The Expression of Large Amplitude Wave equations using Homotopy Analysis Method

Ejinkonye Ifeoma O.

Department of Mathematics and Computer Science, Western Delta University, Oghara, Nigeria.

Abstract *- This work focuses on some mechanisms that concern large amplitude ocean wave event. The existence of the rogue wave is thus partly due to ocean current wave interaction and partly due to the inter-crossing of a large number of quasi-monochromatic wave components with appropriate frequencies, wave numbers and randomly distributed phase angles.*

In this study the equations of water waves are solved by means of an analytic technique, namely the Homotopy Analysis Method (HAM). HAM is a capable and straight forward analytic tool for solving nonlinear differential equations and does not require small/large parameters in the governing equations unlike other well-known analytic approach.

We use HAM to obtain an approximate solution to the governing water wave equations. The free surface displacement $\eta(x,t)$ *and velocity potential,* $\phi(x,z,t)$ *obtained are compared with similar results.*

Key words- *Homotopy Analysis Method, free surface displacement, velocity potential*

I. INTRODUCTION

Rogue waves (also known in literature as freak waves, giant waves, killer waves, monster waves, and extreme waves) are large water waves with large amplitude that often surprisingly appear and disappear from nowhere, on the sea surface sometime unexpected. These large waves are often associated with ocean wave current, energy focusing [1]. These giant waves are thus, threat to ships, ocean liners and onshore engineering structures. As stated in [2], the rogue waves have been noticeable part of marine problem for centuries.

It is expected that the present investigations will enhance the knowledge of the destructive events which manifest as rogue waves for ocean engineering purposes (oil rig platform and other onshore structures)**.** The reason being that rogue waves can appear anywhere on the sea-surface without warning.

The observation of extreme wave such as rogue wave in some area of intense ocean current is fairly regular. The most notable sea areas with such occurrence are that of Agulhas in South Africa and Gulf Stream in North America [2-6]. [7] Observed an area in South Africa, Agulhas current where current energy concentrates sufficient enough to induce rogue wave event.

[8-10] Showed that the second order analytical models for the prediction of extreme event can also be derived by mean of the theory of quasi-determinism. The authors began from the general second order Stokes expansion (using perturbation method) of the surface displacement for long-crested waves, and should that the crest of the non-linear crest (trough) depends upon the initial crest (trough) amplitude. The study of the third order term of the Stokes expansion of wave profile and velocity potential was considered in [11]. The introduction of wave steepness perimeters help to enhance the convergence of the stroke expansion [12].

Liao [13] investigated the steady condition for the nonlinear interaction of two trains of propagating wave in deep water and obtained the solution for both resonant and non-resonant cases. By means of the analytical method called homotopy analysis method (HAM) developed in [14-18] a powerful analytic method for highly nonlinear problems.

In this work, we apply the homotopy analysis method (HAM) to the basic equations governing the dynamics of water waves to derive the mechanism for freak wave generation.

II. BASIC IDEA OF HOMOTOPY ANALYSIS METHOD (HAM)

In this work, we use the homotopy analysis method (HAM) to solve the nonlinear partial differential equations. This method was proposed by a Chinese mathematician [14]. We apply Liao's basic ideas to the nonlinear differential equation. In topology, two continuous functions from one topological space to another are called Homotopic, if one can be "continuously deformed" into the other, such a deformation is called a homotopy between the two functions.

Here, the nonlinear boundary-value problem governed by the PDEs (1-4) is solved by means of the homotopy analysis method.

Traditionally, perturbation methods have been widely applied to solve nonlinear problems related to gravity waves. It is well known that perturbation methods are strongly dependent upon small physical parameters i.e. the so-called perturbation quantities [13].

To overcome the restrictions of perturbation methods and some additional non-perturbation techniques [14-18] developed an analytic technique for highly nonlinear problem, namely the homotopy analysis method (HAM). HAM gives us great freedom in the choice of the initial guess, the equation type of linear sub problems and basic functions of solution.

