

Weighted dom-chromatic number of some classes of Type-I weighted caterpillars

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Abstract

A set D of vertices is a dominating set of G if every vertex not in D is adjacent to at least one member of D . A set D of vertices is said to be dom-chromatic if D is a dominating set and $\chi(\langle D \rangle) = \chi(G)$. A Weighted tree, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \rightarrow R^+$. The weighted domination number $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dominating set D of T . The weighted dom-chromatic number $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dom-chromatic set D of T . A caterpillar is a graph which can be obtained from the path on n vertices by appending x_i pendant vertices to the i^{th} vertex of the path, P_n . The caterpillar with parameters n, x_1, x_2, \dots, x_n will be denoted as $P_n(x_1, x_2, \dots, x_n)$. In this paper, the weighted dom-chromatic numbers are determined for some classes of Type-I weighted caterpillars.

Keywords: dominating set, dom-chromatic set, weighted domination, weighted dom-chromatic number, Type-I weighted labeling

MSC Subject Classification: 05C69

1 Introduction

A set S of vertices is a *dominating set* of G if every vertex not in S is adjacent to at least one member of S . The minimum cardinality of a dominating set in

G is called the *domination number* and is denoted by $\gamma(G)$. The set $\mathcal{D}(G)$ is the collection of all dominating sets of G . A subset S of V is said to be a *dom-chromatic set* (or *dc-set*) if S is a dominating set and $\chi(\langle S \rangle) = \chi(G)$. The minimum cardinality of a dom-chromatic set in G is called the *dom-chromatic number* (or *dc-number*) and is denoted by $\gamma_{ch}(G)$. The set $\mathcal{D}_{ch}(G)$ is the collection of all dom-chromatic sets of G .

A *Weighted tree*, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \rightarrow R^+$. The *weighted domination number* $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dominating set D of T . The *weighted dom-chromatic number* $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dom-chromatic set D of T .

P. Palanikumar and S. Balamurugan [12] has introduced the concept of *Type-I weighted labeling* and they study the weighted dom-chromatic number of a weighted tree. Also, determine the weighted dom-chromatic number of a *Type I* weighted paths.

Theorem 1.1. [12] *Let (T, w) be a weighted tree and $[1, 2, \dots, n]$ be a leaf-first labeling of T Where $w(i) = w_i$ for $i = 1, 2, \dots, n$. If i is a leaf of T then $\eta_{ch}(i) = w_i$; $\theta_{ch}(i) = 0$; $\lambda_{ch}(i) = w_i$; $\mu_{ch}(i) = 0$.*

Definition 1.2. [12] *Let (T, w) be a weighted tree and $[1, 2, \dots, n]$ be a leaf-first labeling of (T, w) . Then L is said to be of *Type-I* if i is the first leaf of $T - \{1, 2, \dots, i - 1\}$ from left.*

Theorem 1.3. [12] *For a path $P_n, (n \geq 3)$ of Type-I,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}(n^2 + n + 6) & \text{if } n \equiv 0(\text{mod } 3) \\ \frac{1}{6}(n^2 + n + 10) & \text{if } n \equiv 1(\text{mod } 3) \\ \frac{1}{6}(n^2 + n + 12) & \text{if } n \equiv 2(\text{mod } 3) \end{cases}$$

2 Caterpillar

A caterpillar is a graph which can be obtained from the path on n vertices by appending x_i pendant vertices to the i^{th} vertex of the path,

P_n . The caterpillar with parameters n, x_1, x_2, \dots, x_n will be denoted as $P_n(x_1, x_2, \dots, x_n)$.

Note, this is a tree with the property that the removal of its leaves and incident edges results in a path P_n called the *spine* of the caterpillar. Let l denote the number of leaves, i.e., $l = \sum_{i=1}^n x_i$. We say a caterpillar is complete if every vertex on the spine of the caterpillar is adjacent to at least one leaf.

Let $P_n(x_1, x_2, \dots, x_n)$ be a caterpillar. We first consider the case of caterpillars where $x_1 = x_n = l$ and $x_i = 0$ for $2 \leq i \leq n - 1$. It is observe that it can have three cases that is a path with n vertices where $n = 3k, 3k + 1, 3k + 2$ to determine $\gamma_{wch}(P_n(l, 0, 0, \dots, 0, l))$.

