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Weighted dom-chromatic number of some classes of Type-I weighted caterpillars

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Abstract

A set D of vertices is a dominating set of G if every vertex not in D is adjacent to at least one member of D. A set D of vertices is said to be dom-chromatic if D is a dominating set and $\chi(\langle D\rangle) = \chi(G)$. A Weighted tree, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \longrightarrow R^+$. The weighted domination number $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v\in D} w(v)$ of a dominating set D of T. The weighted dom-chromatic number $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v\in D} w(v)$ of a dom-chromatic set D of T. A caterpillar is a graph which can be obtained from the path on n vertices by appending x_i pendant vertices to the i^{th} vertex of the path, P_n. The caterpillar with parameters $n, x_1, x_2, \ldots, x_n$ will be denoted as $P_n(x_1, x_2, \ldots, x_n)$. In this paper, the weighted dom-chromatic numbers are determined for some classes of Type-I weighted caterpillars.

Keywords: dominating set, dom-chromatic set, weighted domination, weighted dom-chromatic number, Type-I weighted labeling
MSC Subject Classification: 05C69

1 Introduction

A set S of vertices is a *dominating set* of G if every vertex not in S is adjacent to at least one member of S. The minimum cardinality of a dominating set in

G is called the *domination number* and is denoted by $\gamma(G)$. The set $\mathcal{D}(G)$ is the collection of all dominating sets of G. A subset S of V is said to be a *dom-chromatic set* (or *dc-set*) if S is a dominating set and $\chi(< S >) = \chi(G)$. The minimum cardinality of a dom-chromatic set in G is called the *dom-chromatic number* (or *dc-number*) and is denoted by $\gamma_{ch}(G)$. The set $\mathcal{D}_{ch}(G)$ is the collection of all dom-chromatic sets of G

A *Weighted tree*, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \longrightarrow R^+$. The *weighted domination number* $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dominating set D of T. The *weighted dom-chromatic number* $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dom-chromatic set D of T.

P. Palanikumar and S. Balamurugan [12] has introduced the concept of *Type-I weighted labeling* and they study the weighted dom-chromatic number of a weighted tree. Also, determine the weighted dom-chromatic number of a *Type I* weighted paths.

Theorem 1.1. *[12] Let (T, w) be a weighted tree and $[1, 2, \ldots, n]$ be a leaf-first labeling of T Where $w(i) = w_i$ for $i = 1, 2, \ldots, n$. If i is a leaf of T then $\eta_{ch}(i) = w_i$; $\theta_{ch}(i) = 0$; $\lambda_{ch}(i) = w_i$; $\mu_{ch}(i) = 0$.*

Definition 1.2. *[12] Let (T, w) be a weighted tree and $[1, 2, \ldots, n]$ be a leaf-first labeling of (T, w). Then L is said to be of Type-I if i is the first leaf of $T - \{1, 2, \ldots, i - 1\}$ from left.*

Theorem 1.3. *[12] For a path P_n,$(n \geq 3)$ of Type-I,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}\left(n^2 + n + 6\right) & \textit{if } n \equiv 0(mod\ 3) \\ \frac{1}{6}\left(n^2 + n + 10\right) & \textit{if } n \equiv 1(mod\ 3) \\ \frac{1}{6}\left(n^2 + n + 12\right) & \textit{if } n \equiv 2(mod\ 3) \end{cases}$$

2 Caterpillar

A caterpillar is a graph which can be obtained from the path on n vertices by appending x_i pendant vertices to the i^{th} vertex of the path,

P_n. The caterpillar with parameters $n, x_1, x_2, \ldots, x_n$ will be denoted as $P_n(x_1, x_2, \ldots, x_n)$.

Note, this is a tree with the property that the removal of its leaves and incident edges results in a path P_n called the *spine* of the caterpillar. Let l denote the number of leaves, i.e., $l = \sum_{i=1}^{n} x_i$. We say a caterpillar is complete if every vertex on the spine of the caterpillar is adjacent to at least one leaf.