III. GOVERNING EQUATIONS OF WATER WAVES

We introduce the governing equations of water waves. If we consider the nonlinear interaction of a train of progressive gravity waves of finite depth and assume that the fluid is inviscid and incompressible, the flow is irrotational and the surface tension is neglected. The x-axis positive in the direction of wave propagation and the z-axis points vertically upward from the still- water level, the problem is steady and is periodic in the x- variable. Let the vertical free surface displacement be $\eta(x,t)$ and the velocity potential $\phi(x,z,t)$. Both the velocity potential $\phi(x,z,t)$ and free surface displacement $\eta(x,t)$ have to satisfy the Laplace equation.

$$\nabla^2\phi(x,z,t)=0 \quad z=\eta(x,t) \qquad (1)$$

The velocity potential ϕ is subject to the unknown $\eta(x,t)$ free-surface boundary conditions.

$$\frac{\partial^2\phi(x,z,t)}{\partial t^2}+g\frac{\partial\phi(x,z,t)}{\partial z}+\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi(x,z,t)}{\partial z}\right)^2\right]-g\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)\left(\frac{\partial\eta(x,t)}{\partial x}\right)=0 \qquad (2)$$

$$g\eta(x,t)+\frac{\partial\phi(x,z,t)}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\phi(x,z,t)}{\partial z}\right)^2\right]=0 \qquad (3)$$

The solid boundary condition at the horizontal bottom is given as

$$\frac{\partial\phi(x,z,t)}{\partial z}=0 \text{ at } z=-d \qquad (4)$$

where g is the acceleration of gravity.

Even if the governing equation (1) and the bottom boundary condition (4) are linear, the two nonlinear boundary conditions (2) and (3) are satisfied on the unknown free surface $\eta(x,t)$. Such nonlinear partial differential equations (PDEs) are difficult to solve. In this work we apply the homotopy analysis method (HAM) to solve this boundary-value problem with the nonlinear conditions with an unknown free surface displacement $\eta(x,t)$.

3.2 The Continuous Deformation

In solving the nonlinear boundary-value problem above governed by the PDEs equations (1-4) by means of HAM, first we start with the initial approximations.

Let $\phi_0(x,z,t)$, $\eta_0(x,t)$ denote the initial guesses of the velocity potential $\phi(x,z,t)$ and free surface displacement $\eta(x,t)$ respectively.

Let $p\in[0,1]$ denote an embedding parameter and let h be the so-called convergence-control parameter. Here, both p and h are auxiliary parameter without physical meaning. Instead of solving the nonlinear $PDEs$ in equation $(1-4)$ directly, we first construct a family (with respect to p) of PDEs about two continuous deformations $\phi(x,z,t)$ and $\eta(x,t)$ governed by the so- called zero-order deformation equations,

$$\nabla^2\varphi(x,z\ t;p)=0 \qquad -d<z<\eta(x,t;p) \qquad (5)$$

subject to the two boundary conditions on the unknown free surface $z=\eta(x,t:p)$

$$(1-p)L[\varphi(x,z,t;p)-\phi_0 x(x,z,t)]=phN[\varphi(x,z,t;p)] \qquad (6)$$

$$(1-p)\eta(x,t;p)=phZ[\eta(x,t;p),\varphi(x,z,t;p)] \qquad (7)$$

And the bottom condition

$$\frac{\partial\varphi(x,z,t;p)}{\partial z}=0 \text{ at } z=-d, \tag{8}$$

Where h is an auxiliary parameter and it is important to know that one has great freedom to choose auxiliary parameter h, L is an auxiliary linear operator with the property $L(0)=0,$ N and Z are nonlinear differential operators. If we choose our nonlinear operator to be

$$N(\varphi(x,z,t;p))=\frac{\partial^2\varphi(x,z,t;p)}{\partial t^2}+g\frac{\partial\varphi(x,z,t;p)}{\partial z}+$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t;p)}{\partial z}\right)^2\right]-g\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)\frac{\partial\eta(x,t;p)}{\partial x} \tag{9}$$

and

$$Z[\varphi(x,z,t;p)]=\frac{1}{g}\left(\frac{\partial\varphi(x,z,t;p)}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t,p)}{\partial z}\right)^2\right]\right) \tag{10}$$

We choose the auxiliary linear operations as

$$L[\varphi(x,z,t;p)]=\frac{\partial^2\varphi(x,z,t;p)}{\partial t^2}+g\frac{\partial\varphi(x,z,t;p)}{\partial z} \tag{11}$$

Note that, the definitions of N,Z and L are based on the two boundary conditions equations (2) and (3) respectively.

