

# Weighted dom-chromatic number of Type-I weighted complete caterpillars

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## Abstract

A set  $D$  of vertices is a dominating set of  $G$  if every vertex not in  $D$  is adjacent to at least one member of  $D$ . A set  $D$  of vertices is said to be dom-chromatic if  $D$  is a dominating set and  $\chi(\langle D \rangle) = \chi(G)$ . A Weighted tree,  $(T, w)$  a tree together with a positive weight function on the vertex set  $w : V(T) \rightarrow R^+$ . The weighted domination number  $\gamma_w(T)$  of  $(T, w)$  is the minimum weight  $w(D) = \sum_{v \in D} w(v)$  of a dominating set  $D$  of  $T$ . The weighted dom-chromatic number  $\gamma_{wch}(T)$  of  $(T, w)$  is the minimum weight  $w(D) = \sum_{v \in D} w(v)$  of a dom-chromatic set  $D$  of  $T$ . A caterpillar is a graph which can be obtained from the path on  $n$  vertices by appending  $x_i$  pendant vertices to the  $i^{th}$  vertex of the path,  $P_n$ . The caterpillar with parameters  $n, x_1, x_2, \dots, x_n$  will be denoted as  $P_n(x_1, x_2, \dots, x_n)$ . In this paper, the weighted dom-chromatic numbers are determined for some Type-I weighted complete caterpillars.

**Keywords:** dominating set, dom-chromatic set, weighted domination, weighted dom-chromatic number, Type-I weighted labeling

**MSC Subject Classification:** 05C69

## 1 Introduction

A set  $S$  of vertices is a *dominating set* of  $G$  if every vertex not in  $S$  is adjacent to at least one member of  $S$ . The minimum cardinality of a dominating set in

$G$  is called the *domination number* and is denoted by  $\gamma(G)$ . The set  $\mathcal{D}(G)$  is the collection of all dominating sets of  $G$ . A subset  $S$  of  $V$  is said to be a *dom-chromatic set* (or *dc-set*) if  $S$  is a dominating set and  $\chi(\langle S \rangle) = \chi(G)$ . The minimum cardinality of a dom-chromatic set in  $G$  is called the *dom-chromatic number* (or *dc-number*) and is denoted by  $\gamma_{ch}(G)$ . The set  $\mathcal{D}_{ch}(G)$  is the collection of all dom-chromatic sets of  $G$ .

A *Weighted tree*,  $(T, w)$  a tree together with a positive weight function on the vertex set  $w : V(T) \rightarrow R^+$ . The *weighted domination number*  $\gamma_w(T)$  of  $(T, w)$  is the minimum weight  $w(D) = \sum_{v \in D} w(v)$  of a dominating set  $D$  of  $T$ . The *weighted dom-chromatic number*  $\gamma_{wch}(T)$  of  $(T, w)$  is the minimum weight  $w(D) = \sum_{v \in D} w(v)$  of a dom-chromatic set  $D$  of  $T$ .

P. Palanikumar and S. Balamurugan [13] has introduced the concept of *Type-I weighted labeling* and they study the weighted dom-chromatic number of a weighted tree and determine the weighted dom-chromatic number of a *Type I* weighted paths. Also, they found the Weighted dom-chromatic number of some classes of Type-I weighted caterpillars in [14]. In this paper, the weighted dom-chromatic numbers are determined for some Type-I weighted complete caterpillars.

**Theorem 1.1.** [13] *Let  $(T, w)$  be a weighted tree and  $[1, 2, \dots, n]$  be a leaf-first labeling of  $T$  Where  $w(i) = w_i$  for  $i = 1, 2, \dots, n$ . If  $i$  is a leaf of  $T$  then  $\eta_{ch}(i) = w_i$ ;  $\theta_{ch}(i) = 0$ ;  $\lambda_{ch}(i) = w_i$ ;  $\mu_{ch}(i) = 0$ .*

**Definition 1.2.** [13] *Let  $(T, w)$  be a weighted tree and  $[1, 2, \dots, n]$  be a leaf-first labeling of  $(T, w)$ . Then  $L$  is said to be of *Type-I* if  $i$  is the first leaf of  $T - \{1, 2, \dots, i - 1\}$  from left.*

**Theorem 1.3.** [13] *For a path  $P_n, (n \geq 3)$  of Type-I,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}(n^2 + n + 6) & \text{if } n \equiv 0(\text{mod } 3) \\ \frac{1}{6}(n^2 + n + 10) & \text{if } n \equiv 1(\text{mod } 3) \\ \frac{1}{6}(n^2 + n + 12) & \text{if } n \equiv 2(\text{mod } 3) \end{cases}$$

## 2 Complete Caterpillar

A caterpillar is a graph which can be obtained from the path on  $n$  vertices by appending  $x_i$  pendant vertices to the  $i^{th}$  vertex of the path,  $P_n$ . The caterpillar with parameters  $n, x_1, x_2, \dots, x_n$  will be denoted as  $P_n(x_1, x_2, \dots, x_n)$ .