Let $P_n(x_1, x_2, \ldots, x_n)$ be a caterpillar. We first consider the case of caterpillars where $x_1 = x_n = l$ and $x_i = 0$ for $2 \leq i \leq n-1$. It is observe that it can have three cases that is a path with n vertices where $n = 3k$, $3k+1$, $3k+2$ to determine $\gamma_{wch}(P_n(l, 0, 0, \ldots 0, l))$.

Theorem 2.1. *For a caterpillar, $P_n(x_1, x_2, \ldots, x_n)$, where $x_1 = x_n = l$, $x_i = 0$ for $2 \leq i \leq n-1$ of Type-I, then*

$$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = \begin{cases} \frac{1}{6}\left(n^2 + 3n + 6\right) + \frac{(n+6)l}{3} & \text{if } n \equiv 0 \ (mod\ 3) \\ \frac{1}{6}(n+1)(n+2) + \frac{(n+5)l}{3} & \text{if } n \equiv 1 \ (mod\ 3) \\ \frac{1}{6}(n+1)(n+2) + \frac{(n+7)l}{3} & \text{if } n \equiv 2 \ (mod\ 3) \end{cases}$$

Proof. Let $G = (P_n(x_1, x_2, \ldots, x_n), w)$ be a weighted caterpillar with $x_1 = x_n = l$ and $x_i = 0$ for $2 \leq i \leq n-1$. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$. Attach l pendent vertices, namely $\{r_1, r_2, \ldots, r_l\}$,the left siblings, at v_1 and Attach l pendent vertices, namely $\{s_1, s_2, \ldots, s_l\}$,the right siblings, at v_n as shown in Figure 2.1.

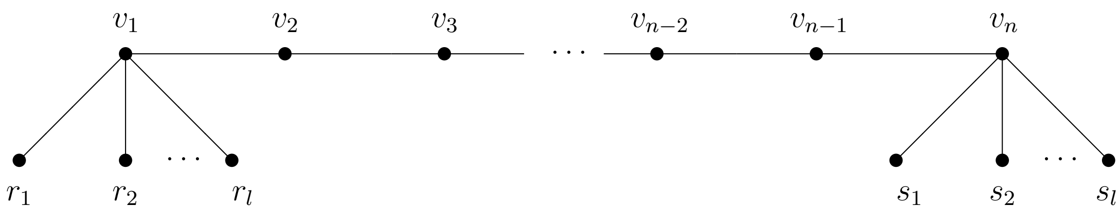

Figure 2.1: A caterpillar $P_n(l, 0, 0, \ldots, 0, l)$

Let $L = [1, 2, \ldots, n]$ be a leaf-first labeling of $(P_n(x_1, x_2, \ldots, x_n), w)$ and L is of Type-I.

If $l = 1$, then $G = P_n(x_1, x_2, \ldots, x_n)$ reduces to a path on $n+2$ vertices. Thus, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = \gamma_{wch}(P_{n+2})$. By Theorem 1.3, the result is obvious.

Now we consider for $l > 1$. To dominate the left sibling vertices $\{r_1, r_2, \ldots, r_l\}$, the minimum weighted vertex is v_1 and to dominate the right sibling vertices $\{s_1, s_2, \ldots, s_l\}$, the minimum weighted vertex is v_n. Thus v_1 and v_n are must be in γ_w- set of G. We consider the following cases.

Case (1): Suppose $n \equiv 0(mod\ 3)$. Then $n = 3k$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertices of P_{3k}. As the maximum weighted vertex v_{3k} dominates the vertex v_{3k-1}, to dominate the maximum weighted vertex v_{3k-2}, choose the minimum weighted vertex v_{3k-3} for the γ_w-set so that the vertices v_{3k-4} and v_{3k-2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-5}, choose the minimum weighted vertex v_{3k-6} for the γ_w-set so that the vertices v_{3k-7} and v_{3k-5} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4, choose the minimum weighted vertex v_3 for the γ_w-set of G. Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k-3}, v_{3k-6}, \ldots, v_6, v_3\}$ belongs to the γ_w-set of G. Since the vertices v_1 and v_{3k} are included in any weighted dominating set of G, the set $D = \{v_1, v_3, v_6, \ldots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Obviously, it is r_1. Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_3, v_6, \ldots, v_{3k-6}, v_{3k-3}, v_{3k}\}$.