3.3 Deformation Equation of High Order

Differentiating the zero-order deformation equations (5-8) m times with respect to p, them dividing them by m! and setting p=0 we have the mth-order deformation equation

$$\nabla^2\phi_m=0 \qquad -d<z<0 \tag{12}$$

Subject to the two boundary condition at $z=0$

$$\frac{\partial^m}{\partial p}\frac{1}{m!}[(1-p)L(\varphi(x,z,t;p)-\phi_0(x,z,t))=X_n phN[\varphi(x,z,t;p)]] \tag{13}$$

and

$$\frac{\partial^m}{\partial p}\frac{1}{m!}[(1-p)\eta(x,t;p)]=X_n phZ[\eta(x,t;p),\ \varphi(x,z,t;p)] \tag{14}$$

and the bottom condition

$$\frac{\partial\phi_0^n(x,z,t)}{\partial z}=0 \qquad z=-d \tag{15}$$

Where

$$X_n=\begin{cases}0 & when \quad n\le 1\\ 1 & when \quad n>1\end{cases} \tag{16}$$

Moreover, differentiating equations (6) and (7) m times with respect to the embedding parameter at p=0 we obtain the respective free-surface boundary conditions defined at $z=\varsigma_0(x,t)$ as

$$\left.\frac{D^nL(\varphi(x,z,t;p))}{Dp^n}\right|_{p=0}=n\left\{X_n\frac{D^{n-1}L(\varphi(x,z,t;p))}{Dp^{n-1}}+h\frac{D^{n-1}N[\varphi(x,z,t;p)]}{Dp^{n-1}}\right\}\Big|_{p=0} \tag{17}$$

and

$$\eta_0^{(n)}(x,t)=n\left\{(X_n+h)\eta_0^{n-1}(x,t)-h\frac{D^{n-1}Z[\varphi(x,z,t;p)]}{Dp^{n-1}}\Big|_{p=0}\right\} \tag{18}$$

where

$$\frac{D^nL[\varphi(x,z,t;p)]}{Dp^n}=\frac{D^n\left(\frac{\partial^2\varphi^n(x,z,t;p)}{\partial t^2}\right)}{Dp^{n-1}}\Big|_{p=0}+g\frac{D^n\left(\frac{\partial\varphi^n(x,z,t;p)}{\partial z}\right)}{Dp^{n-1}}\Big|_{p=0} \tag{19}$$

$$\frac{D^{n-1}N[\varphi(x,z,t;p)]}{Dp^{n-1}}=\frac{D^n\left(\frac{\partial^2\varphi^n(x,z,t;p)}{\partial t^2}\right)}{Dp^{n-1}}\Big|_{p=0}+g\frac{D^n\left(\frac{\partial\varphi^n(x,z,t;p)}{\partial z}\right)}{Dp^{n-1}}\Big|_{p=0}+ \tag{20}$$

and

$$\frac{D^{n-1}Z(\varphi(x,z,t;p))}{Dp^{n-1}}=\frac{D^{n-1}}{Dp^{n-1}}\left[\frac{1}{g}\frac{\partial\varphi(x,z,t;p)}{\partial t}-\frac{1}{2}\left[\left(\frac{\partial\varphi(x,z,t;p)}{\partial x}\right)^2+\left(\frac{\partial\varphi(x,z,t;p)}{\partial z}\right)^2\right]\right] \tag{21}$$

Therefore substituting equations (19) and (20) into (17) we have

$$\left(\frac{\partial^2 \phi^n(x,z,t)}{\partial t^2}\right) + g\left(\frac{\partial \phi^n(x,z,t)}{\partial z}\right) = \tag{22}$$

$$\left[\begin{array}{l} nx_n\left(\frac{\partial^2 \phi^{(n-1)}(x,z,t)}{\partial t^2} + g\,\frac{\partial \phi^{(n-1)}(x,z,t)}{\partial z}\right) - \\ -h\left[\begin{array}{l} \frac{\partial^2 \phi^{(n-1)}(x,z,t)}{\partial t^2} + g\,\frac{\partial \phi^{(n-1)}(x,z,t)}{\partial z} \\ +\frac{1}{2}\frac{\partial}{\partial t}\left[\left(\frac{\partial \phi^{n-1}(x,z,t)}{\partial x}\right)^2 + \left(\frac{\partial \phi^{n-1}(x,z,t)}{\partial z}\right)^2\right] - g\,\frac{\partial \varphi^{n-1}(x,z,t)}{\partial x}\frac{\partial \eta^{n-1}(x,t)}{\partial x} \end{array}\right] \end{array}\right]$$

