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Weighted dom-chromatic number of Type-II weighted paths

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Abstract

A set D of vertices is a dominating set of G if every vertex not in D is adjacent to at least one member of D. A set D of vertices is said to be dom-chromatic if D is a dominating set and $\chi(\langle D \rangle) = \chi(G)$. A Weighted tree, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \longrightarrow R^+$. The weighted domination number $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dominating set D of T. The weighted dom-chromatic number $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dom-chromatic set D of T. In this paper, we will study the weighted dom-chromatic number of weighted paths. Also, we introduce a new way of weights on the vertices called *Type II weighted labeling*. We determine the weighted dom-chromatic number of *Type II* weighted paths.

Keywords: weighted domination, weighted dom-chromatic number, Type-I weighted labeling

MSC Subject Classification: 05C69

1 Introduction

A set S of vertices is a *dominating set* of G if every vertex not in S is adjacent to at least one member of S. The minimum cardinality of a dominating set in G is called the *domination number* and is denoted by $\gamma(G)$. The set $\mathcal{D}(G)$ is the collection of all dominating sets of G. A subset S of V is said to be a *dom-chromatic set* (or *dc-set*) if S is a dominating set and $\chi(< S >) = \chi(G)$. The minimum cardinality of a dom-chromatic set in G is called the *dom-chromatic number* (or *dc-number*) and is denoted by $\gamma_{ch}(G)$. The set $\mathcal{D}_{ch}(G)$ is the collection of all dom-chromatic sets of G

Some totally different dominatin concepts which involving weights on the vertices of the underlying graphs such as *signed domination*([5]), *minus domination* ([4]) or *majority domination* ([2]) etc have been studied. Peter Dankelmann [3] studied a very natural generalization of the classical domination number for weighted graphs. Instead of considering a dominating set of small cardinality they considered dominating sets of small weight and they precisely defined the *minimum weighted dominating set* in [3]. In this paper, we mainly focus on weighted trees.

A *Weighted tree*, (T, w) a tree together with a positive weight function on the vertex set $w : V(T) \longrightarrow R^+$. The *weighted domination number* $\gamma_w(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dominating set D of T. The *weighted dom-chromatic number* $\gamma_{wch}(T)$ of (T, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a dom-chromatic set D of T.

Min-Jen Jou and Jenq-Jong Lin [13] introduced the concept of *leaf-first labeling* and they provided the liner-time algorithms for findng the weighted domination number and weighted independent domination number of a weighted tree. The *leaf-first labeling* $[1, 2, \ldots, n]$ of a tree T is a vertex labeling such that the vertex 1 is a leaf of T and i is a leaf of the subgraph $T - \{1, 2, \ldots, i-1\}$ for $i = 2, 3, \ldots, n-1$. For $i \geq 1$, the subgraph T_i is the maximal connected graph of T including the vertex i and some vertices $j < i$.

P. Palanikumar and S. Balamurugan [15] has introduced the concept of *Type-I weighted labeling* and they study the weighted dom-chromatic number of a weighted tree and determine the weighted dom-chromatic number of a *Type I* weighted paths. Also, they found the Weighted dom-chromatic number of some classes of Type-I weighted caterpillars in [16] and Weighted dom-chromatic number of Type-I weighted complete caterpillars in [17]. In this paper, we will study the weighted dom-chromatic number of a weighted path. Also, we introduce a new way of weights on the vertices called *Type II weighted labeling.* We determine the weighted dom-chromatic number of *Type II* weighted paths.

Theorem 1.1. *[15]*

Let (T, w) *be a weighted tree and* $[1, 2, \ldots, n]$ *be a leaf-first labeling of* T *Where* $w(i) = w_i$ *for* $i = 1, 2, \ldots, n$. *If* i *is a leaf of* T *then* $\eta_{ch}(i) = w_i$; $\theta_{ch}(i) = 0$; $\lambda_{ch}(i) = w_i$; $\mu_{ch}(i) = 0$.

Theorem 1.2. *[15] For a path* P_n,$(n \geq 3)$ *of Type-I,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}\left(n^2 + n + 6\right) & \text{if } n \equiv 0 (mod\ 3) \\ \frac{1}{6}\left(n^2 + n + 10\right) & \text{if } n \equiv 1 (mod\ 3) \\ \frac{1}{6}\left(n^2 + n + 12\right) & \text{if } n \equiv 2 (mod\ 3) \end{cases}$$

2 *Type II* Weighted Tree

In this section, we introduce a new way of weights on the vertices called *Type-II weighted labeling* and obtain the weighted dom-chromatic number of *Type-II* weighted trees and *Type-II* weighted paths.

