CDPU Hypergraphs

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Abstract *- A graph G = (V, E) is complementary distance pattern uniform (CDPU), if there exists M □V(G)such that $f_M(u)$ = {d(u, v) : v ϵ M}, for every u ϵ V(G) - M, is independent of the choice of u ϵ V(G) -M and the set M is called the CDPU set. In this paper, we extend the notion of CDPU sets into hypergraphs. As every graph admits a CDPU set and a graph has more than one CDPU set, we can construct a hypergraph corresponding to that graph with the same vertex set and edge set corresponds to the different CDPU sets of a graph G.*

Keywords — *Complementary distance pattern uniform set, CDPU hypergraph.*

I. INTRODUCTION

For all terminology and notation in graph theory, not defined specifically in this paper, we refer the reader to Harary [8]. Unless mentioned otherwise, all the graphs considered in this paper are simple, self-loop-free and finite. B.D.Acharya [9] define the M− distance pattern of a vertex as follows :
Let G = (V, E) be a (p, q) graph and M be any non-empty subset of V(G). Then, the M-distance pattern of u is the set$f_M(u)$ = {d(u, v): v ϵ M}, where d(u, v) denotes the usual distance between u and v in G. If for a subset M of vertices in a graph G = (V, E), f_M is injective, then the set M is called the distance pattern distinguishing set (DPD-set in short).Germina and Beena[7]defined Complementary Distance Pattern Uniform (CDPU) Graphas, if $f_M(u)$ is independent of the choice of u ϵ V-M, then G is called a Complementary Distance Pattern Uniform (CDPU) Graph and the set M is called the CDPU set. The least cardinality of CDPU set in G is called the CDPU number ofG, denoted σ(G).

II. CDPU HYPERGRAPHS

Let G be a connected graph on n vertices. When |M| = n - 1 and eccentricity of a vertex in M is same as the eccentricity of avertex in V -M, then we say that M is a trivial CDPU set. All other CDPU sets are said to be non-trivial.
Here, we are considering only non-trivial CDPU sets.

Definition 2.1 Let M_1, M_2, …M_k be the non-trivial CDPU sets of a graph with V(G) = {v_1, v_2, …v_n}.The CDPU hypergraph of G, denoted by H_G, is defined V(H_G) = {v_1, v_2, …v_n}and E(H_G) = {M_1, M_2, …M_k}

Since every connected graph possess CDPU sets, we can easilysee that there corresponds a CDPU hypergraph H_G for everygraph G.

CDPU HYPERGRAPHS OF VARIOUS CLASSES OF GRAPHS

Theorem 2.2 The CDPU hypergraph of K_3 is a totally disconnected graph on 3 vertices.

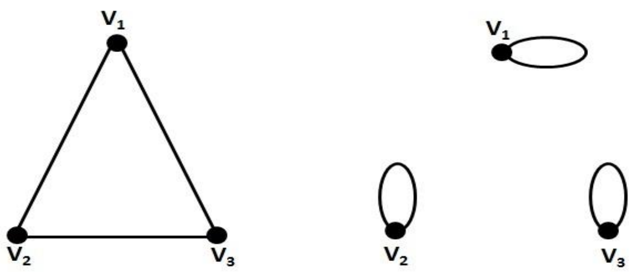

Fig. 1 K_3 and the corresponding CDPU graph

Proof: The CDPU sets for a complete graph K_3 with vertices {v_1,v_2,v_3} are M_1 = {v_1}, M_2 = {v_2} and M_3 = {v_3}. Hence we have three vertices and three edges that are totally disconnected in the corresponding hypergraph.

Theorem 2.3 The CDPU hypergraph of K_4 is $(K_4)_2$.

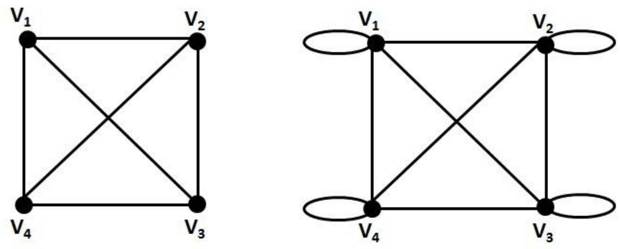

Fig. 2: K_4 and the corresponding CDPU graph

Proof: Let $V(K_4) = \{v_1, v_2, v_3, v_4\}$. The CDPU sets are $\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_1, v_2\},\{v_1, v_3\},\{v_1, v_4\},\{v_2, v_3\},\{v_2, v_4\},\{v_3, v_4\}$. Hencein H_{K_4}, there are four vertices and ten edges which is isomorphic to $(K_4)_2$.