$$\eta^n(x,t) = n\left\{(X_n + h)\eta_0^{n-1} - \frac{h}{g}\left[\frac{\partial \phi^{n-1}(x,z,t)}{\partial t} - \frac{1}{2}\left[\left(\frac{\partial \phi^{n-1}(x,z,t)}{\partial x}\right)^2 + \left(\frac{\partial \phi^{n-1}(x,z,t)}{\partial z}\right)^2\right]\right]\right\} \tag{23}$$

The boundary value problem at the m[th]-order approximation is defined by the governing equation (12) and the boundary conditions (15), (22) and (23). So $\phi^n(x,z,t)$ velocity potential and $\eta^n(x,t)$ free surface displacement can be easily symbolically solved by the computer software MATHEMATICA and directly calculated from equations (22) and (23). From equation (23) we now successfully obtain the equations of $\eta^n(x,t)$ free surface displacement from zero order to higher order as

$$\eta^0 = 0 \tag{24}$$

$$\eta_1 = \frac{1}{2}A_1 h(A_1 k + 2\cos[tw - kx]) \tag{25}$$

Where $A_1 = a$

$$\eta_2 = A_2 h\left(2(1 + h - w^2)\cos[tw - kx] + k(1 + h - w^4)\cos[2tw - 2kx]A_2\right) \tag{26}$$

Where $A_2 = -\frac{A_1^2 h}{4w}$

$$\eta_3 = 3A_2 h\left(\begin{array}{l} 2kw\cos[2tw - 2kx] + 2k^3 w^2 A_3 \\ +(1+h)\left(2(1 + h - w^2)\cos[tw - kx]A_3 + k(1 + h - w^4)\cos[2tw - 2kx]A_3\right) \end{array}\right) \tag{27}$$

Where $A_3 = \frac{6h^2(2+h)A_2^2 - 6h + 8}{13w}$

$$\eta_4 = 2h\left(\begin{array}{l} 4kw\cos[2tw - 2kx]A_3 + 8kw^2\cos[4tw - 4kx]A_4 + \\ 4k^4 w^2(\cos[2tw - 2kx]A_3 + 2\cos[4tw - 4kx]A_4)^2 + 4k^4\sin[2tw - 2kx]^2(A_3 + 3w\cos[2tw - 2kx]A_4)^2 + \\ +6(1+h)A_4\left(2kw\cos[2tw - 2kx] + 2k^3 w^2 A_4 + (1+h)\left(\begin{array}{l} 2(1 + h - w^2)\cos[tw - kx] \\ +k(1 + h - w^4\cos[2tw - 2kx]A_4) \end{array}\right)\right) + \end{array}\right) \tag{28}$$

Where $A_4 = \frac{4h\left(12(2h + 3h^2 + h^3 - kw)A_2^3 - 14\right)}{(9+4h)w}$

$$\eta_5 = 5\left(2h(h+1)\left(\begin{array}{l} 4kw\cos[2tw - 2kx]A_4 + 8kw^2\cos[4tw - 4kx]A_5 + 4k^4 w^2(\cos[2tw - 2kx]A_4 + 2\cos[4tw - 4kx]A_5)^2 + \\ 4k^4\sin[2tw - 2kx]^2(A_4 + 3w\cos[2tw - 2kx]A_5)^2 \\ +6(1+h)A_5\left(\begin{array}{l} 2kw\cos[2tw - 2kx] + 2k^3 w^2 A_5 + \\ (1+h)\left(2(1 + h - w^2)\cos[tw - kw] + k(1 + h - w^4\cos[2tw - 2kx]A_5)\right) \end{array}\right) - \\ h\left(\begin{array}{l} -2kw\cos[2tw - 2kx]A_3 - 4kw^2\cos[4tw - 4kx]A_4 - 4kw^2\cos[4tw - 4kx]A_3 A_4 - 8kw^3\cos[8tw - 8kx]A_5 \\ +\frac{1}{2}\left(\begin{array}{l} -4k^4\left(\cos[2tw - 2kx]A_3 + 2w\cos[4tw - 4kx](1 + A_2)A_4 + 4w^2\cos[8tw - 8kx]A_5\right)^2 \\ +k^4(2\sin[2tw - 2kx]A_3(1 + 3w\cos[2tw - 2kx]A_4) + w\sin[4tw - 4kx](3A_4 + 8w\cos[4tw - 4kx]A_5))^2 \end{array}\right) \end{array}\right) \end{array}\right)\right) \tag{29}$$

