Multiplicative (*a*, *b*)-*KA* Temperature Indices of Certain Nanostructure

V.R.Kulli

Department of Mathematics. Gulbarga University, Gulbarga 585106, India

Abstract: *In Chemical Graph Theory, graph indices are applied to measure the chemical characteristics of chemical compounds. In this study, we introduce the first and second multiplicative (a, b)-KA temperature indices of a chemical graph. Furthermore, we compute these indices for tetrmeric 1,3-adamantane. Also we establish some other multiplicative temperature indices directly as a special case of multiplicative (a, b)-KA temperature indices for some special values of a and b.*

Keywords: *Chemical graph, temperature of a vertex, first and second multiplicative (a, b)-KA indices, tetrameric 1,3-adamantane.*

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*12.

I. Introduction

In this paper, *G* denotes a finite, simple, connected graph, *V*(*G*) and *E*(*G*) denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex *u* is the number of vertices adjacent to *u*. We refer [1] for undefined term and notation.

A chemical graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Graph indices have their applications in various disciplines of Science and Technology, see [2, 3]. For more information about graph indices, see [4]. Recently, some new graph indices were studied in [5, 6, 7, 8, 9, 10, 11, 12, 13].

The temperature of a vertex *u* of a graph *G* is defined by Faitlowicz in [14] as

$$T(u)=\frac{d_G(u)}{1-d_G(u)}, \qquad \text{where } |V(G)| = n.$$

The multiplicative first and second temperature indices and multiplicative first and second hyper temperature indices were introduced by Kulli in [15] and they are defined as

$$TII_1(G)=\prod_{uv\in E(G)}[T(u)+T(v)], \qquad TII_2(G)=\prod_{uv\in E(G)}T(u)T(v).$$

$$HTII_1(G)=\prod_{uv\in E(G)}[T(u)+T(v)]^2, \qquad HTII_2(G)=\prod_{uv\in E(G)}[T(u)T(v)]^2.$$

Recently, some new variants of temperature indices were studied such as first temperature index [16], first and second hyper temperature indices [17], (*a*, *b*)- temperature index [18], harmonic temperature index [19], atom bond connectivity temperature index [20].

In [15], Kulli introduced the general multiplicative first and second temperature indices of a graph *G* and they are defined as

$$TII_1^a(G)=\prod_{uv\in E(G)}[T(u)+T(v)]^a, \qquad TII_2^a(G)=\prod_{uv\in E(G)}[T(u)T(v)]^a.$$

where *a* is a real number.

Also in the same paper [15], the multiplicative sum and product connectivity temperature indices and multiplicative reciprocal product connectivity temperature index were defined as

$$STII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{T(u)+T(v)}}.$$

$$PTII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{T(u)T(v)}}.$$

$$RPTII(G)=\prod_{uv\in E(G)}\sqrt{T(u)T(v)}.$$

Recently some new connectivity indices were studied in [21, 22, 23, 24, 25].

The multiplicative *F*-temperature index and general multiplicative temperature index were introduced in [15] and they are defined as

$$FTII(G)=\prod_{uv\in E(G)}\left[T(u)^2+T(v)^2\right]. \qquad TII_a(G)=\prod_{uv\in E(G)}\left[T(u)^a+T(v)^a\right].$$

We now propose the modified multiplicative first and second temperature indices of a graph *G*, defined as

$$^{m}T_1II(G)=\prod_{uv\in E(G)}\frac{1}{T(u)+T(v)}. \qquad ^{m}T_2II(G)=\prod_{uv\in E(G)}\frac{1}{T(u)T(v)}.$$

We introduce the multiplicative first and second (*a*, *b*)-*KA* temperature indices of a graph *G*, defined as

$$KAT^1_{a,b}II(G)=\prod_{uv\in E(G)}\left[T(u)^a+T(v)^a\right]^b, \qquad KAT^2_{a,b}II(G)=\prod_{uv\in E(G)}\left[T(u)^aT(v)^a\right]^b,$$

where *a*, *b* are real numbers.