Hence, we generalize the case of a CDPU hypergraph corresponding to a complete graph in Theorem 2.4.

Theorem 2.4 $H_{K_n} \cong (K_n)_{n-2}$

Theorem 2.5 The CDPU hypergraph of a path P_n, n evenis an n-2 uniform hypergraph.

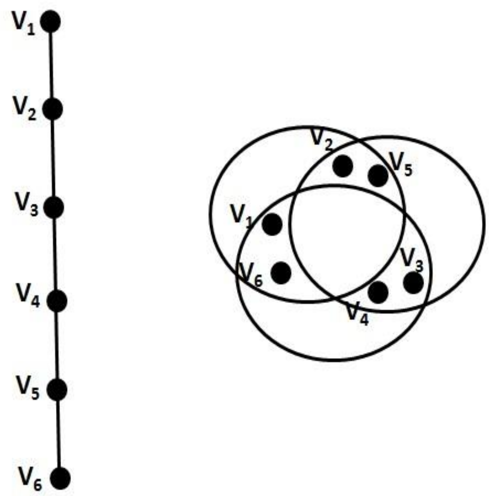

Fig.3: $P_6$and the corresponding CDPU graph

Proof: For a path P_n with n even, there are exactly two vertices with the same eccentricity. Hence, there are exactly $\frac{n}{2}$ different eccentricities for P_n. Let it be$e_1, e_2, \ldots e_{\frac{n}{2}}$. Then, $M_i = V(P_n)$- {vertices corresponding to eccentricity e_i}are the $\frac{n}{2}$different CDPU sets for G. Thus, allM_i's contains n-2 vertices. Hence, H_{P_n} is ann-2 uniform hypergraph.

Remark 2.6 For a path P_n with n odd, there is only one vertex with eccentricity $\frac{d}{2}$, where d is the diameter of P_n and exactly two vertices with the eccentricities $\frac{d}{2}+1, \frac{d}{2}+2, \ldots d$. Hence, there areexactly $\frac{n+1}{2}$different eccentricities for G.Let it be $e_1, e_2, \ldots e_{\frac{n+1}{2}}$. Then, $M_i = V(P_n)$ - {vertices corresponding to eccentricity e_i} are the $\frac{n+1}{2}$ different CDPU sets for G. Thus in H_{P_n}, exactly one hyperedge contains n-1 vertices and all other hyperedges contains n-2 vertices.

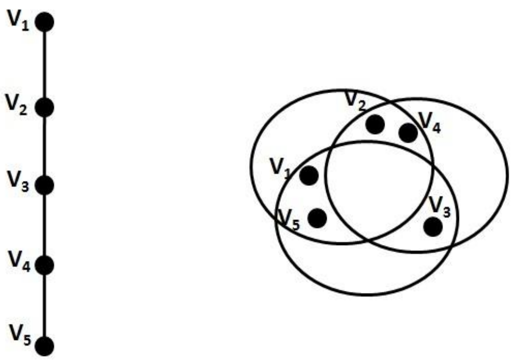

Fig.4: P_5 and the corresponding CDPU hypergraph

Remark 2.7 Let$G \cong K_{1,n}$ with V(G) = {u, v_1, v_2, … v_n} with u as the central vertex. Since {u}, the union of v and every n-j, j = 2, 3,… n-1 combinations and {v_2, v_3, … v_n}are CDPU sets, so are the hyper edges in H_G.

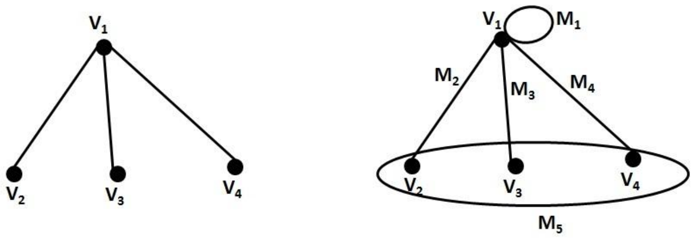

Fig 5: $K_{1,3}$ and the corresponding CDPU hypergraph

Theorem 2.8 The CDPU hypergraph of the cycle C_4 is K_4.

