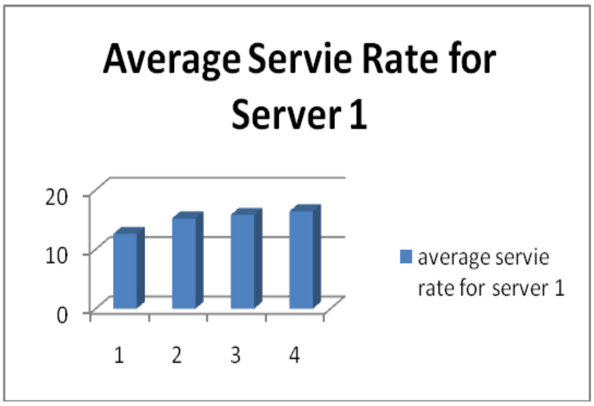
The Probability that an Arrival will have to Wait for Service $(p_w) = \frac{w_q}{w_a} = 0.0435$

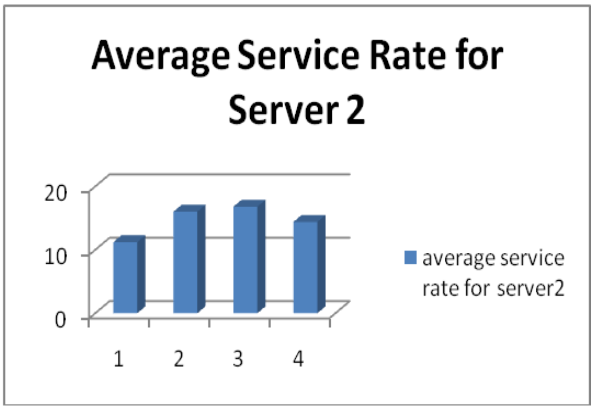
Average Number of Customers in the System (Waiting and being Served) $(L_s) = L_Q + \rho = 1.8337$

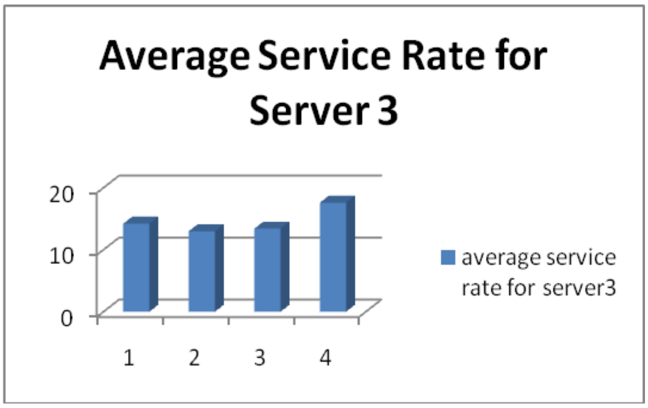
the Average Waiting Time spend in the System $(w_s) = w_q + \frac{1}{\mu} = 0.071655$

System Capacity $= M\mu = 88.2$

System Utilization $(\rho) = \frac{\lambda}{M\mu} = 0.2905$







2. For the cash and digital payments, we have the following observations:

CASH & DIGITAL PAYMENT

Table – 2.1 DAY - 1 (THURSDAY)

	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	17	12	18	12	16	12
5:00- 6:00	20	16	22	15	20	15
6:00- 7:00	22	15	20	16	21	17
7:00- 8:00	18	12	19	12	20	19
8:00- 9:00	15	13	16	10	16	18
Total	**92**	**68**	**95**	**65**	**93**	**81**

Table -2.2 DAY - 2 (FRIDAY)

	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	20	16	22	15	15	10
5:00- 6:00	25	22	27	17	26	18
6:00- 7:00	30	17	31	25	28	21
7:00- 8:00	20	15	23	18	24	17
8:00- 9:00	25	15	24	20	26	19
Total	**120**	**85**	**127**	**95**	**119**	**85**

Table – 2.3 DAY - 3 (SATURDAY)

	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	25	15	27	15	28	15
5:00- 6:00	29	21	30	20	29	14
6:00- 7:00	30	20	33	25	31	17
7:00- 8:00	32	20	29	22	28	16
8:00- 9:00	33	22	28	17	29	20
Total	**149**	**98**	**147**	**99**	**145**	**82**

Table – 2.4 DAY - 4 (SUNDAY)

	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	30	16	29	15	28	14
5:00- 6:00	31	20	28	14	26	19
6:00- 7:00	32	23	30	17	31	21
7:00- 8:00	29	19	33	19	32	23
8:00- 9:00	**32**	**17**	**29**	**22**	**31**	**18**
Total	154	95	149	87	148	95

Table 2.5						
	Counter-1		**Counter-2**		**Counter-3**	
	Arrival	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
Day 1	92	68	95	65	93	81
	18.4	13.6	19	13	18.6	16.2
Day 2	120	85	127	95	119	85
	24	17	25.4	19	23.8	17
Day 3	149	98	147	99	145	82
	29.8	19.6	29.4	19.8	29	16.4
Day 4	154	95	149	87	148	95
	30.8	19	29.8	17.4	29.6	19