Hence, the minimum weight of a dom-chromatic set is,$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 1+(l+1)+[(l+3)+(l+6)+\ldots+(l+3k-3)]+(2l+3k-1) = \frac{1}{6}(n^2+3n+6) + \frac{(n+6)l}{3}$

Case(2): Suppose $n \equiv 1\ (mod\ 3)$. Then $n = 3k+1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertices of P_{3k+1}. As the maximum weighted vertex v_{3k+1} dominates the vertex v_{3k}, to dominate the maximum weighted vertex v_{3k-1}, choose the minimum weighted vertex v_{3k-2} for the γ_w-set so that the vertices v_{3k-3} and v_{3k-1} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-4}, choose the minimum weighted vertex v_{3k-5} for the γ_w-set so that the vertices v_{3k-6} and v_{3k-4} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_5, choose the minimum weighted vertex v_4 for the γ_w-set of G. Then the vertices v_3 and v_5 are dominated. Thus the set of vertices $\{v_{3k-2}, v_{3k-5}, \ldots, v_7, v_4\}$ belongs to the γ_w-set of G. Since the vertices v_1 and v_{3k+1} are included in any weighted dominating set of G, the set $\{v_1, v_4, v_7, \ldots, v_{3k-2}, v_{3k+1}\}$ will be a minimum weighted dominating set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally, it is r_1. Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_4, \ldots, v_{3k-2}, v_{3k+1}\}$.

Hence, the minimum weight of a dom-chromatic set is,$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 1+[(l+1)+(l+4)+(l+7)+\ldots+(l+3k-2)]+(2l+3k) = \frac{1}{6}(n+1)(n+2) + \frac{(n+5)l}{3}$

Case(3): Suppose $n \equiv 2 \ (mod\ 3)$. Then $n = 3k+2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+2}\}$ be the vertices of P_{3k+2}. As the maximum weighted vertex v_{3k+2} dominates the vertex v_{3k+1}, to dominate the maximum weighted vertex v_{3k}, choose the minimum weighted vertex v_{3k-1} for the γ_w-set so that the vertices v_{3k-2} and v_{3k} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3}, choose the minimum weighted vertex v_{3k-4} for the γ_w-set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3, choose the minimum weighted vertex v_2 for the γ_w-set of G. Then the vertices v_1 and v_3 are dominated. Thus the set of vertices $\{v_{3k-1}, v_{3k-4}, \ldots, v_6, v_2\}$ belongs to the γ_w-set of G. Since the vertices v_1 and v_{3k+2} are included in any weighted dominating set of G, the set $D = \{v_1, v_2, v_5, \ldots, v_{3k}, v_{3k+2}\}$ will be a minimum weighted dominating set of G.

Clearly, the set D preserves the chromticity of least weight in G. Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = (l+1)+[(l+2)+(l+5)+\ldots+(l+3k-1)]+(2l+3k+1) = \frac{1}{6}(n+1)(n+2) + \frac{(n+7)l}{3}$. $\square$

We next consider the case of caterpillars where $x_1 = l$ and $x_i = 0$ for $2 \leq i \leq n$. We observe that it can have three cases that is a path with n vertices where $n = 3k$, $3k+1$,$3k+2$ to determine $\gamma_{wch}(P_n(l, 0, 0, \dots 0, 0))$.