Recently, some (*a*, *b*)-*KA* indices were studied in [26, 27, 28].

In this paper, the first and second multiplicative (*a*, *b*)-*KA* temperature indices for tetrameric 1,3-adamantane are computed.

II. Observations

We observe the following

1. $TII_1(G)=KAT^1_{1,1}II(G)$.
2. $HTII_1(G)=KAT^1_{1,2}II(G)$.
3. $^{m}T_1II(G)=KAT^1_{1,-1}II(G)$.
4. $TII_1^a(G)=KAT^1_{1,a}II(G)$.
5. $STII(G)=KAT^1_{1,-\frac{1}{2}}II(G)$.
6. $FTII(G)=KAT^1_{2,1}II(G)$.
7. $TII_a(G)=KAT^1_{a,1}II(G)$.

We also observe that

1. $TII_2(G)=KAT^2_{1,1}II(G)$.
2. $HTII_2(G)=KAT^2_{1,2}II(G)$.
3. $^{m}T_2II(G)=KAT^2_{1,-1}II(G)$.
4. $TII_2^a(G)=KAT^2_{1,a}II(G)$.
5. $PTII(G)=KAT^2_{1,-\frac{1}{2}}II(G)$.
6. $RPTII(G)=KAT^2_{1,\frac{1}{2}}II(G)$.

Clearly, we obtain some other multiplicative temperature indices directly as a special case of multiplicative (*a*, *b*)-*KA* temperature indices for some special values of *a* and *b*.

III. Results for Tetrameric 1,3-Adamantane

In Chemistry, diamondoids are variants of the carbon cage known as adamantine $C_{10}H_{16}$, the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular graph structure of the family of the tetrameric 1,3-adamantane, symbolized by *TA*[*n*]. The graph of tetrameric 1,3-adamantane *TA*[4] is shown in Figure 1.

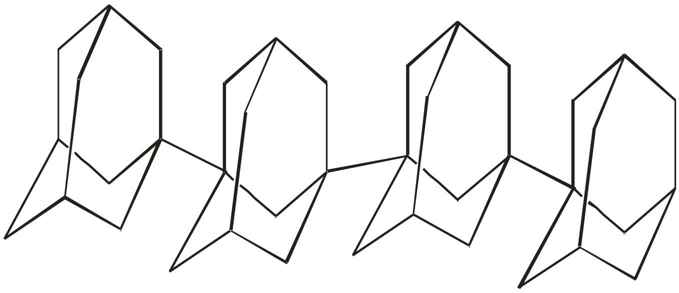

Figure 1.

Let G be the graph of tetrameric 1,3-adamantane $TA[n]$. By calculation , G has $10n$ vertices and $13n - 1$ edges.

By calculation, we obtain three types of edges based on the degrees of the end vertices of each edge as follows:

$E_1=\{uv\in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_1| = 6n + 6$.

$E_2=\{uv\in E(G) \mid d_G(u)=2, d_G(v)=4\}$, $|E_2| = 6n - 6$.

$E_3=\{uv\in E(G) \mid d_G(u)=d_G(v) = 4\}$, $|E_3| = n - 1$.

Thus in $TA[n]$, there are three types of edges based on the temperature of end vertices of each edge as given in Table 1.

$T(u), T(v)\backslash uv \in E(G)$	$\left(\frac{2}{10n-2},\frac{3}{10n-3}\right)$	$\left(\frac{2}{10n-2},\frac{4}{10n-4}\right)$	$\left(\frac{4}{10n-4},\frac{4}{10n-4}\right)$
Number of edges	$6n + 6$	$6n - 6$	$n - 1$

Table 1. Temperature edge partition of G

Theorem 1. The multiplicative first (a, b)-KA temperature index of a tetrameric 1,3-adamantane $TA[n]$ is

$$KAT_{a,b}^{1}(TA[n])=\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{2}{5n-2}\right)^a\right]^{b(6n-6)}\times\left[2\left(\frac{2}{5n-2}\right)^a\right]^{b(n-1)}.$$