Definition 2.1. *Let* (T, w) *be a weighted tree and* $[1, 2, \ldots, n]$ *be a leaf-first labeling of* (T, w). *Then* L *is said to be of Type-II weighted labeling is defined as follows. Let* P_1 *be the set of all leaves of the tree* T. *Label the leaves in* P_1 *from left to right with the numbers* $1, 2, 3, \ldots, |P_1|$. *Let* $T_1 = T - P_1$. *Let* P_2 *be the set of leaves of* T_1. *Label the leaves in* P_2 *from left to right with numbers* $|P_1| + 1, |P_1| + 2, \ldots, |P_1| + |P_2|$. *Proceeding in this way, let at the* i^{th} *stage,* $T_i = T - (P_1 \cup P_2 \cup \ldots \cup P_i)$ *label the leaves* P_{i+1} *of* T_i *from left to right with numbers* $|P_1| + |P_2| + \ldots + |P_i| + 1$ *to* $|P_1| + |P_2| + \ldots + |P_i| + |P_{i+1}|$. *Proceed in this way, till all vertices are labeled.*

Here, we find the weighted dom-chromatic number of Type-II weighted tree.

Example 2.2. *Consider the following Type-II weighted tree. We now find the weighted dom-chromatic number of given weighted tree.*

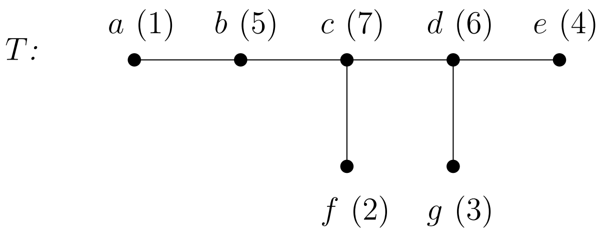

Figure 2.1: Example of Type-II weighted tree

As an example of Type-II weighted tree, shown in Figure 2.1 Let T have vertices $\{a, b, c, d, e, f, g\}$ as shown in Figure 2. The dom-chromatic vertex sets of T are $\{a, c, d\}$, $\{b, c, d\}$, $\{b, c, e, g\}$,$\{a, f, d, g\}$, $\{a, f, d, e\}$, $\{a, b, f, d\}$ and $\{a, b, f, e, g\}$ and thus $\gamma_{ch}(T) = 3$. The weighted dom-chromatic number of T can be found by checking their weights. They are $14, 18, 19, 12, 13, 14$ and 15 respectively. Thus $\gamma_{wch}(T) = 12$, the minimum weight of these dom-chromatic set, which can be found using $\{a, f, d, g\}$.

Next, we find the weighted dom-chromatic number of Type II weighted paths P_n. The path graph P_n is a tree with two end vertices of degree 1, and the other $n - 2$ vertices of degree 2. we consider a path P_n with $n = 3k$ for some integer $k > 0$. In this case, we have a pattern for odd and even vertices for a path graph P_n where $n = 3k$ to determine $\gamma_{wch}(P_n)$. From the above pattern, we have the followng theorem.

Theorem 2.3. *For a path P_n, where $n \equiv 0(mod\ 3)$ of Type-II,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}\left(n^2 + n + 6\right) & \text{if } n \text{ is even} \\ \frac{1}{6}\left(n^2 + n + 12\right) & \text{if } n \text{ is odd} \end{cases}$$

Proof. Let (P_n, w) be a weighted path and $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ such that $w(v_i) = j$, for all $i, j = 1, 2, \ldots, n$ and let $L = [1, 2, \ldots, n]$ be a leaf-first labeling of (P_n, w) and L is of Type-II. Let $n \equiv 0$ $(mod\ 3)$. Then $n = 3k$ for some integer $k \geq 1$. we consider the following cases.

