Travelling Salesman Problem Using Branch And Bound Technique

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Abstract: *In this paper Branch and bound technique is applied to solve the Travelling Salesman Problem (TSP) whose objective is to minimize the cost. Given a set of cities and the distance between every pair of cities, the problem of finding the shortest route between a set of points and locations that must be visited. Here fuzzy cost is considered as an interval number and further they are transformed into a crisp problem that is solved by branch and bound technique. This is illustrated by means of a numerical example.*

Keywords: *Interval numbers, Hexagonal numbers, Branch and Bound technique, ranking function.*

I. INTRODUCTION

The branch and bound technique can be viewed as a generalization of backtracking for the optimization problems. A salesman has a given tour of a specified number of cities with distances between every pair of cities, starting from any one of these cities; he must make a tour, visiting each of the other cities only once and reach the final city. This task should be achieved in such a way that the total distance travelled should be minimized. The traveling salesman problem (TSP) is an algorithmic problem when focused on optimization; TSP is often used in computer science to find the most efficient route for data to travel between various nodes. Several applications like identifying network or hardware optimization methods etc. and so on. TSP was first developed by W.R. Hamilton, Irish mathematician, and Thomas Kirkman a British mathematician in the 1800s through the creation of a game that was solvable by finding a Hamilton cycle, which is a non-overlapping path between all nodes.TSP, has been studied by several researchers for decades and many solutions have been theorized. The simplest solution is to try all possibilities, but this is also the most time consuming and expensive method. Many problems include heuristics methods which provide probability outcomes. However, the results are approximate and not always optimal. In this paper we used branch and bound method to find the optimal solution.

II. PRELIMINARIES

Definition: **2.1**

An interval number A is defined as $A= [a, b] =\{x / a \le x \le b, x\in\Re \}$. Here $a, b\in\Re$ are the lower and upper bound of the intervals.

Definition: 2.2 (Hexagonal fuzzy number)

A fuzzy number $\overline{A}_H$ is a hexagonal fuzzy number denoted by $\overline{A}_H =(a_1,a_2,a_3,a_4,a_5,a_6)$ where a_1,a_2,a_3,a_4,a_5,a_6 are real numbers and , its membership function $\mu_{\overline{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ 1 & a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ 0 & x > a_6 \end{cases}$$

Definition: 2.3 (Ranking Function)

We define a ranking function R: F(R)⟶ R which maps each fuzzy numbers to real line F(R) represent the set of all hexagonal fuzzy numbers. If R be any linear ranking functions, then

$$R(\overline{A}_H) = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}$$

Definition: 2.4

Given a fuzzy set 'A' defined on 'X' and any number α ∈ [0,1], the α –cut is denoted by A(α) and is defined by the crisp set $A(\alpha) = \{x: A(x) \geq \alpha\}$. i.e. A (α) ={x: μ(x) ≥ α, α ∈ [0, 1]}

Definition: 2.5 (Fuzzification method)

A new approach is used to fuzzify the given interval data into a hexagonal fuzzy number. Consider an interval number [L, U]. The difference of this interval is d = $\frac{U-L}{5}$.The required hexagonal fuzzy number will be in arithmetic progression.

III. PROPOSED METHOD

The branch and bound technique is used to solve the travelling salesman problem. The proposed method is as follows.

We grow a tree of partial solutions. At each node of the recursion tree, we check whether the current partial solution can be extended to a solution which is better than the best solution found so far. If not, don't continue this branch.

The cost matrix is defined by

$$C(i,j) = w(i,j), \text{ if there is a direct path from } c_i \text{ to } c_j$$
$$= \infty \text{ otherwise.}$$

when we go from city i to city j,

Cost of node j=cost of parent node i+ cost of the edge (i,j)+lower bound of the path starting at node j.

The root node is the first node that has no parent so the cost will be only the LB of the path starting at the root.

To find LB:

1. Reduce the minimum value in each row from each element in that row. Similarly for column also do the same procedure.

2.The total expected cost of the root node is the sum of all reductions.

Remaining calculations are explained in the numerical example.

IV. NUMERICAL EXAMPLE

Consider an interval integer transportation problem

	A	B	C	D	E
A	∞	[19,21]	[29,31]	[9,11]	[10,12]
B	[14,16]	∞	[15,17]	[3,5]	[1,3]
C	[2,4]	[4,6]	∞	[1,3]	[3,5]
D	[18,20]	[5,7]	[17,19]	∞	[2,4]
E	[15,17]	[3,5]	[6,8]	[15,17]	∞

Solution:

Converting interval numbers into hexagonal numbers using [2.5]

[19,21]	(19,19.4,19.8,20.2,20.6,21)	[9,11]	(9,9.4,9.8,10.2,10.6,11)
[29,31]	(29,29.4,29.8,30.2,30.6,31)	[3,5]	(3,3.4,3.8,4.2,4.6,5)
[10,12]	(10,10.4,10.8,11.2,11.6,12)	[1,3]	(1,1.4,1.8,2.2,2.6,3)
[14,16]	(14,14.4,14.8,15.2,15.6,16)	[2,4]	(2,2.4,2.8,3.2,3.6,4)
[15,17]	(15,15.4,15.8,16.2,16.6,17)	[4,6]	(4,4.4,4.8,5.2,5.6,6)
[18,20]	(18,18.4,18.8,19.2,19.6,20)	[5,7]	(5,5.4,5.8,6.2,6.6,7)
[17,19]	(17,17.4,17.8,18.2,18.6,19)	[6,8]	(6,6.4,6.8,7.2,7.6,8)

