On πg^*b – Continuous Functions

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Abstract : *The aim of this paper is to characterize πg*b-closure and πg*b-interior, πg*b-continuous functions. Further the concept of almost πg*b-continuous and their properties are discussed.*

Key words: *πg*b-cl(A), πg*b-int(A), πg*b-continuous and almost πg*b-continuous.*

I. Introduction

Levine [8] introduced the concept of generalized closed sets in topological spaces. Andrijevic[1] introduced the concept of generalized open sets called b-open sets. Since then many authors have contributed to the study of generalized b-closed sets. In 1968 Zaitsev [19] defined π-closed sets. Dontchev and Noiri [4] introduced the notion of πg-closed sets. Veerakumar[17] introduced the notion of g*-closed sets. Sreeja and C.Janaki[13] introduced the concept of πgb-closed sets and πgb-continuity in topological spaces.

Hussain(1966) [6], M.K.Singal and A.R. Singal(1968) introduced the concept of almost continuity in topological spaces. Recently K.Geethapadmini and C.Janaki [5] introduced and studied the properties of πg*b-closed sets in topological spaces. The purpose of this paper is to study πg*b-closure, πg*b-interior, πg*b-continuous functions and almost πg*b-continuous functions and some of its basic properties.

II. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represents topological spaces on which no separation axioms are discussed. (X,τ) will be replaced by X if there is no chance of confusion.

Definition 2.1 : A subset A of a topological space X is said to be

1) a α- closed set [10]if $cl(int(cl(A))) \subset A$
2) a pre-closed set [9] if $cl(int(A)) \subset A$
3) a regular closed set[11] if $A = cl(int(A))$
4) b-closed set[1] if $int(cl(A)) \cap cl(int(A)) \subset A$
5) π-open [19] set if A is a finite union of regular open sets.

Definition 2.2 : A subset A of a space (X, τ) is called

1) a generalized closed (briefly g-closed) [8] if $cl(A) \subset U$ whenever $A \subset U$ and U is open.
2) a generalized * closed (briefly g*-closed)[17] if $cl(A) \subset U$ whenever $A \subset U$ and U is g- open.
3) a generalized *b-closed (briefly g*b-closed)[18] if $bcl(A) \subset U$ whenever $A \subset U$ and U is g-open.
4) πg- closed[4] if $cl(A) \subset U$ whenever $A \subset U$ and U π- open.
5) πgp- closed[12] if $pcl(A) \subset U$ whenever $A \subset U$ and U π- open.
6) πgα- closed[14] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U π- open.
7) πgs -closed[2] if $scl(A) \subset U$ whenever $A \subset U$ and U π- open.
8) πgb- closed[13] if $bcl(A) \subset U$ whenever $A \subset U$ and U π- open.
9) πg*p- closed[15] if $pcl(A) \subset U$ whenever $A \subset U$ and U πg- open.
10) πg*s- closed[16] if $scl(A) \subset U$ whenever $A \subset U$ and U πg- open.
11) πg*b- closed[5] if $bcl(A) \subset U$ whenever $A \subset U$ and U πg- open.

Definition 2.3 : A function $f : (X, \tau) \to (Y, \sigma)$ is called continuous (resp. α- continuous, pre- continuous, b-continuous, g*b- continuous, πgb- continuous, πg*p- continuous, πg*s- continuous, πg*b- continuous) if $f^{-1}(V)$ is closed (resp. α-closed, pre-closed, b-closed, g*b-closed, πgb-closed, πg*p-closed, πg*s- closed, πg*b-closed) in (X, τ) for every closed set V in (Y, σ).

Theorem 2.4:[5] Every closed, α-closed, pre-closed, b-closed, πg^*p- closed, πg^*s- closed sets are πg^*b- closed and the converse need not be true.

Theorem 2.5 :[5] Every πg^*b- closed set is πgb- closed and g^*b-closed and the converse need not be true.

Definition 2.6 : [5] A function $f : (X, \tau) \to (Y, \sigma)$ is called πg^*b-irresolute if $f^{-1}(V)$ is πg^*b-closed in X for every πg^*b-closed set V of Y.

III. πg^*b-Closure and Interior

Definition 3.1: For any set $A \subseteq X$, the πg^*b-closure of A is defined as the intersection of all πg^*b-closed sets containing A and is denoted by $\pi g^*b\text{-cl}(A)$.