Where $A_5 = \frac{5h\left(12(2h + 3h^2 + h^3 - kw)A_3^2 - 14(8h^2 + 32h^3 - kw)A_4\right)}{(15+4h)w}$

Therefore the summation free surface displacement i.e.

$$\eta^n(x,t) = \eta^0 + \eta^1 + \eta^2 + \eta^3 + \eta^4 + \eta^5 + \ldots$$

$$=\frac{1}{2}A_1h(A_1k+2\cos[tw-kx])+A_2h\left(2(1+h-w^2)\cos[tw-kx]+k(1+h-w^4)\cos[2tw-2kx]A_2\right)+$$
$$3A_2h\begin{pmatrix}2kw\cos[2tw-2kx]+2k^3w^2A_3\\ +(1+h)(2(1+h-w^2)\cos[tw-kx]A_3+k(1+h-w^4)\cos[2tw-2kx]A_3))\end{pmatrix}+$$
$$2h\begin{pmatrix}4kw\cos[2tw-2kx]A_3+8kw^2\cos[4tw-4kx]A_4+\\ 4k^4w^2(\cos[2tw-2kx]A_3+2\cos[4tw-4kx]A_4)^2+4k^4\sin[2tw-2kx]^2(A_3+3w\cos[2tw-2kx]A_4)^2+\\ +6(1+h)A_4\left(2kw\cos[2tw-2kx]+2k^3w^2A_4+(1+h)\begin{pmatrix}2(1+h-w^2)\cos[tw-kx]\\ +k(1+h-w^4\cos[2tw-2kx]A_4))\end{pmatrix}\right)+\end{pmatrix}+$$
$$5\left(2h(h+1)\begin{pmatrix}4kw\cos[2tw-2kx]A_4+8kw^2\cos[4tw-4kx]A_5+4k^4w^2(\cos[2tw-2kx]A_4+2\cos[4tw-4kx]A_5)^2+\\ 4k^4\sin[2tw-2kx]^2(A_4+3w\cos[2tw-2kx]A_5)^2\\ +6(1+h)A_5\begin{pmatrix}2kw\cos[2tw-2kx]+2k^3w^2A_5+\\ (1+h)(2(1+h-w^2)\cos[tw-kw]+k(1+h-w^4\cos[2tw-2kx]A_5))\end{pmatrix}-\\ h\begin{pmatrix}-2kw\cos[2tw-2kx]A_3-4kw^2\cos[4tw-4kx]A_4-4kw^2\cos[4tw-4kx]A_3A_4-8kw^3\cos[8tw-8kx]A_5\\ +\frac{1}{2}\begin{pmatrix}-4k^4(\cos[2tw-2kx]A_3+2w\cos[4tw-4kx](1+A_2)A_4+4w^2\cos[8tw-8kx]A_5)^2\\ +k^4(2\sin[2tw-2kx]A_3(1+3w\cos[2tw-2kx]A_4)+w\sin[4tw-4kx](3A_4+8w\cos[4tw-4kx]A_5))^2\end{pmatrix}\end{pmatrix}\end{pmatrix}\right)$$

(30)

Similarly, from equation (22) we have the equations of $\phi^n(x,z,t)$ velocity potential from zero order to higher order as

$$\phi^0=Agw^{-1}e^{kz}\sin[kx-wt] \qquad (31)$$

Where $A=1$

$$\phi^1=A_1gw^{-1}e^{kz}\sin[kx-wt] \qquad (32)$$

Where $A_1=a$

$$\phi_0^2=A_2w^2e^{2kz}\sin[2wt-2kx] \qquad (33)$$

Where $A_2=-\frac{hA_1^2}{4w}$

$$\phi_0^3=-A_2w^2e^{2kz}\sin[2wt-2k]-A_3w^3e^{3kz}\sin[4wt-4kx] \qquad (34)$$

Where $A_3=\frac{6hA_2^2(2+h)-6h+8}{13we^{kz}}$

$$\phi_0^4=A_2w^2e^{2kz}\sin[2kx-2wt]+A_2A_3w^3e^{3kz}\sin[4kx-4wt]+A_3w^3e^{3kz}\sin[4kx-4wt]+A_4w^4e^{4kz}\sin[8kx-8wt] \qquad (35)$$