M	1	2	3	4	5	6	7	8	9
P_0	-0.48	0.149425	0.21544	0.22559	0.2273	0.227585	0.22763	0.227637	0.227667

Total Arrival rate for server 1 = 25.75

Total Arrival rate for server 2 = 25.90

Total Arrival rate for server 3 = 25.25

Total service rate for server 1 = 17.30

Total service rate for server 2 = 17.30

Total service rate for server 3 = 17.15

Number of Servers = 5

Average Utilization Factor $(\rho) = \frac{\lambda}{\mu} = \frac{25.63}{17.25} = 1.48$

Probability of Zero Units in the System $(P_0) = \sum_{n=0}^{M-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^M}{M!\left(1 - \frac{\lambda}{M\mu}\right)} = 0.227300$

Average Number of Customers in the Queue $(L_q) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)(\mu M - \lambda)^2} = 0.04854$

Average Waiting Time for an Arrival $(w_a) = \frac{1}{M\mu - \lambda} = 0.0164\ hr$

Average Waiting Time of Customer Wait in the Queue $(w_q) = \frac{L_q}{\lambda} = 0.0018938$

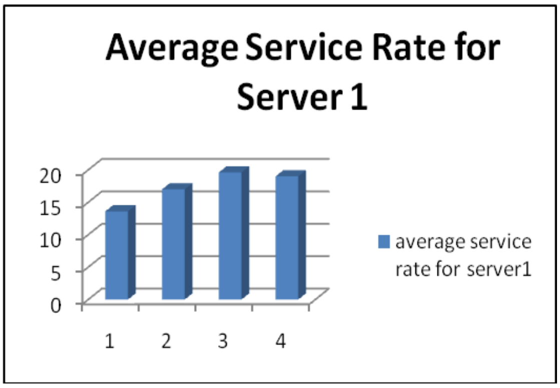
The Probability that an Arrival will have to Wait for Service $(p_w) = \frac{w_q}{w_a} = 0.11548$

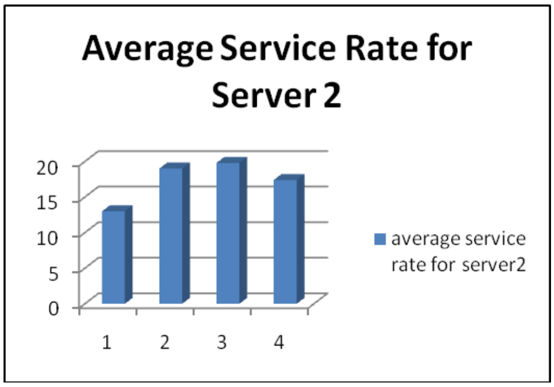
Average Number of Customers in the System (Waiting and being Served) $(L_s) = L_Q + \rho = 1.52854$

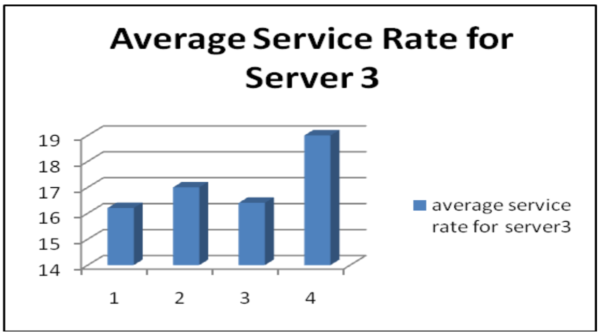
the Average Waiting Time spend in the System $(w_s) = w_q + \frac{1}{\mu} = 0.05986$

System Capacity $= M\mu = 86.25$

System Utilization $(\rho) = \frac{\lambda}{M\mu} = 0.2971$







3. For the digital payments, we have the following observations:

DIGITAL PAYMENT

Table – 3.1 DAY - 1 (THURSDAY)						
	Counter-1		Counter-2		Counter-3	
Time	Arrival	Server	Arrival	Server	Arrival	Server
4:00- 5:00	17	15	18	14	16	15
5:00- 6:00	20	18	22	18	20	18
6:00- 7:00	22	20	20	16	21	18
7:00- 8:00	18	15	19	15	20	19
8:00- 9:00	15	14	16	13	16	18
Total	92	82	95	76	93	88

Table – 3.2 DAY - 2 (FRIDAY)						
	Counter-1		Conter-2		Counter-3	
Time	Arrival	Server	Arrival	Server	Arrival	Server
4:00- 5:00	20	18	22	17	15	13
5:00- 6:00	25	23	27	20	26	20
6:00- 7:00	30	19	31	28	28	22
7:00- 8:00	20	16	23	21	24	18
8:00- 9:00	25	20	24	20	26	19
Total	120	96	127	106	119	92

Table – 3.3 DAY - 3 (SATURDAY)						
	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	25	18	27	17	28	19
5:00- 6:00	29	19	30	23	29	25
6:00- 7:00	30	24	33	28	31	24
7:00- 8:00	32	26	29	25	28	20
8:00- 9:00	33	25	28	20	29	23
Total	**149**	**112**	**147**	**113**	**145**	**111**