Converting Hexagonal numbers into crisp numbers using [2.3]

	A	B	C	D	E
A	∞	20	30	10	11
B	15	∞	16	4	2
C	3	5	∞	2	4
D	19	6	16	∞	3
E	16	4	7	16	∞

Step: 1

After row reduction and column reduction we have reduced matrix as

	A	B	C	D	E	Reduction
A	∞	10	17	0	1	10
B	12	∞	11	2	0	2
C	0	3	∞	0	2	2
D	5	3	10	∞	0	3
E	11	0	0	12	∞	4
Reduction	1	0	3	0	0	

Cost of node [1] = Σ(reduction) ⇒**25**

Step:2

Path A → B

From the reduced matrix of step: 1, V [A, B] =10

Set row A column B to ∞

Set V [A, B] = ∞

Cost of node [2] = cost of node [1] + V [A, B] + reduction cost.

= 25 + 10 + 0

⇒ 35

Similarly

Path A→ C

Cost of node [3] = 25+17+11⇒53

Path A→ D

Cost of node [4] = 25+0+0⇒**25**

Path A→ E

Cost of node [5] = 25+1+5 ⇒31.

Reduced matrix after step: 2 is

	A	B	C	D	E
A	∞	∞	∞	∞	∞
B	12	∞	11	∞	0
C	0	3	∞	∞	2
D	∞	3	12	∞	0
E	11	0	0	∞	∞

Step: 3

Path A→ D → B

Set rows A, D& column B to ∞

Set V [D, A] =∞, V [B, A] = ∞

Cost of node [6] = cost of node [4] + V [D, B] + reduction cost.

= 25+3+0

⇒**28**

Similarly,

Path A→ D → C

Cost of node [7] = 25+12+13⇒ 50

Path A→ D → E

Cost of node [8] = 25+0+11⇒ 36

The reduced matrix after step: 3 is

	A	B	C	D	E
A	∞	∞	∞	∞	∞
B	∞	∞	11	∞	0
C	0	∞	∞	∞	2
D	∞	∞	∞	∞	∞
E	11	∞	0	∞	∞

Step: 4

Path A→ D → B→C

Set rows A, D, B& column C to ∞

Set V [D, A] =∞, V [B, A] = ∞, V[C, A] = ∞

Cost of node [9] = cost of node [6] + V [B, C] + reduction cost.

= 28+11+13

⇒52

Similarly,

Path A→ D → B→E

Cost of node [10] = 28+0+0 ⇒**28**

After step: 4 the reduced matrix is

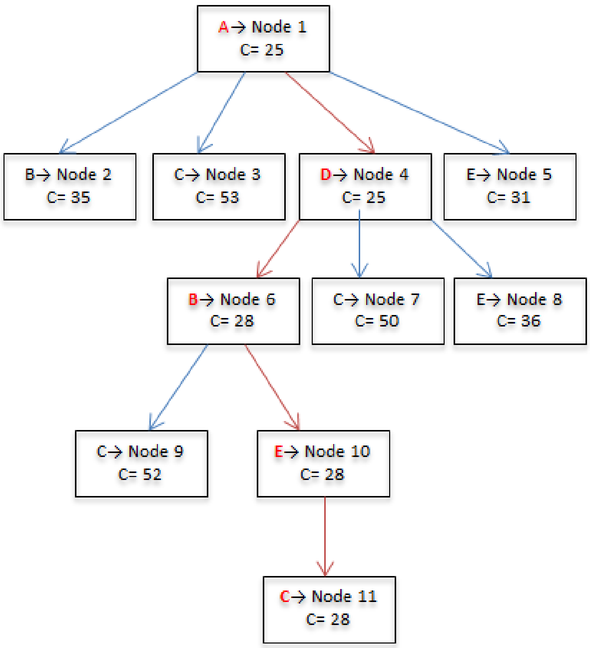
	A	B	C	D	E
A	∞	∞	∞	∞	∞
B	∞	∞	∞	∞	∞
C	0	∞	∞	∞	∞
D	∞	∞	∞	∞	∞
E	∞	∞	0	∞	∞

Step: 5

Path A→ D → B→E→C

Set rows A, D, B, E& column C to ∞
Set V[D,A] =∞ ,V[B,A] = ∞,V[E,A]= ∞,V[C,A]= ∞
Cost of node [11] = cost of node [10] + V [E, C] + reduction cost.
= 28+0+0
⇒**28**
The optimal route isA→ D → B→E→C→A.Hence the route condition is satisfied.
The optimal cost = 28.

Fig: 1



V. CONCLUSION

Several methods are available for solving the Travelling salesman problem. In this paper Branch and bound technique is considered for finding the optimal route. At each step the cost matrix is calculated. The initial stage cost which is not the exact cost but it gives some idea to the remaining problem is calculated first. Using this we find the cost of remaining nodes and identify the optimum one. This is verified by means of a numerical example.

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