Where $A_4=\frac{4h(12(2h+3h^2+h^3-kw)A_2^3-14)}{(9+4h)we^{kz}}$

$$\phi_0^5=5w^4e^{2kz}(1+h)\left(2\sin[2wt-2kx]A_2(1+13e^{kz}w\sin[2wt-2kx]A_3)+e^{kz}w\begin{pmatrix}13\sin[4tw-4kx]A_3\\ +60e^{kz}w\sin[8wt-8kx]A_4\end{pmatrix}\right)+$$

Where

$$2e^{5kz}k^2w^6\begin{pmatrix}(-3\sin[6tw-6kx]-7\sin[2wt-2kx])A_2(1+A_2)A_3-4e^{2kz}w^2(11\sin[4wt-4kx]+15\sin[12(tw-kx)])\\ (1+A_2)A_3A_4-e^{kz}w\begin{pmatrix}7\sin[8wt-8kx](1+A_2)^2A_3^2+4(9\sin[6wt-6kx]+5\sin[10(tw-kx)])A_2A_4\\ -96w^3e^{3kz}\sin[16(tw-kx)]A_5\end{pmatrix}\end{pmatrix} \qquad (36)$$

$$A_5=\frac{5h(12(2h+3h^2+h^3-kw)A_2^3-14(8h^2+32h^3-kw)A_4)}{(15+4h)we^{kz}}$$

Therefore summation of velocity potential

$$
\begin{aligned}
\phi(x,z,t) &= \phi_0 + \phi_0^1 + \phi_0^2 + \phi_0^3 + \phi_0^4 + \phi_0^5 + \dots \\
&= Agkw^{-1}e^{kz}\cos[wt-kx] + A_1gkw^{-1}e^{kz}\cos[wt-kx] - A_2w^2e^{2kz}\sin[2wt-2kx] - 2A_3w^3e^{3kz}\sin[4wt-4kx] \\
&- A_2A_3w^3e^{3kz}\sin[4wt-4kx] - A_4w^4e^{4kz}\sin[8wt-8kx] \\
&+ 5w^4e^{2kz}(1+h)\left(2\sin[2wt-2kx]A_2\left(1+13e^{kz}w\sin[2wt-2kx]A_3\right) + e^{kz}w\begin{pmatrix}13\sin[4tw-4kx]A_3\\ +60e^{kz}w\sin[8wt-8kx]A_4\end{pmatrix}\right) + \\
&2e^{5kz}k^2w^6\begin{pmatrix}(-3\sin[6tw-6kx]-7\sin[2wt-2kx])A_2(1+A_2)A_3 - 4e^{2kz}w^2(11\sin[4wt-4kx]+15\sin[12(tw-kx)]) \\ (1+A_2)A_3A_4 - e^{kz}w\begin{pmatrix}7\sin[8wt-8kx](1+A_2)^2A_3^2 + 4(9\sin[6wt-6kx]+5\sin[10(tw-kx)])A_2A_4 \\ -96w^3e^{3kz}\sin[16(tw-kx)]A_5\end{pmatrix}\end{pmatrix}
\end{aligned}
\quad (37)
$$

Where k is the wave number and in deep water $k = \frac{\omega^2}{g}$, ω is the frequency. The coefficient a is amplitude of the wave component. We should remember that both $\phi(x,z,t)$ velocity potential and $\eta(x,t)$ free surface displacement are dependent of $\hbar$ an auxiliary parameter which no physical meaning but it is convergence-control parameter.
The necessary condition for the series to be convergent is $|1+h| < 1$.

It is interesting that the convergence region of the solution series depends upon the value of h. The closer the value of h $(-2 \le h < 0)$ to zero, the larger the convergence region of the series.