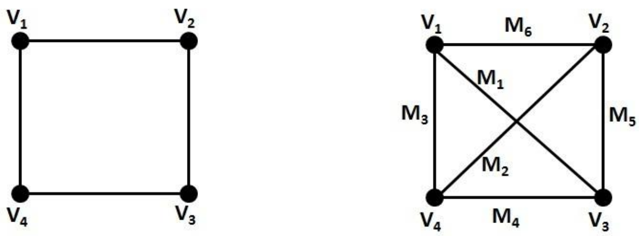

Fig.6: C_4 and the corresponding CDPU hypergraph

Proof: The CDPU sets for a cycle C_4 with vertices {v_1, v_2, v_3, v_4} areM_1 = {v_1, v_3}, M_2 = {v_2, v_4}, M_3 ={v_1,v_4}, M_4 = {v_3, v_4}, M_5 ={v_2, v_3}, M_6 = {v_1,v_2}. There are four vertices and six edges in H_{C_4} witheach of the vertices have degree three which implies that$H_{C_4} \cong K_4$.

Theorem 2.9 The CDPU hypergraph of C_5 is a 3- uniform hypergraph.

Proof : Let V(C_5) = { v_1, v_2, v_3, v_4, v_5}.The CDPU sets are M_1 = {v_1, v_2, v_3}; M_2 = {v_2, v_3, v_4}; M_3 = {v_3, v_4, v_5}; M_4={v_4, v_5, v_1};M_5 = {v_5, v_1, v_2}. Thus in H_{C_5}, there are five vertices and five edges in which each of the vertices have degree three, impliesthat H_{C_5} is a3-uniform hypergraph.

Theorem2.10 H_G is a 1-uniform hypergraph if and only if G ≅ K_2 or K_3.

Proof: Let G ≅ K_2 and V(G) = {v_1, v_2}. Then M_1 = {v_1} and M_2 = {v_2} are the CDPU sets which implies in H_G, there are two vertices with loop in each vertex. Let G ≅ K_3 and V(K_3) = {v_1, v_2, v_3}. Then M_1 = {v_1}, M_2 ={v_2}and M_3 = {v_3} are the CDPU sets which implies in H_G, there are three vertices with loop in each vertex. Hence, if G ≅ K_2 or K_3, then H_G is a 1-uniform hypergraph.

Conversely, assume that H_G is a 1-uniform hypergraph. Thus, there exists loop on each vertex. Clearly, H_G is a disconnected graph. Also, each M_i contains exactly one vertex implies that every vertex in G is a full degree vertex. Hence, $G \cong K_n$. When $G \cong K_n$, $n \geq 4$, there should be CDPU sets with $|M| = 1, 2, \ldots, n-2$.
Hence, $G \cong K_2$ or K_3

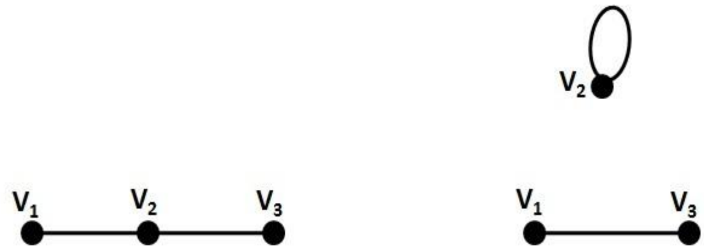

Fig.7: P_3 and the corresponding CDPU hypergraph

Remark 2.11 Let Gbe a connected graph with three vertices. Then $G \cong P_3$or K_3. When $G \cong P_3$, with $V(P_3) = \{v_1, v_2, v_3\}$, then $M_1 = \{v_2\}$ and $M_2 = \{v_1, v_3\}$ are two CDPU sets for G. In H_G, M_1 and M_2 are two hyperedges that are not connected [see Fig. 7]. So H_G is disconnected. Then $G \cong K_3$ with $V(K_3) = \{u_1, u_2, u_3\}$, then $M_1 = \{u_1\}$, $M_2 = \{u_2\}$, $M_3 = \{v_3\}$ are three CDPU sets for G [seeFig. 1]. Hence, H_G is disconnected.