Case(1): Suppose n is even. Then $n = 3k$ for some even integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertices of P_{3k}. By the definition of Type II weighted labeling, $v_{\frac{3k}{2}+1}$ admits the maximum weight of G and hence to dominate the maximum weighted vertex $v_{\frac{3k}{2}+1}$, choose the minimum weighted vertex $v_{\frac{3k}{2}+2}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k}{2}+1}$ and $v_{\frac{3k}{2}+3}$ are dominated. Similarly to dominate the maximum weighted vertex $v_{\frac{3k}{2}+4}$, choose the minimum weighted vertex $v_{\frac{3k}{2}+5}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k}{2}+4}$ and $v_{\frac{3k}{2}+6}$ are dominated.

Proceeding like this, to dominate the maximum weighted vertex v_{3k-2}, choose the minimum weighted vertex v_{3k-1} for the $\gamma_w(G)$-set of G. Then the vertices v_{3k-2} and v_{3k} are dominated. Thus the set of vertices $\left\{v_{\frac{3k}{2}+2}, v_{\frac{3k}{2}+5}, \ldots, v_{3k-1}\right\}$ belongs to the γ_w-set of G.

To dominate the maximum weighted vertex $v_{\frac{3k}{2}}$, choose the minimum weighted vertex $v_{\frac{3k}{2}-1}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k}{2}-2}$ and $v_{\frac{3k}{2}}$ are dominated. Similarly to dominate the maximum weighted vertex $v_{\frac{3k}{2}-3}$, choose the minimum weighted vertex $v_{\frac{3k}{2}-4}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k}{2}-5}$ and $v_{\frac{3k}{2}-3}$ are dominated.

Proceeding like this, to dominate the maximum weighted vertex v_3, choose the minimum weighted vertex v_2 for the γ_w-set of G. Then the vertices v_1 and v_3 are dominated. Thus the set of vertices $\left\{v_{\frac{3k}{2}-1}, v_{\frac{3k}{2}-4}, \ldots, v_5, , v_2\right\}$ belongs to the γ_w-set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally it is v_1. Therefore the least weight dom chromatic set is $\left\{v_1, v_2, v_5, \ldots, v_{\frac{3k}{2}-4}, v_{\frac{3k}{2}-1}, v_{\frac{3k}{2}+2}, \ldots, v_{3k-4}, v_{3k-1}\right\}$. Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n) = w(D) = \sum w(v_i) = 1 + (3+4) + (9+10) + \ldots + (3k-3) + (3k-2) = \frac{1}{6}\left(n^2 + n + 6\right)$.

Case(2): Suppose n is odd. Then $n = 3k$ for some odd integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertices of P_{3k}. It is easily observed that a vertex of maximum weight is included in any γ_w-set of G. Thus, the maximum weighted vertex $v_{\frac{3k+1}{2}}$ is included in γ_w-set of G. Then, the the vertices $v_{\frac{3k-1}{2}}$ and $v_{\frac{3k+3}{2}}$ are dominated. Similarly, choose the vertex $v_{\frac{3k+7}{2}}$ for the γ_w-set of G, then the vertices $v_{\frac{3k+5}{2}}$ and $v_{\frac{3k+9}{2}}$ are dominated.

Proceeding in the same way, choose the vertex v_{3k-1} for the γ_w-set of G, then the vertics v_{3k-2} and v_{3k} are dominated. Thus, the set of vertices $\{v_{\frac{3k+1}{2}}, v_{\frac{3k+7}{2}}, \ldots, v_{3k-1}\}$ belongs to the γ_w-set of G.

Also, since the vertex $v_{\frac{3k+1}{2}}$ dominates $v_{\frac{3k-1}{2}}$, choose the vertex $v_{\frac{3k-5}{2}}$ for the γ_w-set of G, then the vertices $v_{\frac{3k-7}{2}}$ and $v_{\frac{3k-3}{2}}$ are dominated. Similarly, choose the vertex $v_{\frac{3k-11}{2}}$ for the γ_w-set of G, then the vertices $v_{\frac{3k-13}{2}}$ and $v_{\frac{3k-9}{2}}$ are dominated.

Proceeding in the same way, choose the vertex v_2 for the γ_w-set of G, then the vertices v_1 and v_3 are dominated. Thus, the set of vertices $\{v_{\frac{3k-5}{2}}, v_{\frac{3k-11}{2}}, \ldots, v_5, v_2\}$ belongs to the γ_w-set of G. Therefore, the minimum weighted dominating set D of P_n is $\{v_2, v_5, \ldots, v_{\frac{3k-5}{2}}, v_{\frac{3k+1}{2}}, \ldots, v_{3k-1}\}$.