Theorem 2.1. For a caterpillar, $P_n(x_1, x_2, \dots, x_n)$, where $x_1 = x_n = l, x_i = 0$ for $2 \leq i \leq n - 1$ of Type-I, then

$$\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \begin{cases} \frac{1}{6}(n^2 + 3n + 6) + \frac{(n+6)l}{3} & \text{if } n \equiv 0 \pmod{3} \\ \frac{1}{6}(n + 1)(n + 2) + \frac{(n+5)l}{3} & \text{if } n \equiv 1 \pmod{3} \\ \frac{1}{6}(n + 1)(n + 2) + \frac{(n+7)l}{3} & \text{if } n \equiv 2 \pmod{3} \end{cases}$$

Proof. Let $G = (P_n(x_1, x_2, \dots, x_n), w)$ be a weighted caterpillar with $x_1 = x_n = l$ and $x_i = 0$ for $2 \leq i \leq n - 1$. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$. Attach l pendent vertices, namely $\{r_1, r_2, \dots, r_l\}$, the left siblings, at v_1 and Attach l pendent vertices, namely $\{s_1, s_2, \dots, s_l\}$, the right siblings, at v_n as shown in Figure 2.1.

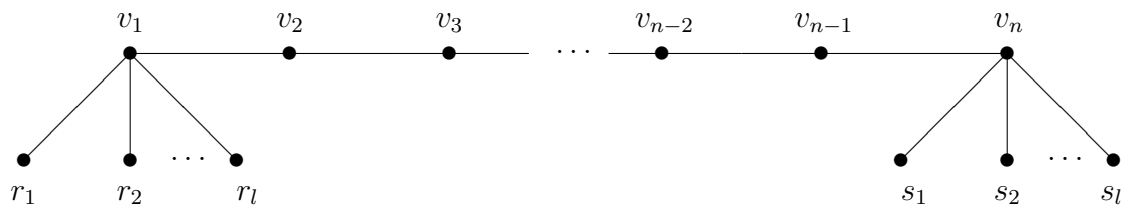


Figure 2.1: A caterpillar $P_n(l, 0, 0, \dots, 0, l)$

Let $L = [1, 2, \dots, n]$ be a leaf-first labeling of $(P_n(x_1, x_2, \dots, x_n), w)$ and L is of Type-I.

If $l = 1$, then $G = P_n(x_1, x_2, \dots, x_n)$ reduces to a path on $n + 2$ vertices. Thus, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \gamma_{wch}(P_{n+2})$. By Theorem 1.3, the result is obvious.

Now we consider for $l > 1$. To dominate the left sibling vertices $\{r_1, r_2, \dots, r_l\}$, the minimum weighted vertex is v_1 and to dominate the right sibling vertices $\{s_1, s_2, \dots, s_l\}$, the minimum weighted vertex is v_n . Thus v_1 and v_n are must be in γ_w -set of G . We consider the following cases.

Case (1): Suppose $n \equiv 0(mod 3)$. Then $n = 3k$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k}\}$ be the vertices of P_{3k} . As the maximum weighted vertex v_{3k} dominates the vertex v_{3k-1} , to dominate the maximum weighted vertex v_{3k-2} , choose the minimum weighted vertex v_{3k-3} for the γ_w -set so that the vertices v_{3k-4} and v_{3k-2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-5} , choose the minimum weighted vertex v_{3k-6} for the γ_w -set so that the vertices v_{3k-7} and v_{3k-5} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4 , choose the minimum weighted vertex v_3 for the γ_w -set of G . Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k-3}, v_{3k-6}, \dots, v_6, v_3\}$ belongs to the γ_w -set of G . Since the vertices v_1 and v_{3k} are included in any weighted dominating set of G , the set $D = \{v_1, v_3, v_6, \dots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G .