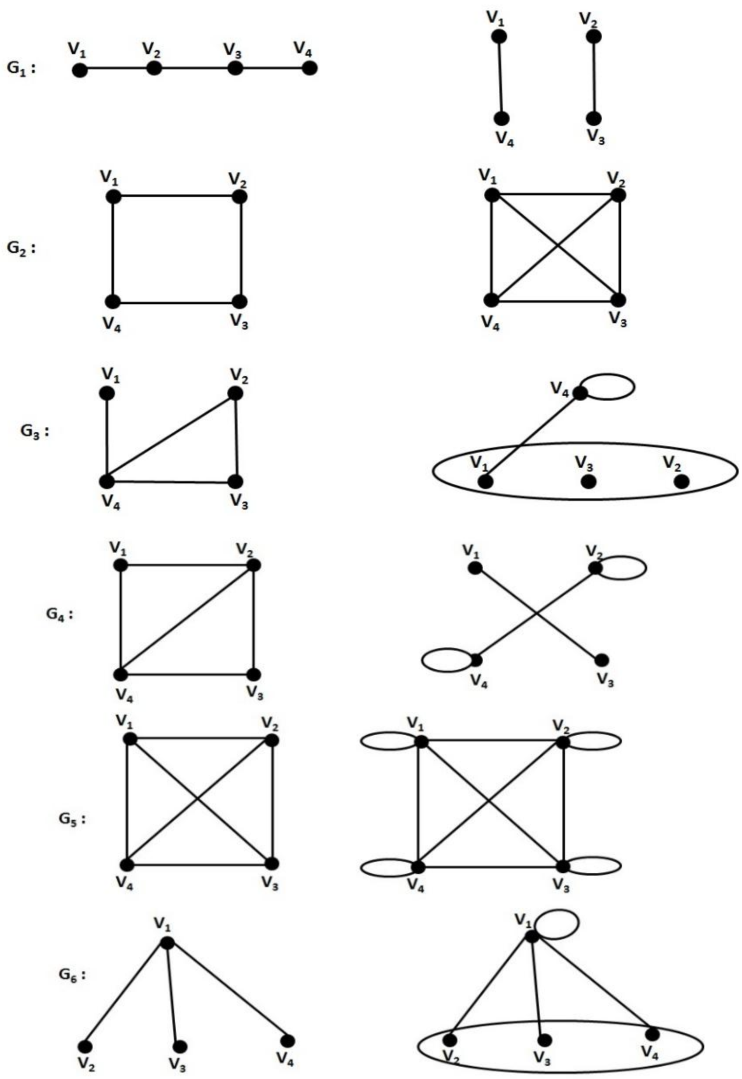

Fig.8: Graphs and corresponding CDPU graphs

Theorem 2.12 Let G be a connected graph with 4 vertices. Then, H_G is disconnected if and only if $G \cong P_4$ or K_4-x

Proof: Fig.8 shows the different connected graphs on 4 vertices and their corresponding CDPU hypergraphs.

When $G \cong P_4$ with $V(P_4) = \{v_1, v_2, v_3, v_4\}$, $M_1 = \{v_2, v_3\}$ and $M_2 = \{v_1, v_4\}$ are the CDPU sets which gives a disconnected CDPU hypergraph.
When $G \cong K_4$-x with $V(K_4\text{-}x) = \{v_1, v_2, v_3, v_4\}$, $M_1 = \{v_2, v_4\}$, $M_2 = \{v_2\}$, $M_3 = \{v_4\}$ and $M_4 = \{v_1, v_3\}$ are the CDPU sets which gives a disconnected CDPU hypergraph.
Conversely, assume that G is not isomorphic to P_4 or K_4-x. $G \cong C_4$ implies H_G, a connected hypergraph. When $G \cong G_2$, G_5 and G_6 implies H_G, a connected hypergraph.

Remark 2.13 Let G be a graph on $p \geq 5$ vertices. Then, H_G is connected.

Theorem 2.14 H_G is disconnected if and only if $G \cong K_2$, K_3, P_3, P_4 and K_4-x.

Theorem 2.15 H_G is a2-uniform hypergraph if and only if $G \cong P_4$ or C_4.

Proof: When $G \cong P_4$ or C_4, clearly H_G is a 2-uniform hypergraph.

Conversely, assume that H_G is a 2-uniform hypergraph. Then, there exists M_i's such that $|M_i| = 2$, for every i. Thus, the diameter of G is atleast two [see Fig.8]. For graphs with diameter two, except C_4 has some M with $|M| > 2$. When diameter is three, all but P_4, has M such that $|M| > 2$. If diameter is greater than three, then clearly $|M| \geq 3$.

Remark 2.16 If v is a vertex of full degree in a graph G, then there exist an M with M = {v} and hence, there is a loop incident with the vertex v and conversely. Thus, anedge in H_G is a loop if and only if G has a full degreevertex.

Remark 2.17 Every vertex in H_G is incident with a loopif and only if $G \cong K_n$.

Remark 2.18 For a graph G with CDPU sets M_i, i = 1, 2, …,k and $M_i \cap M_j \neq \emptyset$, for every i and j; $\cap_{i=1}^{k} M_i = \emptyset$. Thus CDPUhypergraph does not follow Helly property.

CONCLUSIONS

Every connected graph possess CDPU sets. In this paper, an idea of using these CDPU sets for constructing a new hypergraph is introduced. CDPU hypergraph of various classes of graphs and various properties of these hypergraphs are established.

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