For chromaticity, we add neighbor of least weight vertex to this set. Naturally, it is v_1. Therefore, the least weight dom chromatic set is $\{v_1, v_2, v_5, \ldots, v_{\frac{3k-5}{2}}, v_{\frac{3k+1}{2}}, \ldots, v_{3k-1}\}$.

Hence, the minimum weight of a dom-chromatic set is, $\gamma_{wch}(P_n) = w(D) = \sum w(v_i) = 1+(3+4)+(9+10)+\ldots+(3k-6)+(3k-5)+3k = \frac{1}{6}\left(n^2+n+12\right)$ $\square$

Next, we find the weighted dom chromatic number of Type-II weighted path graph P_n with $n = 3k+1$ for some integer $k > 0$. In this case, we have a pattern for odd and even vertices for a path graph P_n where $n = 3k+1$ for some integer $k > 0$ to determine $\gamma_{wch}(P_n)$. From the above pattern, we have the followng theorem.

Theorem 2.4. *For a path P_n, where $n \equiv 1(mod\ 3)$ of Type-II,* $\gamma_{wch}(P_n) = \frac{1}{6}\left(n^2+n+16\right)$

Proof. Let (P_n, w) be a weighted path and $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ such that $w(v_i) = j$, for all $i, j = 1, 2, \ldots, n$ and let $L = [1, 2, \ldots, n]$ be a leaf-first labeling of (P_n, w) and L is of Type-II. Let $n \equiv 1\ (mod\ 3)$. Then $n = 3k + 1$ for some integer $k > 0$. we consider the following cases.

Case(1): Suppose n is even. Then $n = 3k + 1$ for some odd integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertices of P_{3k+1}. By the definition of Type II weighted labeling, $v_{\frac{3k+3}{2}}$ admits the maximum weight of G and hence to dominate the maximum weight vertex $v_{\frac{3k+3}{2}}$, choose the minimum weight vertex $v_{\frac{3k+5}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k+3}{2}}$ and $v_{\frac{3k+7}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k+9}{2}}$, choose the minimum weight vertex $v_{\frac{3k+11}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k+9}{2}}$ and $v_{\frac{3k+13}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_{3k}, choose the minimum weight vertex v_{3k+1} for the $\gamma_w(G)$-set of G. Then the vertex v_{3k+1} is dominated. Thus the set of vertices $\left\{v_{\frac{3k+5}{2}}, v_{\frac{3k+11}{2}}, \ldots, v_{3k+1}\right\}$ belong to the γ_w-set of G.

Also, to dominate the maximum weight vertex $v_{\frac{3k+1}{2}}$, choose the minimum weight vertex $v_{\frac{3k-1}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-3}{2}}$ and $v_{\frac{3k+1}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k-5}{2}}$, choose the minimum weight vertex $v_{\frac{3k-7}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-9}{2}}$ and $v_{\frac{3k-5}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_2, choose the minimum weight vertex v_1 for the γ_w-set of G. Then, the vertices v_2 is dominated. Thus, the set of vertices $\left\{v_{\frac{3k-1}{2}}, v_{\frac{3k-7}{2}}, \ldots, v_4, v_1\right\}$ belong to the γ_w-set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally it is v_2. Therefore, the least weight dom chromatic set is $\left\{v_1, v_2, v_4, v_7, \ldots, v_{\frac{3k-1}{2}}, v_{\frac{3k+5}{2}}, \ldots, v_{3k-2}, v_{3k+1}\right\}$ Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n) = w(D) = \sum w(v_i) =$
$[1 + 7 + 13 + \ldots + (3k-2)]+[2 + 8 + 14 + \ldots + (3k-1)]+3 = \frac{1}{6}(n^2 + n + 16)$.

Case(2): Suppose n is odd. Then $n = 3k + 1$ for some even integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertices of P_{3k+1}. By the definition of Type II weighted labeling, $v_{\frac{3k+2}{2}}$ admits the maximum weight of G and hence to dominate the maximum weight vertex $v_{\frac{3k+2}{2}}$, choose the minimum weight vertex $v_{\frac{3k}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-2}{2}}$ and $v_{\frac{3k+2}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k+4}{2}}$, choose the minimum weight vertex $v_{\frac{3k+6}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k+4}{2}}$ and $v_{\frac{3k+8}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_{3k-1}, choose the minimum weight vertex v_{3k} for the $\gamma_w(G)$-set of G. Then the vertices v_{3k-1} and v_{3k+1} are dominated. Thus the set of vertices $\left\{v_{\frac{3k}{2}}, v_{\frac{3k+6}{2}}, \ldots, v_{3k-3}, v_{3k}\right\}$ belong to the γ_w-set of G.