For chromatic preserving, add a neighbor of least weight vertex to this set. Obviously, it is r_1 . Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_3, v_6, \dots, v_{3k-6}, v_{3k-3}, v_{3k}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 1 + (l+1) + [(l+3) + (l+6) + \dots + (l+3k-3)] + (2l+3k-1) = \frac{1}{6}(n^2 + 3n + 6) + \frac{(n+6)l}{3}$

Case(2): Suppose $n \equiv 1(mod 3)$. Then $n = 3k + 1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+1}\}$ be the vertices of P_{3k+1} . As the maximum weighted vertex v_{3k+1} dominates the vertex v_{3k} , to dominate the maximum weighted vertex v_{3k-1} , choose the minimum weighted vertex v_{3k-2} for the γ_w -set so that the vertices v_{3k-3} and v_{3k-1} are dominated. Similarly, to dominate

the maximum weighted vertex v_{3k-4} , choose the minimum weighted vertex v_{3k-5} for the γ_w -set so that the vertices v_{3k-6} and v_{3k-4} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_5 , choose the minimum weighted vertex v_4 for the γ_w -set of G . Then the vertices v_3 and v_5 are dominated. Thus the set of vertices $\{v_{3k-2}, v_{3k-5}, \dots, v_7, v_4\}$ belongs to the γ_w -set of G . Since the vertices v_1 and v_{3k+1} are included in any weighted dominating set of G , the set $\{v_1, v_4, v_7, \dots, v_{3k-2}, v_{3k+1}\}$ will be a minimum weighted dominating set of G .

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally, it is r_1 . Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_4, \dots, v_{3k-2}, v_{3k+1}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 1 + [(l+1) + (l+4) + (l+7) + \dots + (l+3k-2)] + (2l+3k) = \frac{1}{6}(n+1)(n+2) + \frac{(n+5)l}{3}$

Case(3): Suppose $n \equiv 2 \pmod{3}$. Then $n = 3k + 2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+2}\}$ be the vertices of P_{3k+2} . As the maximum weighted vertex v_{3k+2} dominates the vertex v_{3k+1} , to dominate the maximum weighted vertex v_{3k} , choose the minimum weighted vertex v_{3k-1} for the γ_w -set so that the vertices v_{3k-2} and v_{3k} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3} , choose the minimum weighted vertex v_{3k-4} for the γ_w -set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3 , choose the minimum weighted vertex v_2 for the γ_w -set of G . Then the vertices v_1 and v_3 are dominated. Thus the set of vertices $\{v_{3k-1}, v_{3k-4}, \dots, v_6, v_2\}$ belongs to the γ_w -set of G . Since the vertices v_1 and v_{3k+2} are included in any weighted dominating set of G , the set $D = \{v_1, v_2, v_5, \dots, v_{3k}, v_{3k+2}\}$ will be a minimum weighted dominating set of G .

Clearly, the set D preserves the chromaticity of least weight in G . Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = (l+1) + [(l+2) + (l+5) + \dots + (l+3k-1)] + (2l+3k+1) = \frac{1}{6}(n+1)(n+2) + \frac{(n+7)l}{3}$. □

We next consider the case of caterpillars where $x_1 = l$ and $x_i = 0$ for $2 \leq i \leq n$. We observe that it can have three cases that is a path with n vertices where $n = 3k, 3k + 1, 3k + 2$ to determine $\gamma_{wch}(P_n(l, 0, 0, \dots, 0, 0))$.

Theorem 2.2. *For a caterpillar, $P_n(x_1, x_2, \dots, x_n)$, where $x_1 = l, x_i = 0$ for $2 \leq i \leq n$ of Type-I, then*

$$\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \begin{cases} \frac{1}{6}(n^2 + n + 6) + \frac{(n+3)l}{3} & \text{if } n \equiv 0 \pmod{3} \\ \frac{1}{6}(n^2 + n + 10) + \frac{(n+2)l}{3} & \text{if } n \equiv 1 \pmod{3} \\ \frac{1}{6}(n^2 + n + 6) + \frac{(n+1)l}{3} & \text{if } n \equiv 2 \pmod{3} \end{cases}$$

Proof. Let $G = (P_n(x_1, x_2, \dots, x_n), w)$ be a weighted caterpillar with $x_1 = l$ and $x_i = 0$ for $2 \leq i \leq n$. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$. Attach l pendent vertices, namely $\{r_1, r_2, \dots, r_l\}$, the left siblings, at v_1 as shown in Figure 2.2.