Proof: By definition and by using Table 1, we deduce

$$KAT_{a,b}^{1}II(G)=\prod_{uv\in E(G)}\left[T(u)^a+T(v)^a\right]^b$$

$$=\left[\left(\frac{2}{10n-2}\right)^a+\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{2}{10n-2}\right)^a+\left(\frac{4}{10n-4}\right)^a\right]^{b(6n-6)}$$

$$\times\left[\left(\frac{4}{10n-4}\right)^a+\left(\frac{4}{10n-4}\right)^a\right]^{b(n-1)}$$

$$=\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{2}{5n-2}\right)^a\right]^{b(6n-6)}\times\left[2\left(\frac{2}{5n-2}\right)^a\right]^{b(n-1)}.$$

By using observations and from Theorem 1, we obtain the following results.

Corollary 1.1. The multiplicative first temperature index of $TA[n]$ is

$$TII_1(TA[n])=KAT_{1,1}^{1}II(TA[n])$$

$$=\left[\frac{25n-6}{(5n-1)(10n-3)}\right]^{(6n+6)}\times\left[\frac{15n-4}{(5n-1)(5n-2)}\right]^{(6n-6)}\times\left(\frac{4}{5n-2}\right)^{(n-1)}$$

Corollary 1.2. The multiplicative first hyper temperature index of $TA[n]$ is

$$HTII_1(TA[n])=KAT_{1,2}^{1}II(TA[n])$$

$$=\left[\frac{25n-6}{(5n-1)(10n-3)}\right]^{(12n+12)}\times\left[\frac{15n-4}{(5n-1)(5n-2)}\right]^{(12n-12)}\times\left(\frac{4}{5n-2}\right)^{(2n-2)}$$

Corollary 1.3. The modified multiplicative first temperature index of $TA[n]$ is

$${}^{m}T_1II(TA[n])=KAT^1_{1,-1}II(TA[n])$$

$$=\left[\frac{25n-6}{(5n-1)(10n-3)}\right]^{-(6n+6)}\times\left[\frac{15n-4}{(5n-1)(5n-2)}\right]^{-(6n-6)}\times\left(\frac{4}{5n-2}\right)^{-(n-1)}$$

Corollary 1.4. The general multiplicative first temperature index of $TA[n]$ is

$$TII_1^a(TA[n])=KAT^1_{1,a}II(TA[n])$$

$$=\left[\frac{25n-6}{(5n-1)(10n-3)}\right]^{a(6n+6)}\times\left[\frac{15n-4}{(5n-1)(5n-2)}\right]^{a(6n-6)}\times\left(\frac{4}{5n-2}\right)^{a(n-1)}$$

Corollary 1.5. The multiplicative sum connectivity temperature index of $TA[n]$ is

$$STII(TA[n])=KAT^1_{1,-\frac{1}{2}}II(TA[n])$$

$$=\left[\frac{25n-6}{(5n-1)(10n-3)}\right]^{-(3n+3)}\times\left[\frac{15n-4}{(5n-1)(5n-2)}\right]^{-(3n-3)}\times\left(\frac{4}{5n-2}\right)^{-\frac{1}{2}(n-1)}$$

Corollary 1.6. The multiplicative *F*-temperature index of $TA[n]$ is

$$FTII(TA[n])=KAT^1_{2,1}II(TA[n])$$

$$=\left[\frac{1}{(5n-1)^2}+\frac{9}{(10n-3)^2}\right]^{6n+6}\times\left[\frac{1}{(5n-1)^2}+\frac{4}{(5n-1)^2}\right]^{6n-6}\times\left(\frac{4}{(5n-2)^2}\right)^{n-1}$$

Corollary 1.7. The general multiplicative temperature index of $TA[n]$ is

$$TII_a(TA[n])=KAT^1_{a,1}II(TA[n])$$

$$=\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{3}{10n-3}\right)^a\right]^{6n+6}\times\left[\left(\frac{1}{5n-1}\right)^a+\left(\frac{2}{5n-2}\right)^a\right]^{6n-6}\times\left[2\left(\frac{2}{5n-2}\right)^a\right]^{n-1}$$