Note, this is a tree with the property that the removal of its leaves and incident edges results in a path  $P_n$  called the *spine* of the caterpillar. Let  $l$  denote the number of leaves, i.e.,  $l = \sum_{i=1}^n x_i$ . We say a caterpillar is complete if every vertex on the spine of the caterpillar is adjacent to at least one leaf.

Also, a caterpillar  $P_n(x_1, x_2, \dots, x_n)$ , is complete if the  $x_i > 0$  for  $1 \leq i \leq n$ . First, let us consider the class of complete caterpillars where  $x_i = 1$  for  $1 \leq i \leq n$ , that is, having only one leaf attached to each vertex on the spine of the caterpillar. Examples are given in Figure 2.1.

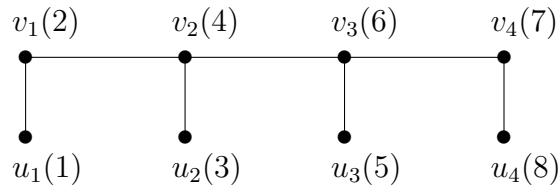


Figure 2.1: A complete caterpillar  $P_4(1, 1, 1, 1)$

As an example of Type -I weighted complete caterpillar  $P_4(1, 1, 1, 1)$ , shown in Figure 2.1. Let  $P_4(1, 1, 1, 1)$  have the vertices  $\{v_1, v_2, v_3, v_4, u_1, u_2, u_3, u_4\}$  as shown in Figure 2.1. The minimum weighted dom-chromatic set is  $D = \{v_1, u_1, u_2, u_3, v_4\}$  and  $w(D) = 1+2+3+5+7 = 18$ . Thus  $\gamma_{wch}(P_4(1, 1, 1, 1)) = 18$ .

Observe that the weighted dom chromatic number of the above caterpillars is the sum of the weight of the pendent vertices of  $P_4$  and weight of the leaves except the maximum weight of the leaf.

Now, we are ready to prove Proposition.

**Theorem 2.1.** For a caterpillar,  $P_n(x_1, x_2, \dots, x_n)$ , where  $x_i = 1$  for  $1 \leq i \leq n$  of Type-I, then  $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = n^2 + 2$

*Proof.* Let  $G = (P_n(x_1, x_2, \dots, x_n), w)$  be a weighted complete caterpillar with  $x_i = 1$  for  $1 \leq i \leq n$ . Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ . Attach one pendent vertex  $u_j$  for  $1 \leq j \leq n$  at each  $v_i$  for  $1 \leq i \leq n$  as shown in Figure 3.2. Then  $V((P_n(x_1, x_2, \dots, x_n))) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ . Thus  $|V((P_n(x_1, x_2, \dots, x_n)))| = 2n$ .

let  $L = [1, 2, \dots, n]$  be a leaf-first labeling of  $(P_n(x_1, x_2, \dots, x_n), w)$  and  $L$  is of Type-I.

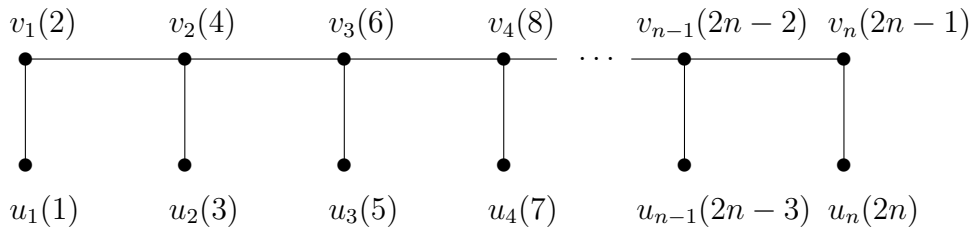


Figure 2.2: A complete caterpillar  $P_n(1, 1, \dots, 1)$

To dominate the maximum weighted vertex  $u_n$ , choose the minimum weighted vertex is  $v_n$  for the  $\gamma_w$ -set of  $G$ . A dominating set of  $G$  either contains a non-end vertex  $v$  or the end vertex of  $v$ . This implies that the weighted vertex  $v_n$  unioned with the set of leaves except  $u_n$  will be a minimum weighted dominating set of  $G$ . Thus, the minimum weighted dominating set of  $G$  is  $\{u_1, u_2, \dots, u_n, v_n\}$ . For chromatic preservation, add a neighbor of least weight vertex to this set. Naturally, it is  $v_1$ . Therefore, the least weight dom-chromatic set is  $D = \{v_1, u_1, u_2, \dots, u_n, v_n\}$ . Hence the minimum weight of a dom-chromatic set is,  $\gamma_w(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = (1 + 3 + 5 + \dots + 2n - 3 + 2n - 1) + 2 = n^2 + 2$ .  $\square$

Our next result of interest gives the class of complete caterpillars with  $x_i = l$  where  $l > 1$  for  $1 \leq i \leq n$ , that is , having  $k$  leaves attached to each vertex on the spine of the caterpillar. We observe that the weighted dom-chromatic number of the above caterpillars is the sum of the weights of the vertices of  $P_n$ .