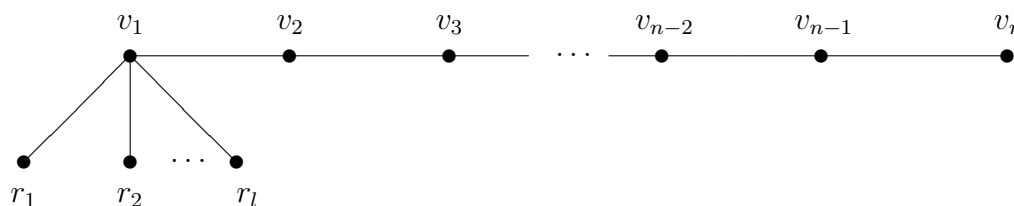


Figure 2.2: A caterpillar $P_n(l, 0, 0, \dots, 0, 0)$

Let $L = [1, 2, \dots, n]$ be a leaf-first labeling of $(P_n(x_1, x_2, \dots, x_n), w)$ and L is of Type-I.

If $l = 1$, then $G = P_n(x_1, x_2, \dots, x_n)$ reduces to a path on $n + 1$ vertices. Thus, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \gamma_{wch}(P_{n+1})$. By Theorem 1.3, the result is obvious.

Now we consider for $l > 1$. To dominate the left sibling vertices $\{r_1, r_2, \dots, r_l\}$, the minimum weighted vertex is v_1 . Thus v_1 must be in γ_w -set of G . We consider the following cases.

Case (1): Suppose $n \equiv 0 \pmod{3}$. Then $n = 3k$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k}\}$ be the vertices of P_{3k} . It is obvious that v_{3k} admits the maximum weight in G . Hence to dominate the vertex v_{3k} , choose the minimum weighted vertex v_{3k-1} for the γ_w -set. Then the vertices v_{3k-2} and v_{3k}

are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3} , choose the minimum weighted vertex v_{3k-4} for the γ_w -set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3 , choose the minimum weighted vertex v_2 for the γ_w -set of G . Then the vertices v_1 and v_3 are dominated. Thus the set of vertices $\{v_{3k-1}, v_{3k-4}, \dots, v_5, v_2\}$ belongs to the γ_w -set of G . Since the vertex v_1 is included in any weighted dominating set of G , the set $D = \{v_1, v_2, v_6, \dots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G .

Naturally, the set D preserves the chromaticity of least weight in G . Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = (l + 1) + [(l + 2) + (l + 5) + \dots + (l + 3k - 1)] = \frac{1}{6}(n^2 + n + 6) + \frac{(n+3)l}{3}$.

Case (2): Suppose $n \equiv 1 \pmod{3}$. Then $n = 3k + 1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+1}\}$ be the vertices of P_{3k+1} . It is obvious that v_{3k+1} admits the maximum weight in G . Hence to dominate the vertex v_{3k+1} , choose the minimum weighted vertex v_{3k} for the γ_w -set. Then the vertices v_{3k-1} and v_{3k+1} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-2} , choose the minimum weighted vertex v_{3k-3} for the γ_w -set so that the vertices v_{3k-4} and v_{3k-2} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4 , choose the minimum weighted vertex v_3 for the γ_w -set of G . Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k}, v_{3k-3}, \dots, v_6, v_3\}$ belongs to the γ_w -set of G . Since the vertex v_1 is included in any weighted dominating set of G , the set $\{v_1, v_3, v_6, \dots, v_{3k-3}, v_{3k}\}$ will be a minimum weighted dominating set of G .