Theorem 2. The multiplicative second (a, b)-KA temperature index a tetrameric 1,3-adamantane $TA[n]$ is

$$KAT^2_{a,b}(TA[n])=\left[\left(\frac{1}{5n-1}\right)^a\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{1}{5n-1}\right)^a\left(\frac{2}{5n-2}\right)^a\right]^{b(6n-6)}\times\left(\frac{2}{5n-2}\right)^{2ab(n-1)}.$$

Proof: To compute $KAT^2_{a,b}II(TA[n])$, we see that

$$KAT^2_{a,b}II(G)=\prod_{uv\in E(G)}\left[T(u)^aT(v)^a\right]^b$$

$$=\left[\left(\frac{2}{10n-2}\right)^a\times\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{2}{10n-2}\right)^a\times\left(\frac{4}{10n-4}\right)^a\right]^{b(6n-6)}$$

$$\times\left[\left(\frac{4}{10n-4}\right)^a\times\left(\frac{4}{10n-4}\right)^a\right]^{b(n-1)}$$

$$=\left[\left(\frac{1}{5n-1}\right)^a\times\left(\frac{3}{10n-3}\right)^a\right]^{b(6n+6)}\times\left[\left(\frac{1}{5n-1}\right)^a\times\left(\frac{2}{5n-2}\right)^a\right]^{b(6n-6)}\times\left(\frac{2}{5n-2}\right)^{2ab(n-1)}.$$

From Theorem 2 and observations, we establish the following results.

Corollary 2.1. The multiplicative second temperature index of $TA\,[n]$ is

$$T_2II(TA[n])=KAT^2_{1,1}II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{(6n+6)}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{(6n-6)}\times\left(\frac{2}{5n-2}\right)^{2(n-1)}.$$

Corollary 2.2. The multiplicative second hyper temperature index of $TA[n]$ is

$$HT_2II(TA[n])=KAT_{1,2}^2II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{12n+12}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{12n-12}\times\left(\frac{2}{5n-2}\right)^{4(n-1)}$$

Corollary 2.3. The modified multiplicative second temperature index of $TA[n]$ is

$${}^mT_2II(TA[n])=KAT_{1,-1}^2II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{-(6n+6)}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{-(6n-6)}\times\left(\frac{2}{5n-2}\right)^{-2(n-1)}.$$

Corollary 2.4. The general multiplicative second temperature index of TA $[n]$ is

$$T_2^aII(TA[n])=KAT_{1,a}^2II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{a(6n+6)}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{a(6n-6)}\times\left(\frac{2}{5n-2}\right)^{2a(n-1)}.$$

Corollary 2.5. The multiplicative product connectivity temperature index of TA $[n]$ is

$$PTII(TA[n])=KAT^2_{1,-\frac{1}{2}}II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{-(3n+3)}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{-(3n-3)}\times\left(\frac{2}{5n-2}\right)^{-(n-1)}.$$

Corollary 2.6. The multiplicative reciprocal product connectivity temperature index $TA[n]$ is

$$RPTII(TA[n])=KAT^2_{1,\frac{1}{2}}II(TA[n])$$

$$=\left[\frac{3}{(5n-1)(10n-3)}\right]^{3n+3}\times\left[\frac{2}{(5n-1)(5n-2)}\right]^{3n-3}\times\left(\frac{2}{5n-2}\right)^{n-1}.$$

References

[1] V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).

[2] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).

[3] V.R.Kulli, *Multiplicative Connectivity Indices of Nanostructures*, LAP LEMBERT Academic Publishing, (2018).

[4] V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal, S. Samanta and A. Pal, (eds.) IGI Global, USA (2019) 66-91.

[5] V.R.Kulli, Computing Banhatti indices of networks, *International Journal of Advances in Mathematics*, 2018(1) (2018) 31-40.

[6] V.R.Kulli, Two new arithmetic-geometric ve-degree indices, *Annals of Pure and Applied Mathematics*, 17(1) (2018) 107-112.