We write $\pi g^*b\text{-cl}(A) = \cap \{ F : A \subseteq F \text{ is } \pi g^*b\text{-closed in } X \}$

Theorem 3.2: For any $x \in X$, $x \in \pi g^*b\text{-cl}(A)$ iff $V \cap A \neq \phi$ for every πg^*b-open set V containing x.

Proof : Let us assume that there exists a πg^*b-open set V containing x such that $V \cap A = \phi$. Since $A \subseteq X - V$, $\pi g^*b\text{-cl}(A) \subseteq X - V$. This implies $x \notin \pi g^*b\text{-cl}(A)$, which is a contradiction to the fact that $x \in \pi g^*b\text{-cl}(A)$. Hence, $V \cap A \neq \phi$ for every πg^*b-open set V containing x.

On the other hand, let $x \notin \pi g^*b\text{-cl}(A)$. Then there exists a $x \notin \pi g^*b$-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and $x - F$ is πg^*b-open. Also $(X-F) \cap A \neq \phi$ which is a contradiction. Hence the lemma.

Lemma 3.3: Let A and B be subsets of (X, τ) Then

(i) $\pi g^*b\text{-cl}(\phi) = \phi$, $\pi g^*b\text{-cl}(X) = X$

(ii) if $A \subseteq B$, $\pi g^*b\text{-cl}(A) \subseteq \pi g^*b\text{-cl}(B)$

(iii) $A \subseteq \pi g^*b\text{-cl}(A)$

(iv) $\pi g^*b\text{-cl}(A) \subseteq \pi g^*b\text{-cl}(\pi g^*b\text{-cl}(A))$

Proof : Straight forward

Theorem 3.4 : If $A \subseteq X$, is πg^*b-closed, then $\pi g^*b\text{-cl}(A) = A$.

Proof : Follows from the definition.

Remark 3.5 : The converse of the above theorem need not be true as seen by the following example.

Example 3.6 : Let $X = \{ a,b,c,d \}$. $\tau = \{ \phi, \{a\}, \{c\}, \{a,c\}, \{c,d\}, \{a,c,d\}, X \}$. Here $A = \{c\}$. $\pi g^*b\text{-cl}(A) = A$ but A is not πg^*b-closed.

Definition 3.7: For any set $A \subseteq X$, the πg^*b-interior of A is defined as the union of all πg^*b-open sets contained in A and is denoted by $\pi g^*b\text{-int}(A)$.

We write $\pi g^*b\text{-int}(A) = \cup \{ G : G \text{ is } \pi g^*b\text{-open and } G \subseteq A \}$.

Theorem 3.8 : Let A and B be subsets of X. Then

(i) $\pi g^*b\text{-int}(\phi) = \phi$, $\pi g^*b\text{-int}(X) = X$

(ii) $\pi g^*b\text{-int}(A) \subseteq A$

(iii) If B is any πg^*b-open set contained in A, then $B \subseteq \pi g^*b\text{-int}(A)$

(iv) If A □ B, πg*b-int(A) □ πg*b-int(B)

(v) πg*b-int (πg*b-int(A)) = πg*b-int(A).

Proof : Straight forward

Theorem 3.9 : If A□ X is πg*b-open, then πg*b-int(A) = A.

Proof : Straight forward

Remark 3.10 : The converse of the above theorem need not be true as seen by the following example.

Example 3.11: Let X = { a, b, c, d }. τ = { ϕ, {a}, {c}, {a ,c}, {c, d}, {a, c, d}, X }. Here A = {a, b, d}. πg*b-int (A) = A but A is not πg*b-open.

IV. πg*b- Continuous Functions

Theorem 4.1 : Every continuous function is πg*b-continuous.

Proof : Let f : (X, τ) → (Y, σ) be a continuous function. Let V be a closed set in Y. Since f is a continuous function, $f^{-1}(V)$ is closed in X. As every closed set is πg*b-closed, $f^{-1}(V)$ is πg*b-closed. Hence, f is πg*b-continuous.

Theorem 4.2 :

(i) Every α-continuous function is πg*b-continuous.

(ii) Every b-continuous function is πg*b-continuous.

(iii) Every pre-continuous function is πg*b-continuous.

(iv) Every is πg*p-continuous function is πg*b-continuous.