For chromatic preserving, a least weight vertex is to be added which is r_1 . Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_3, v_6, \dots, v_{3k-3}, v_{3k}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 1 + (l + 1) + [(l + 3) + (l + 6) + \dots + (l + 3k)] = \frac{1}{6}(n^2 + n + 10) + \frac{(n+2)l}{3}$

Case (3): Suppose $n \equiv 2 \pmod{3}$. Then $n = 3k + 2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+2}\}$ be the vertices of P_{3k+2} . It is obvious that v_{3k+2} admits the maximum weight in G . Hence to dominate the vertex v_{3k+2} , choose the minimum weighted vertex v_{3k+1} for the γ_w -set. Then the vertices v_{3k} and v_{3k+2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-1} , choose the minimum weighted vertex v_{3k-2} for the γ_w -set so that the vertices v_{3k-3} and v_{3k-1} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_5 , choose the minimum weighted vertex v_4 for the γ_w -set of G . Then the vertices v_3 and v_5 are dominated. Thus the set of vertices $\{v_{3k+1}, v_{3k-2}, \dots, v_7, v_4\}$ belongs to the γ_w -set of G . Since the vertex v_1 is included in any weighted dominating set of G , the set $\{v_1, v_4, v_7, \dots, v_{3k-2}, v_{3k+1}\}$ will be a minimum weighted dominating set of G .

For chromatic preserving, a least weight vertex is to be added, Naturally, it is r_1 . Thus the least weight dom-chromatic set D is $\{r_1, v_1, v_4, v_7, \dots, v_{3k-2}, v_{3k+1}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 1 + [(l+1) + (l+4) + \dots + (l+3k+1)] = \frac{1}{6}(n^2 + n + 6) + \frac{(n+1)l}{3}$

□

Next let us consider the case of caterpillars where $x_n = l$ and $x_i = 0$ for $1 \leq i \leq n - 1$. We observe that it can have three cases that is a path with n vertices where $n = 3k, 3k + 1, 3k + 2$ to determine $\gamma_{wch}(P_n(0, 0, \dots, 0, l))$.

Theorem 2.3. *For a caterpillar, $P_n(x_1, x_2, \dots, x_n)$, where $x_n = l, x_i = 0$ for $1 \leq i \leq n - 1$ of Type-I, then*

$$\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \begin{cases} \frac{1}{6}(n^2 + 3n + 6) + l & \text{if } n \equiv 0 \pmod{3} \\ \frac{1}{6}(n^2 + 3n + 8) + l & \text{if } n \equiv 1 \pmod{3} \\ \frac{1}{6}(n^2 + 3n + 2) + l & \text{if } n \equiv 2 \pmod{3} \end{cases}$$

Proof. : Let $G = (P_n(x_1, x_2, \dots, x_n), w)$ be a weighted caterpillar with $x_n = l$ and $x_i = 0$ for $1 \leq i \leq n - 1$. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$. Attach l

pendent vertices, namely $\{s_1, s_2, \dots, s_l\}$, the right siblings, at v_n as shown in Figure 2.3

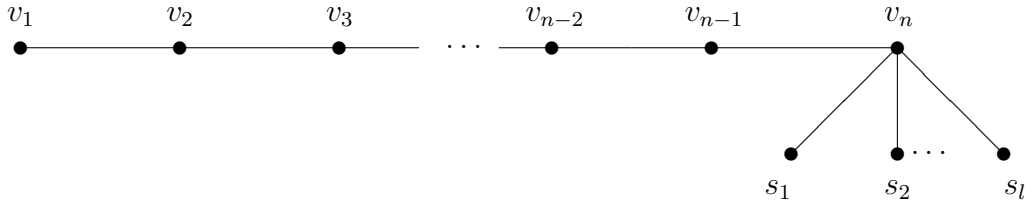


Figure 2.3: A caterpillar $P_n(0, 0, \dots, 0, l)$

Let $L = [1, 2, \dots, n]$ be a leaf-first labeling of $(P_n(x_1, x_2, \dots, x_n), w)$ and L is of Type-I.

If $l = 1$, then $G = P_n(x_1, x_2, \dots, x_n)$ reduces to a path on $n + 1$ vertices. Thus, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = \gamma_{wch}(P_{n+1})$. By Theorem 1.3, the result is obvious.

Now we consider for $l > 1$. To dominate the right sibling vertices $\{s_1, s_2, \dots, s_l\}$, the minimum weighted vertex is v_n . Thus v_n must be in γ_w -set of G . We consider the following cases.