Theorem 2.2. *For a caterpillar, $P_n(x_1, x_2, \dots, x_n)$, where $x_1 = l$, $x_i = 0$ for $2 \leq i \leq n$ of Type-I, then*

$$\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \begin{cases} \frac{1}{6}(n^2+n+6) + \frac{(n+3)l}{3} & \text{if } n \equiv 0 (mod\ 3) \\ \frac{1}{6}(n^2+n+10) + \frac{(n+2)l}{3} & \text{if } n \equiv 1\ (mod\ 3) \\ \frac{1}{6}(n^2+n+6) + \frac{(n+1)l}{3} & \text{if } n \equiv 2\ (mod\ 3) \end{cases}$$

Proof. Let $G = (P_n(x_1, x_2, \dots, x_n), w)$ be a weighted caterpillar with $x_1 = l$ and $x_i = 0$ for $2 \leq i \leq n$. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$. Attach l pendent vertices, namely $\{r_1, r_2, \dots, r_l\}$,the left siblings, at v_1 as shown in Figure 2.2.

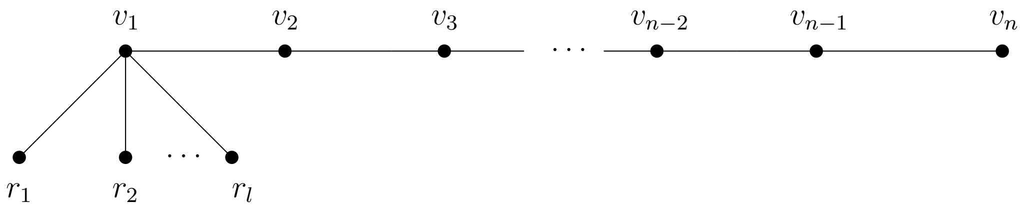

Figure 2.2: A caterpillar $P_n(l, 0, 0, \dots, 0, 0)$

Let $L = [1, 2, \dots, n]$ be a leaf-first labeling of $(P_n(x_1, x_2, \dots, x_n), w)$ and L is of Type-I.

If $l = 1$, then $G = P_n(x_1, x_2, \dots, x_n)$ reduces to a path on $n+1$ vertices. Thus, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \gamma_{wch}(P_{n+1})$. By Theorem 1.3, the result is obvious.

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are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3}, choose the minimum weighted vertex v_{3k-4} for the γ_w-set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3, choose the minimum weighted vertex v_2 for the γ_w-set of G. Then the vertices v_1 and v_3 are dominated. Thus the set of vertices $\{v_{3k-1}, v_{3k-4}, \ldots, v_5, v_2\}$ belongs to the γ_w-set of G. Since the vertex v_1 is included in any weighted dominating set of G, the set $D = \{v_1, v_2, v_6, \ldots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G.

Naturally, the set D preserves the chromaticity of least weight in G. Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = (l+1) + [(l+2) + (l+5) + \ldots + (l+3k-1)] = \frac{1}{6}(n^2+n+6) + \frac{(n+3)l}{3}$.

Case (2): Suppose $n \equiv 1\ (mod\ 3)$. Then $n = 3k+1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertices of P_{3k+1}. It is obvious that v_{3k+1} admits the maximum weight in G. Hence to dominate the vertex v_{3k+1}, choose the minimum weighted vertex v_{3k} for the γ_w-set. Then the vertices v_{3k-1} and v_{3k+1} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-2}, choose the minimum weighted vertex v_{3k-3} for the γ_w-set so that the vertices v_{3k-4} and v_{3k-2} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4, choose the minimum weighted vertex v_3 for the γ_w-set of G. Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k}, v_{3k-3}, \ldots, v_6, v_3\}$ belongs to the γ_w-set of G. Since the vertex v_1 is included in any weighted dominating set of G, the set $\{v_1, v_3, v_6, \ldots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G.