Table – 3.4 DAY - 4 (SUNDAY)						
	Counter-1		Counter-2		Counter-3	
Time	**Arrival**	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
4:00- 5:00	30	20	29	17	28	17
5:00- 6:00	31	20	28	24	26	23
6:00- 7:00	32	25	30	23	31	24
7:00- 8:00	29	26	33	29	32	26
8:00- 9:00	32	20	29	25	31	21
Total	**154**	**111**	**149**	**118**	**148**	**111**

Table 3.5						
	Counter-1		Counter-2		Counter-3	
	Arrival	**Server**	**Arrival**	**Server**	**Arrival**	**Server**
Day 1	92	82	95	76	93	88
	18.4	16.4	19	15.2	18.6	17.6
Day 2	120	96	127	106	119	92
	24	19.2	25.4	21.2	23.8	18.4
Day 3	149	112	147	113	145	111
	29.8	22.4	29.4	22.6	29	22.2
Day 4	154	111	149	118	148	111
	30.8	22.2	29.8	23.6	29.6	22.2

M	1	2	3	4	5	6	7	8	9
P_0	-0.26	0.226994	0.275587	0.282445	0.283479	0.28363	0.283651	0.227637	0.283667

Total Arrival rate for server 1 = 25.75

Total Arrival rate for server 2 = 25.90

Total Arrival rate for server 3 = 25.25

Total service rate for server 1 = 20.05

Total service rate for server 2 = 20.65

Total service rate for server 3 = 20.10

Number of Servers = 4

Average Utilization Factor $(\rho) = \frac{\lambda}{\mu} = \frac{25.63}{14.7} = 1.26$

Probability of Zero Units in the System $(P_0) = \sum_{n=0}^{M-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{!n} + \frac{\left(\frac{\lambda}{\mu}\right)^M}{!M\left(1 - \frac{\lambda}{M\mu}\right)} = 0.2828$

Average Number of Customers in the Queue $(L_q) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)(\mu M - \lambda)^2} = 0.04017$

Average Waiting Time for an Arrival $(w_a) = \frac{1}{M\mu - \lambda} = 0.0180\ hr$

Average Waiting Time of Customer Wait in the Queue $(w_q) = \frac{L_q}{\lambda} = 0.00156$

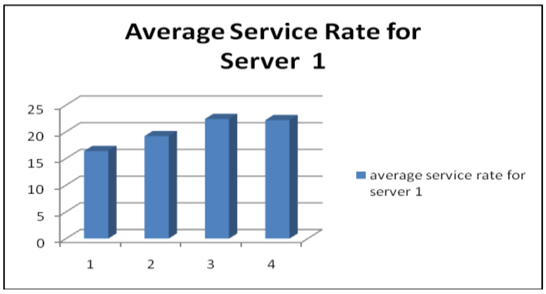
The Probability that an Arrival will have to Wait for Service $(p_w) = \frac{w_q}{w_a} = 0.0866$

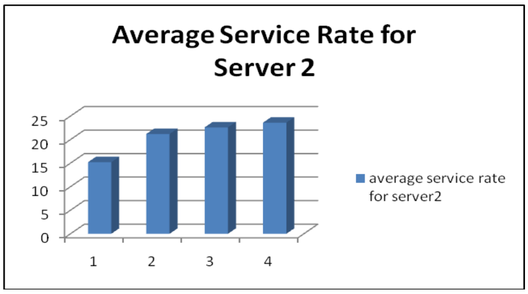
Average Number of Customers in the System (Waiting and being Served) $(L_s) = L_Q + \rho = 1.30017$

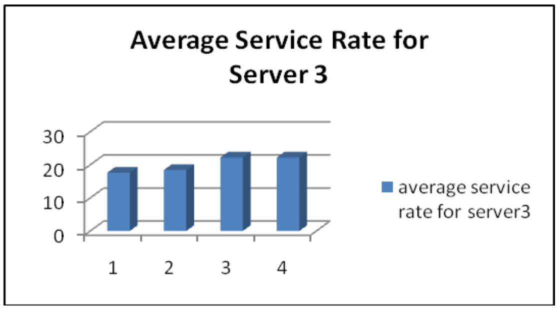
Average Waiting Time spend in the System $(w_s) = w_q + \frac{1}{\mu} = 0.5091$

System Capacity $= M\mu = 81.04$

System Utilization $\rho = \frac{\lambda}{M\mu} = 0.31059$







IV. CONCLUSION

The evaluation of queueing system in an establishment is necessary for the betterment of the establishment. analysis of this queueing system shows that the supermarket needs to increase the number of their channels or servers up to six (6) if all payment mode will be only cash, it has to increase the number of servers up to five (5), if the payment mode will be both digital and cash and it has to increase the service channels up to four (4) if the all payment mode will be digital. It will reduce the waiting time of customers. This will also increase the efficiency of the establishment due to the appreciation in their serve to the customers as and at when due.

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