Case (1): Suppose $n \equiv 0(mod 3)$. Then $n = 3k$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k}\}$ be the vertices of P_{3k} . Since v_{3k} dominates v_{3k-1} , to dominate the maximum weighted vertex v_{3k-2} , choose the minimum weighted vertex v_{3k-3} for the γ_w -set. Then the vertices v_{3k-4} and v_{3k-2} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-5} , choose the minimum weighted vertex v_{3k-6} for the γ_w -set so that the vertices v_{3k-7} and v_{3k-5} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_4 , choose the minimum weighted vertex v_3 for the γ_w -set of G . Then the vertices v_2 and v_4 are dominated. Thus the set of vertices $\{v_{3k}, v_{3k-3}, \dots, v_6, v_3\}$ belongs to the γ_w -set of G .

Now, by choosing the minimum weighted vertex v_1 to the γ_w -set, it is necessary to select the vertex v_2 for the chromatic preservation. Hence, the weighted vertices v_1, v_2 alongwith v_3 contributes a weight of 6 to the γ_{wch} -set. While, we choose the weighted vertex v_2 for γ_w -set, it dominates

v_1 and the γ_w -set unioned with v_2 preserves the chromaticity. Moreover, it contributes a minimum weight of 5 to the γ_{wch} -set. Hence, any γ_w -set unioned with v_2 will be the least weight dom-chromatic set of G . Thus, $D = \{v_2, v_3, v_6, \dots, v_{3k-3}, v_{3k}\}$ is a minimum weighted dom-chromatic set in G .

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 2 + (3 + 6 + \dots + 3k - 3) + (3k - 1 + l) = l + \frac{1}{6}(n^2 + 3n + 6)$

Case (2): Suppose $n \equiv 1 \pmod{3}$. Then $n = 3k + 1$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+1}\}$ be the vertices of P_{3k+1} . Since v_{3k+1} dominates v_{3k} , to dominate the maximum weighted vertex v_{3k-1} , choose the minimum weighted vertex v_{3k-2} for the γ_w -set. Then the vertices v_{3k-3} and v_{3k-1} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-4} , choose the minimum weighted vertex v_{3k-5} for the γ_w -set so that the vertices v_{3k-6} and v_{3k-4} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_2 , choose the minimum weighted vertex v_1 for the γ_w -set of G . Then the vertex v_2 is dominated. Thus the set of vertices $\{v_{3k+1}, v_{3k-2}, \dots, v_4, v_1\}$ belongs to the γ_w -set of G .

For chromatic preserving, a least weight vertex is to be added which is v_2 . Therefore the least weight dom-chromatic set D is $\{v_1, v_2, v_4, \dots, v_{3k-2}, v_{3k+1}\}$. Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 2 + (1 + 4 + \dots + 3k - 2) + 3k + l = l + \frac{1}{6}(n^2 + 3n + 8)$.

Case (3): Suppose $n \equiv 2 \pmod{3}$. Then $n = 3k + 2$ for some integer $k \geq 1$. Let $\{v_1, v_2, \dots, v_{3k+2}\}$ be the vertices of P_{3k+2} . Since v_{3k+2} dominates v_{3k+1} , to dominate the maximum weighted vertex v_{3k} , choose the minimum weighted vertex v_{3k-1} for the γ_w -set. Then the vertices v_{3k-2} and v_{3k} are dominated. Similarly, to dominate the maximum weighted vertex v_{3k-3} , choose the minimum weighted vertex v_{3k-4} for the γ_w -set so that the vertices v_{3k-5} and v_{3k-3} are dominated.

Proceeding in this way, to dominate the maximum weighted vertex v_3 , choose the minimum weighted vertex v_2 for the γ_w -set of G . Then the vertices

v_1 and v_3 is dominated. Thus the set of vertices $\{v_{3k+2}, v_{3k-1}, \dots, v_5, v_2\}$ belongs to the γ_w -set of G .

For chromatic preserving, a least weight vertex is to be added which is v_1 . Therefore the least weight dom-chromatic set D is $\{v_1, v_2, v_5, \dots, v_{3k-1}, v_{3k+2}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = 1 + (2 + 5 + \dots + 3k - 1) + 3k + 1 + l = l + \frac{1}{6}(n^2 + 3n + 2)$. \square

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