(iii) Every is πg*s-continuous function is πg*b-continuous.

(iv) Every is πg*b-continuous function is πgb-continuous.

(vii) Every is πg*b-continuous function is g*b-continuous.

Proof : Straight forward

Remark 4.3 : The converse of the above theorem need not be true as seen by the following examples.

Example 4.4 : Let X = Y = { a,b,c,d }. τ = { ϕ, {a}, {c}, {a,c}, {c,d}, {a,c,d}, X } and σ = { ϕ, {c}, {a,b}, {c,d}, {a,b,c}, X }. Define f : (X, τ) → (Y, σ) by f(a) = c, f(b) = d, f(c) = a, f(d) = b. Here f is πg*b –continuous but not continuous, α-continuous, pre-continuous and πg*p-continuous.

Example 4.5 : Let X = Y = { a, b, c }. τ = { ϕ, {a}, {b, c}, X }and σ = { ϕ, {a}, {b}, {a, b}, {b, c}, X }. Define f : (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c. Here f is πg*b –continuous but not πg*s-continuous.

Example 4.6 : Let X = Y = { a, b ,c }, τ = { ϕ, {a}, {b}, {a, b}, {b, c}, X } and σ = { ϕ, {a}, {b}, {a, b}, X }. Define f : (X, τ) → (Y, σ) by f(a) = a, f(b) = c, f(c) = b. Here f is πgb -continuous but not πg*b-continuous.

Example 4.7 : Let X = Y = { a, b ,c ,d }. τ = { ϕ, {a}, {c}, {a, c}, {c, d}, {a c, d}, X } and σ = { ϕ, {a}, {b}, {a, b}, X }. Define f : (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = d, f(d) = a . f is πg*b -continuous but not b-continuous.

Remark 4.8 : Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then

(i) πg^*b-continuous and $\pi g\alpha$-continuous

(ii) πg^*b-continuous and πgp-continuous

(iii) πg^*b-continuous and πgs-continuous

(iv) πg^*b-continuous and g-continuous

(v) πg^*b-continuous and g^*-continuous

are independent concepts.

Example 4.9 : Let $X = Y = \{ a, b, c \}$, $\tau = \{ \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X \}$, $\sigma = \{ \phi, \{a\}, \{b, c\}, X \}$. The function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = b$, $f(b) = c$, $f(c) = a$ is $\pi g\alpha$-continuous , πgp-continuous, πgs-continuous but not πg^*b-continuous.

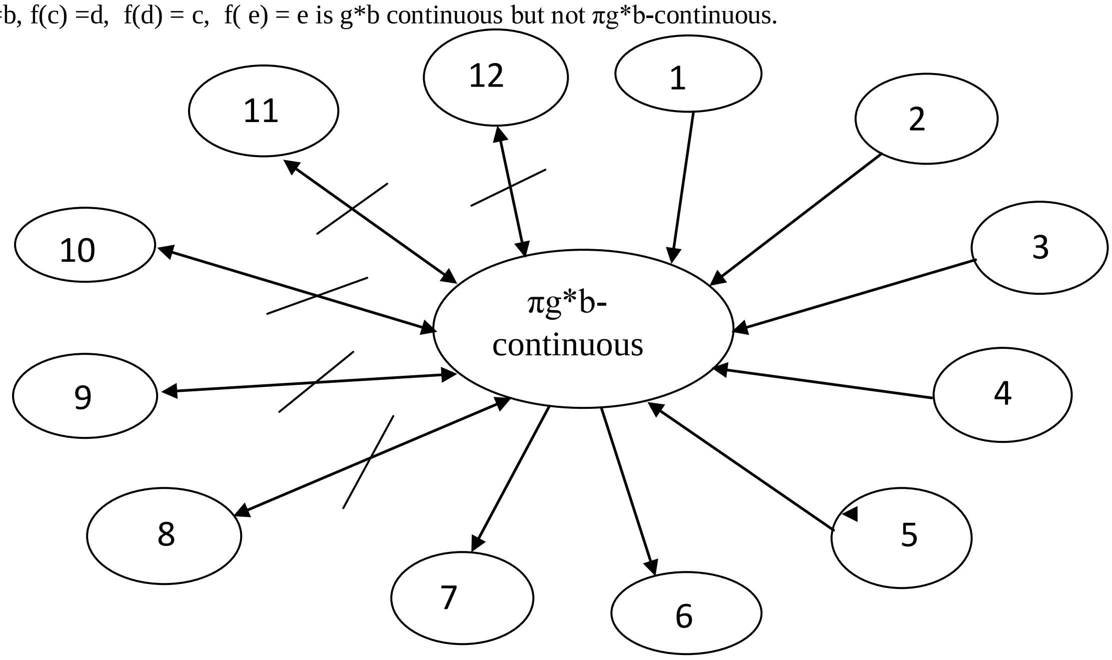
Example 4.10 : Let $X = Y = \{ a, b, c \}$, $\tau = \{ \phi, \{a\}, \{b\}, \{a, b\}, X \}$, $\sigma = \{ \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X \}$. The function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = b$, $f(b) = a$, $f(c) = c$ is πg^*b-continuous but not $\pi g\alpha$-continuous , πgp-continuous, πgs-continuous, g-continuous , and g^*-continuous.