Now, we are ready to prove our theorem.

**Theorem 2.2.** For a caterpillar,  $P_n(x_1, x_2, \dots, x_n)$ , where  $x_i = l$ , where  $l > 1$  for  $1 \leq i \leq n$  of Type-I, then  $\gamma_{wch}(P_n(x_1, x_2, \dots, x_n)) = (n+1)\frac{|V(G)|}{2} - 1$

*Proof.* Let  $G = (P_n(x_1, x_2, \dots, x_n), w)$  be a weighted complete caterpillar with  $x_i = l$ , where  $l > 1$  for  $1 \leq i \leq n$ . Let  $V(P_n) = \{v_1, v_2, \dots, v_n\}$ . Attach  $l$  pendent vertex  $u_{i,1}, u_{i,2}, \dots, u_{i,l}$  at each  $v_i$  for  $1 \leq i \leq n$  as shown in Figure 2.3. Then  $V((P_n(x_1, x_2, \dots, x_n))) = \{v_i, u_{i,j} : 1 \leq i \leq n ; 1 \leq j \leq l\}$ . Thus  $|V((P_n(x_1, x_2, \dots, x_n)))| = n(l+1)$ .

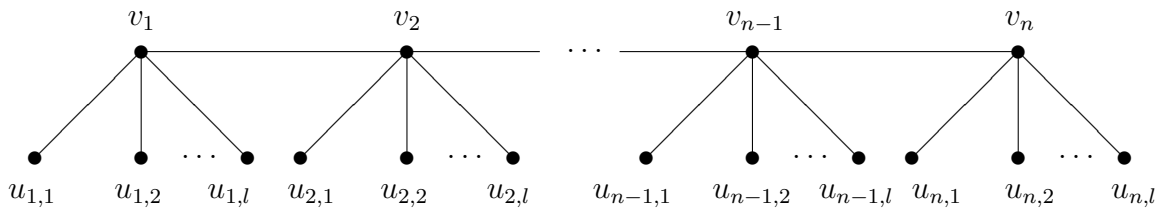


Figure 2.3: A complete caterpillar  $P_n(l, l, \dots, l)$

Let  $L = [1, 2, \dots, n]$  be a leaf-first labeling of  $(P_n(x_1, x_2, \dots, x_n), w)$  and  $L$  is of Type-I as shown in Figure 2.4

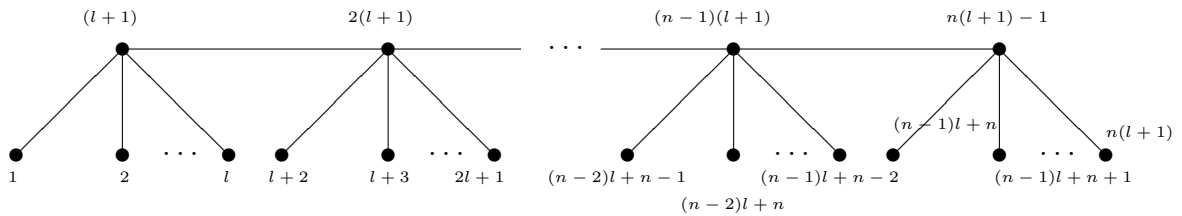


Figure 2.4: A weighted complete caterpillar  $P_n(l, l, \dots, l)$

A dominating set of  $G$  either contains a non-end vertex or the end vertex. It is easily observe that the set of vertices of  $P_n$  is a minimum weighted dominating set of  $G$ . Also, this set preserves the minimum weighed dom-chromatic in  $G$ . Thus, the minimum weighted dom-chromatic set of  $G$  is  $D = \{v_1, v_2, \dots, v_n\}$ . Hence the minimum weight of a dom-chromatic set is,  $\gamma_w(P_n(x_1, x_2, \dots, x_n)) = w(D) = \sum w(v_i) = (l+1) + 2(l+1) + 3(l+1) + \dots + (l+1)(n-1) + [n(l+1) - 1] = (n+1)\frac{|V(G)|}{2} - 1$ .  $\square$

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