[7] V.R.Kulli, On reduced second Zagreb indices of polycyclic aromatic hydrocarbons and benzenoid systems, *Annals of Pure and Applied Mathematics*, 18(1) (2018) 73-78.

[8] V.R.Kulli, ABC Banhatti and augmented Banhatti indices of chemical networks, *Journal of Chemistry and Chemical Sciences*, 8(8) (2018) 1018-1025.

[9] V.R.Kulli, Dakshayani indices, *Annals of Pure and Applied Mathematics*, 18(2) (2018) 139-146.

[10] V.R.Kulli, Neighborhood indices of nanostructures, *International Journal of Current Research in Life Sciences*, 8(1) (2019) 2998-3006.

[11] V.R.Kulli, Neighborhood indices of nanostructures, *International Journal of Current Research in Science and Technology*, 5(3) (2019) 1-14.

[12] V.R.Kulli, F-indices of chemical networks, *International Journal of Mathematics Archive*, 10(3) (2019) 21-30.

[13] V.R.Kulli, Some new topological indices of graphs, *International Journal of Mathematical Archive,* 10(5) (2019) 62-70.

[14] S. Fajtolowicz, On conjectures of Grafitti, Discrete Math. 72(1988) 113-118.

[15] V.R.Kulli, Some multiplicative temperature indices of $HC_5C_7[p,q]$ nanotubes, *International Journal of Fuzzy Mathematical Archive*, 17(2) (2019) 91-98.

[16] P.N.Kisori and S. Dickson, On temperature index of certain nanostructures, (preprint).

[17] V.R.Kulli, Computation of some temperature indices of $HC_5C_5[p,q]$ nanotubes, *Annals of Pure and Applied Mathematics*, 20(2) (2019) 69-74.

[18] V.R. Kulli, The (a, b)-temperature index of H-Naphtalenic nanotubes, *Annals of Pure and Applied Mathematics*, 20(2) (2019) 85-90.

[19] T.K.Atework, P. N. Kisori, T.K. Afework and S. Dickson, Harmonic temperature index of certain nanostructures, *International Journal of Mathematics Trends and Technology*, 56(3) (2018) 159-164.

[20] T.K.Atework, P. N. Kisori, and S. Dickson, Atom bond connectivity temperature index, *Journal of Mathematical Nanoscience*, 8(2) (2018) 67-75.

[21] V.R.Kulli, Multiplicative atom bond connectivity and multiplicative geometric-arithmetic indices of chemical structures in drugs, *International Journal of Mathematical Archive*, 9(6) (2018) 155-163.

[22] V.R.Kulli, Degree based connectivity F-indices of nanotubes, *Annals of Pure and Applied Mathematics*, 18(2) (2018) 201-206.

[23] V.R.Kulli, Some new Multiplicative connectivity Kulli-Basava indices, *International Journal of Mathematics Trends and Technology*, 65(9) (2019) 18-23.

[24] V.R.Kulli, B. Chaluvaraju and T.V. Asha, Multiplicative product connectivity and sum connectivity indices of chemical structures in drugs, *Research Review International Journal of Multidisciplinary*, 4(2) (2019) 949-953.

[25] V.R.Kulli, B.Chaluvaraju and H.S.Baregowda, Some bounds of sum connectivity Banhatti index of graphs, *Palestine Journal of Mathematics*, 8(2) (2019) 355-364.

[26] V.R.Kulli, The (*a, b*)-Kulli-Basava index of graphs, *International Journal of Engineering Sciences and Research Technology*, 8(12) (2019) 103-112.

[27] V.R.Kulli, The (a, b)-KA indices of polycyclic aromatic hydrocarbons, and benzenoid systems, *International Journal of Mathematics Trends and Technology*, 65(11) (2019) 115-120.

[28] V.R.Kulli, Multiplicative (a, b)-KA indices of certain dendrimer nanostars, *International Journal of Recent Scientific Research*, 10, 11(E) (2019) 36010-36014.