For chromatic preserving, a least weight vertex is to be added which is r_1. Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_3, v_6, \ldots, v_{3k-3}, v_{3k}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 1 + (l+1) + [(l+3) + (l+6) + \ldots + (l+3k)] = \frac{1}{6}(n^2+n+10) + \frac{(n+2)l}{3}$

Case (3): Suppose $n \equiv 2\ (mod\ 3)$. Then $n = 3k+2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+2}\}$ be the vertices of P_{3k+2}. It is obvious that v_{3k+2} admits the maximum weight in G. Hence to dominate the vertex v_{3k+2}, choose the minimum weighted vertex v_{3k+1} for the γ_w-set. Then the vertices v_{3k} and v_{3k+2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-1}, choose the minimum weighted vertex v_{3k-2} for the γ_w-set so that the vertices v_{3k-3} and v_{3k-1} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_5, choose the minimum weighted vertex v_4 for the γ_w-set of G. Then the vertices v_3 and v_5 are dominated. Thus the set of vertices $\{v_{3k+1}, v_{3k-2}, \ldots, v_7, v_4\}$ belongs to the γ_w-set of G. Since the vertex v_1 is included in any weighted dominating set of G, the set $\{v_1, v_4, v_7, \ldots, v_{3k-2}, v_{3k+1}\}$ will be a minimum weighted dominating set of G.

For chromatic preserving, a least weight vertex is to be added, Naturally, it is r_1. Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_4, v_7, \ldots, v_{3k-2}, v_{3k+1}\}$.

Hence, the minimum weight of a dom-chromatic set is,$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 1 + [(l+1)+(l+4)+\ldots+(l+3k+1)] = \frac{1}{6}(n^2+n+6)+\frac{(n+1)l}{3}$

□

Next let us consider the case of caterpillars where $x_n = l$ and $x_i = 0$ for $1 \leq i \leq n-1$. We observe that it can have three cases that is a path with n vertices where $n = 3k$, $3k+1$,$3k+2$ to determine $\gamma_{wch}(P_n(0, 0, \ldots 0, l))$.

Theorem 2.3. *For a caterpillar, $P_n(x_1, x_2, \ldots, x_n)$, where $x_n = l$, $x_i = 0$ for $1 \leq i \leq n-1$ of Type-I, then*

$$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = \begin{cases} \frac{1}{6}(n^2+3n+6)+l & if\ n \equiv 0\ (mod\ 3) \\ \frac{1}{6}(n^2+3n+8)+l & if\ n \equiv 1\ (mod\ 3) \\ \frac{1}{6}(n^2+3n+2)+l & if\ n \equiv 2\ (mod\ 3) \end{cases}$$

Proof. : Let $G = (P_n(x_1, x_2, \ldots, x_n), w)$ be a weighted caterpillar with $x_n = l$ and $x_i = 0$ for $1 \leq i \leq n-1$. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$. Attach l

pendent vertices, namely $\{s_1, s_2, \ldots, s_l\}$,the right siblings, at v_n as shown in Figure 2.3

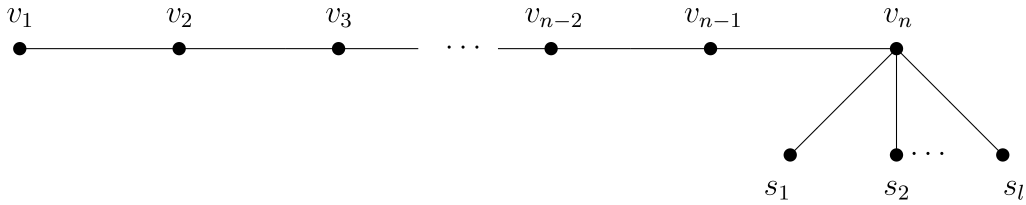

Figure 2.3: A caterpillar $P_n\,(0, 0, \ldots, 0, l)$

Let $L = [1, 2, \ldots, n]$ be a leaf-first labeling of $(P_n\,(x_1, x_2, \ldots, x_n)\,, w)$ and L is of Type-I.

If $l = 1$, then $G = P_n\,(x_1, x_2, \ldots, x_n)$ reduces to a path on $n + 1$ vertices. Thus, $\gamma_{wch}(P_n\,(x_1, x_2, \ldots, x_n)) = \gamma_{wch}(P_{n+1})$. By Theorem 1.3, the result is obvious.

Now we consider for $l > 1$. To dominate the right sibling vertices $\{s_1, s_2, \ldots, s_l\}$, the minimum weighted vertex is v_n . Thus v_n must be in γ_w-set of G. We consider the following cases.