Also, to dominate the maximum weight vertex $v_{\frac{3k-4}{2}}$, choose the minimum weight vertex $v_{\frac{3k-6}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-8}{2}}$ and $v_{\frac{3k-4}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k-10}{2}}$, choose the minimum weight vertex $v_{\frac{3k-12}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-14}{2}}$ and $v_{\frac{3k-10}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_4, choose the minimum weight vertex v_3 for the γ_w-set of G. Then the vertices v_2 and v_4 are dominated. Also since v_1 admits the minimum weight, choose v_1 for the γ_w-set of G. Thus the set of vertices $\left\{v_{\frac{3k-6}{2}}, v_{\frac{3k-12}{2}}, \ldots, v_6, v_3, v_1\right\}$ belong to the γ_w-set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally it is v_{3k+1}. Therefore the least weight dom chromatic set is $\left\{v_1, v_3, v_6, \ldots, v_{\frac{3k-6}{2}}, v_{\frac{3k}{2}}, v_{\frac{3k+6}{2}}, \ldots, v_{3k}, v_{3k+1}\right\}$.

Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n) = w(D) = \sum w(v_i) = 1+[5 + 11 + 17 + \ldots + (3k - 1)]+[4 + 10 + 16 + \ldots + (3k - 2)]+ 2 = \frac{1}{6}\left(n^2 + n + 16\right)$. □

Next, we find the weighted dom chromatic number of Type II weighted path graph P_n with $n = 3k + 2$ for some integer $k > 0$. In this case, we have a pattern for odd and even vertices for a path graph P_n where $n = 3k + 2$

for some integer $k > 0$ to determine $\gamma_{wch}(P_n)$. From the above pattern, we have the followng theorem.

Theorem 2.5. *For a path P_n, where $n \equiv 2(mod\ 3)$ of Type-II,*

$$\gamma_{wch}(P_n) = \begin{cases} \frac{1}{6}\left(n^2+n+6\right) & \text{if } n \text{ is odd} \\ \frac{1}{6}\left(n^2+n+12\right) & \text{if } n \text{ is even} \end{cases}$$

Proof. Let (P_n, w) be a weighted path and $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ such that $w(v_i) = j$, for all $i, j = 1, 2, \ldots, n$ and let $L = [1, 2, \ldots, n]$ be a leaf-first labeling of (P_n, w) and L is of Type-II. Let $n \equiv 2\ (mod\ 3)$. Then $n = 3k + 2$ for some integer $k > 0$. we consider the following cases.

Case(1): Suppose n is odd. Then $n = 3k + 2$ for some odd integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k+2}\}$ be the vertices of P_{3k+2}. By the definition of Type II weighted labeling, $v_{\frac{3k+3}{2}}$ admits the maximum weight of G and hence to dominate the maximum weight vertex $v_{\frac{3k+3}{2}}$, choose the minimum weight vertex $v_{\frac{3k+1}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-1}{2}}$ and $v_{\frac{3k+3}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k-3}{2}}$, choose the minimum weight vertex $v_{\frac{3k-5}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-7}{2}}$ and $v_{\frac{3k-3}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_3, choose the minimum weight vertex v_2 for the $\gamma_w(G)$-set of G. Then the vertex v_1 and v_3 are dominated. Thus the set of vertices $\left\{v_{\frac{3k+1}{2}}, v_{\frac{3k-5}{2}}, \ldots, v_5, v_2\right\}$ belong to the γ_w-set of G.

Also, to dominate the maximum weight vertex $v_{\frac{3k+5}{2}}$, choose the minimum weight vertex $v_{\frac{3k+7}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k+5}{2}}$ and $v_{\frac{3k+9}{2}}$ are dominated. Similarly to dominate the maximum weight vertex $v_{\frac{3k+11}{2}}$, choose the minimum weight vertex $v_{\frac{3k+13}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k+11}{2}}$ and $v_{\frac{3k+15}{2}}$ are dominated.