Example 4.11 : Let $X = Y = \{ a, b, c, d \}$, $\tau = \{ \phi, \{a\}, \{b\}, \{a, b\}, X \}$, $\sigma = \{ \phi, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X \}$. The function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = c$, $f(b) = d$, $f(c) = a$, $f(d) = b$ is g-continuous but not πg^*b-continuous.

Example 4.12 : Let $X = Y = \{ a, b, c, d \}$, $\tau = \{ \phi, \{c\}, \{a, b\}, \{c, d\}, \{a, b, c\}, X \}$,

$\sigma = \{ \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. The identity function $f : (X, \tau) \to (Y, \sigma)$ is g^*-continuous but not πg^*b-continuous.

Example 4.13 : Let $X = Y = \{ a, b, c, d, e \}$, $\tau = \{ \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, c, d\}, X \}$, $\sigma = \{ \phi, \{b\}, \{c\}, \{b, c\}, \{a, b, c\}, \{a, b, c, d\}, \{a, b, c, e\}, X \}$. The function defined by $f(a) = a$, $f(b) = b$, $f(c) = d$, $f(d) = c$, $f(e) = e$ is g^*b continuous but not πg^*b-continuous.



1. α-continuous 2. b-continuous 3.pre-continuous 4. πg^*p-continuous 5. πg^*s-continuous
6. πgb-continuous 7. g^*b-continuous 8. $\pi g\alpha$-continuous 9. πgp-continuous 10. πgs-continuous
11. g-continuous 12. g^*-continuous

Fig.1

V. Almost πg*b- Continuous Functions

Definition 5.1 : A function $f : (X, \tau) \to (Y, \sigma)$ is called almost πg*b-continuous if $f^{-1}(V)$ is πg*b-closed in X for every regular closed set V of Y.

Theorem 5.2 : For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent.

(i) f is almost πg*b-continuous

(ii) $f^{-1}(V)$ is πg*b-open in X for every regular open set V of Y

(iii) $f^{-1}(int(cl(V))$ is πg*b-open in X for every open set V of Y.

(iv) $f^{-1}(cl(int(V))$ is πg*b-closed in X for every closed set V of Y.

Proof : (i) ⇒(ii)

Suppose f is almost πg*b-continuous. Let V be a regular open subset of Y. Since Y-V is regular closed and f is almost πg*b-continuous, $f^{-1}(Y-V) = X - f^{-1}(V)$ is πg*b-closed in X. Hence $f^{-1}(V)$ πg*b-open in X.

(ii) ⇒ (i)

Let V be a regular closed subset of Y. Then Y – V is regular open. By hypothesis, $f^{-1}(Y-V) = X - f^{-1}(V)$ is πg*b-open in X. Therefore $f^{-1}(V)$ is πg*b-closed. Hence f is almost πg*b-continuous.

(ii) ⇒ (iii)

Let V be an open subset of Y. Then int (cl(V)) is regular open in Y. By hypothesis, $f^{-1}(int(cl(V))$ is πg*b-open in X.

(iii) ⇒ (ii)

Let V be a regular open subset of Y. Since V = int(cl(V)) and every regular open set is open then $f^{-1}(V)$ πg*b-open in X.

(iii) ⇒ (iv)

Let V be a closed subset of Y. Then Y – V is open in Y.