Case (1): Suppose $n \equiv 0(mod\ 3)$. Then $n = 3k$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertices of P_{3k}. Since v_{3k} dominates v_{3k-1}, to dominate the maximum weighted vertex v_{3k-2}, choose the minimum weighted vertex v_{3k-3} for the γ_w-set. Then the vertices v_{3k-4} and v_{3k-2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-5}, choose the minimum weighted vertex v_{3k-6} for the γ_w-set so that the vertices v_{3k-7} and v_{3k-5} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4, choose the minimum weighted vertex v_3 for the γ_w-set of G. Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k}, v_{3k-3}, \ldots, v_6, v_3\}$ belongs to the γ_w-set of G.

Now, by choosing the minimum weighted vertex v_1 to the γ_w-set, it is necessary to select the vertex v_2 for the chromatic preservation. Hence, the weighted vertices v_1, v_2 alongwith v_3 contributes a weight of 6 to the γ_{wch}-set. While, we choose the weighted vertex v_2 for γ_w-set, it dominates

v_1 and the γ_w-set unioned with v_2 preserves the chromaticity. Moreover, it contributes a minimum weight of 5 to the γ_{wch}-set. Hence, any γ_w-set unioned with v_2 will be the least weight dom-chromatic set of G. Thus, $D = \{v_2, v_3, v_6, \ldots, v_{3k-3}, v_{3k}\}$ is a minimum weighted dom-chromatic set in G.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 2+(3+6+\ldots+3k-3)+(3k-1+l) = l+\frac{1}{6}(n^2+3n+6)$

Case (2): Suppose $n \equiv 1\ (mod\ 3)$. Then $n = 3k+1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertices of P_{3k+1}. Since v_{3k+1} dominates v_{3k}, to dominate the maximum weighted vertex v_{3k-1}, choose the minimum weighted vertex v_{3k-2} for the γ_w-set. Then the vertices v_{3k-3} and v_{3k-1} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-4}, choose the minimum weighted vertex v_{3k-5} for the γ_w-set so that the vertices v_{3k-6} and v_{3k-4} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_2, choose the minimum weighted vertex v_1 for the γ_w-set of G. Then the vertex v_2 is dominated. Thus the set of vertices $\{v_{3k+1}, v_{3k-2}, \ldots, v_4, v_1\}$ belongs to the γ_w-set of G.

For chromatic preserving, a least weight vertex is to be added which is v_2. Therefore the least weight dom-chromatic set D is $\{v_1, v_2, v_4, \ldots, v_{3k-2}, v_{3k+1}\}$. Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 2 + (1 + 4 + \ldots + 3k - 2) + 3k + l = l + \frac{1}{6}(n^2 + 3n + 8)$.

Case (3): Suppose $n \equiv 2\ (mod\ 3)$. Then $n = 3k + 2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \ldots, v_{3k+2}\}$ be the vertices of P_{3k+2}. Since v_{3k+2} dominates v_{3k+1}, to dominate the maximum weighted vertex v_{3k}, choose the minimum weighted vertex v_{3k-1} for the γ_w-set. Then the vertices v_{3k-2} and v_{3k} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3}, choose the minimum weighted vertex v_{3k-4} for the γ_w-set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3, choose the minimum weighted vertex v_2 for the γ_w-set of G. Then the vertices

v_1 and v_3 is dominated. Thus the set of vertices $\{v_{3k+2}, v_{3k-1}, \ldots, v_5, v_2\}$ belongs to the γ_w-set of G.

For chromatic preserving, a least weight vertex is to be added which is v_1. Therefore the least weight dom-chromatic set D is $\{v_1, v_2, v_5, \ldots, v_{3k-1}, v_{3k+2}\}$.

Hence, the minimum weight of a dom-chromatic set is,$\gamma_{wch}(P_n(x_1, x_2, \ldots, x_n)) = w(D) = \sum w(v_i) = 1+(2+5+\ldots+3k-1)+3k+1+l = l+\frac{1}{6}(n^2+3n+2)$. $\square$

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