Proceeding like this, to dominate the maximum weight vertex v_{3k+1}, choose the minimum weight vertex v_{3k+2} for the γ_w-set of G. Then the vertices v_{3k+1} is dominated. Thus the set of vertices $\left\{v_{\frac{3k+7}{2}}, v_{\frac{3k+13}{2}}, \ldots, v_{3k-1}, v_{3k+2}\right\}$ belong to the γ_w-set of G.

For chromatic preserving, add a neighbor of least weight vertex to this set. Naturally it is v_1. Therefore the least weight dom chromatic set is $\left\{v_1, v_2, v_5, v_8, \ldots, v_{\frac{3k+1}{2}}, v_{\frac{3k+7}{2}}, \ldots, v_{3k-1}, v_{3k+2}\right\}$ Hence, the minimum weight of a dom chromatic set is,$\gamma_w(P_n) = w(D) = \sum w(v_i) = 1+(2+3)+(8+9)+\ldots+(3k-1)+(3k) = \frac{1}{6}(n^2+n+12)$

Case(2): Suppose n is even. Then $n = 3k+2$ for some even integer $k > 0$. Let $\{v_1, v_2, \ldots, v_{3k+2}\}$ be the vertices of P_{3k+2}. From the above example [ ], we observed that a vertex of maximum weight are in any γ_w-set of G. Thus,the vertex $v_{\frac{3k+4}{2}}$ is belongs to the γ_w-set of G. Then, the the vertices $v_{\frac{3k+2}{2}}$ and $v_{\frac{3k+6}{2}}$ are dominated. Now to dominate the maimum weighted vertex $v_{\frac{3k+8}{2}}$, choose the minimum weighted vertex $v_{\frac{3k+10}{2}}$ for the γ_w-set of G, then the vertices $v_{\frac{3k+10}{2}}$ and $v_{\frac{3k+12}{2}}$ are dominated.

Proceeding in the same way, to dominate the maximum weighted vertex v_{3k+1}, choose the minimum weighted vertex v_{3k+2} for the γ_w-set of G, then the vertex v_{3k+1} is dominated. Thus, the set of minimum weighted vertices $\{v_{\frac{3k+4}{2}}, v_{\frac{3k+10}{2}}, \ldots, v_{3k+2}\}$ belongs to the γ_w-set of G.

Also, since the vertex $v_{\frac{3k+4}{2}}$ dominates $v_{\frac{3k+2}{2}}$, to dominate the maximum weighted vertex $v_{\frac{3k}{2}}$, choose the minimum weighted vertex $v_{\frac{3k-2}{2}}$ for the γ_w-set of G. Then the vertices $v_{\frac{3k-4}{2}}$ and $v_{\frac{3k}{2}}$ are dominated. Similarly, to dominate the maximum weighted vertex $v_{\frac{3k-6}{2}}$, choose the minimum weighted vertex $v_{\frac{3k-8}{2}}$ for the γ_w-set of G, then the vertices $v_{\frac{3k-10}{2}}$ and $v_{\frac{3k-6}{2}}$ are dominated.

Proceeding in the same way, to dominate the maximum weighted vertex v_3, choose the minimum weighted vertex v_2 for the γ_w-set of G, then the vertices v_1 and v_3 are dominated. Thus, the set of vertices $\{v_{\frac{3k-2}{2}}, v_{\frac{3k-8}{2}}, \ldots, v_5, v_2\}$ belongs to the γ_w-set of G. Therefore, the minimum weighted dominating set D of P_n is $\{v_2, v_5, \ldots, v_{\frac{3k-2}{2}}, v_{\frac{3k+4}{2}}, \ldots, v_{3k+2}\}$.

For chromaticity, we add neighbor of least weight vertex to this set. Naturally, it is v_1. Therefore, the least weight dom chromatic set is
$\{v_1, v_2, v_5, \ldots, v_{\frac{3k-2}{2}}, v_{\frac{3k+4}{2}}, \ldots, v_{3k+2}\}$.

Hence, the minimum weight of a dom chromatic set is, $\gamma_{wch}(P_n) = w(D) =$

$\sum w(v_i) = 1 + [3 + 9 + 15 + \ldots + (3k - 3)] + [2 + 8 + 14 + \ldots + (3k + 2)] = \frac{1}{6}(n^2 + n + 12)$

□

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