By hypothesis, $f^{-1}(int(cl((Y - V))) = f^{-1}(Y - cl(int(V))) = X - f^{-1}(cl(int(V)))$ is πg*b-open in X. Hence, $f^{-1}(cl(int(V)))$ is πg*b-closed in X.

(iv)⇒ (iii)

Let V be an open subset of Y. Then Y-V is closed in Y.

By hypothesis, $f^{-1}(cl(int(Y-V))) = f^{-1}(Y - int(cl(V))) = X - f\text{-}1(int(cl(V)))$ is πg*b-closed in X.

Hence $f^{-1}(int(cl(V))$ is πg*b-open in X.

Theorem 5.3: Every πg*b-continuous function is almost πg*b-continuous.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be πg*b-continuous. Let V be a regular closed set in Y. Then V is closed in Y. since f is πg*b-continuous f-1(V) is πg*b-closed in X. Hence f is almost πg*b-continuous.

Remark 5.4: The converse of the above theorem need not be true as seen in the following example.

Example 5.5 : Let X = { a,b,c }, τ = { ϕ, {a}, {b}, {a,b}, }, X } and σ = { ϕ, {a}, {b}, {a,b}, {b,c , X }. Define $f : (X, \tau) \to (Y, \sigma)$ by f(a) = a, f(b) = c, f(c) = b. Here f is almost πg*b-continuous but not πg*b-continuous.

Theorem 5.6 : An R-map is almost πg*b-continuous.

Proof : Let $f : X \rightarrow Y$ is an R-map and V be a regular closed subset in Y. Therefore $f^{-1}(V)$ is a regular closed set in X. Since every regular closed set closed, $f^{-1}(V)$ is closed in X. Thus, $f^{-1}(V)$ is πg*b-closed in X. Hence f is almost πg*b-continuous.

Remark 5.7 : The converse of the above theorem need not be true as seen in the following example.

Example: Let X = Y= { a, b, c, d }, τ = { ϕ, {c}, { a, b}, {c, d}, {a, b, c}, X } and σ= { ϕ, {a}, {d}, {a, d}, {a, b, c}, X}. The function $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by f(a) = a, f(b) = d, f(c) = a, f(d) = c is almost πg*b-continuous but not an R-map.

Theorem5.8: If $f : X \rightarrow Y$ is almost b-continuous then f is $f : X \rightarrow Y$ is almost πg*b-continuous.

Proof : Let $f : X \rightarrow Y$ be an almost b-continuous. Let V be a closed set in Y. Then $f^{-1}(V)$ is b-closed in X. since every b-closed set is πg*b-closed, $f^{-1}(V)$ is πg*b-closed. Hence f is almost πg*b-continuous.

Remark 5.9: The converse of the above theorem need not be true as seen in the following example

Example 5.10 : Let X = Y = {a,b.c}, τ = {ϕ, {a}, { b, c }, X } and σ = {ϕ, {a}, {b}, {a, b }, {b, c}, X }. The identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is almost πg*b-continuous but not b-continuous.

Theorem 5.11: Let X be a πg*b-$T_{½}$ space. Then $f : X \rightarrow Y$ is almost πg*b-continuous iff f is almost b-continuous.

Proof : Suppose $f : X \rightarrow Y$ is almost πg*b-continuous. Let V be a regular closed subset in Y. Then $f^{-1}(V)$ is πg*b-closed in X. since X is πg*b-$T_{½}$ space, $f^{-1}(V)$ is b-closed in X.

Therefore f is almost b-continuous.

Conversely, suppose that $f : X \rightarrow Y$ is almost b-continuous. Let V be a regular closed subset in Y. Then $f^{-1}(V)$ is b-closed in X. Since every b-closed set is πg*b-closed, $f^{-1}(V)$ is πg*b-closed. Therefore f is almost πg*b - continuous.

Theorem 5.12 : Every πg*b-irresolute function is almost πg*b-continuous.

Proof : Straight forward.

Remark 5.13 : The converse of the above theorem need not be true as seen in the following example

Example: 5.14 Let X ={ a,b,c}, τ = { ϕ, {a}, {b}, {a,b},{b,c}, X }, σ = { ϕ, {b,c}, X } . Define $f : (X, \tau) \rightarrow (X, \sigma)$ by f(a) = a, f(b) = c, f(c) = b. Then f is almost πg*b-continuous but not πg*b irresolute.

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