IV. RESULTS AND DISCUSSIONS

In this study the new analytical approach (HAM) Homotopy Analysis Method was used to solve the nonlinear rogue wave equations for the first time.
The higher order surface wave elevations were obtained. The pictorial implication on the crest height and trough depth was better understood. The higher order wave elevation clearing showed a very fine picture of freak wave phenomenon. We can see that in fig 1, when h= -1 the graph of equation (30) are the same with [9] and [11]. But fig 2, when h=-1.5 showed more increase in the wave elevation. In fig 3, when h=-2 showed more improve result than that of fig 1 and fig 2. Using HAM with the aid of the control convergence parameter $\hbar$, it enhance the convergence of the surface wave elevation.
Constructive interference suggests that several different wave trains travelling roughly in the same direction meet at some point and build on top of each other.

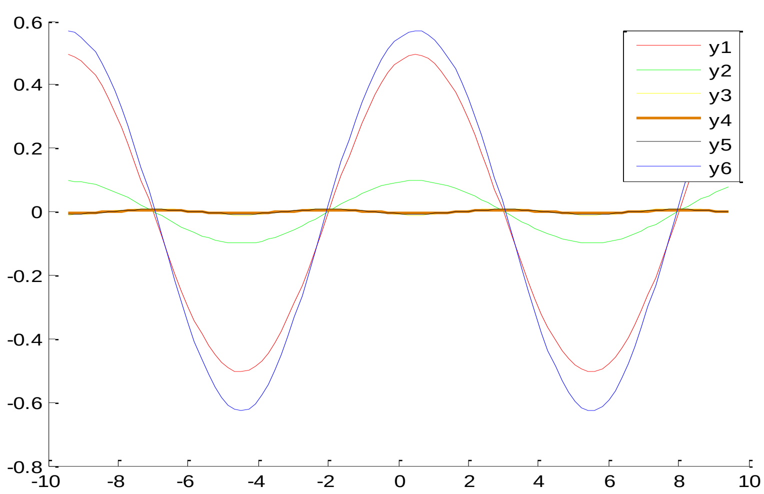

Fig. 1 This is when the convergence control parameter h=-1, y1 = $\eta^1(x,t)$, y2 = $\eta^2(x,t)$, y3 = $\eta^3(x,t)$, y4 = $\eta^4(x,t)$, y5 = $\eta^5(x,t)$and y6 = $\eta^n(x,t)$. This the same compare with Fedele& Arena (2005) and Ejinkonye (2011, 2018).

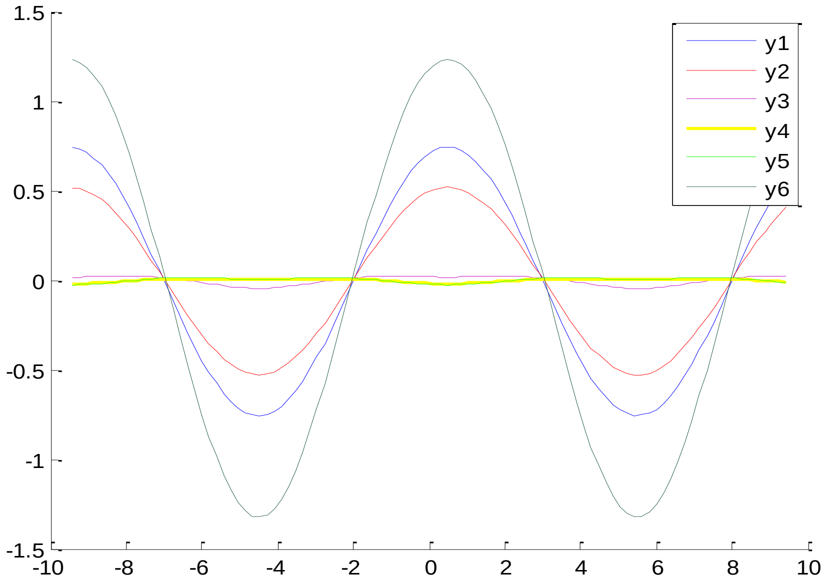

Fig. 2 This is when the convergence control parameter h=-1.5, y1 = $\eta^1(x,t)$, y2 = $\eta^2(x,t)$, y3 = $\eta^3(x,t)$, y4 = $\eta^4(x,t)$, y5 = $\eta^5(x,t)$ and y6 = $\eta^n(x,t)$.

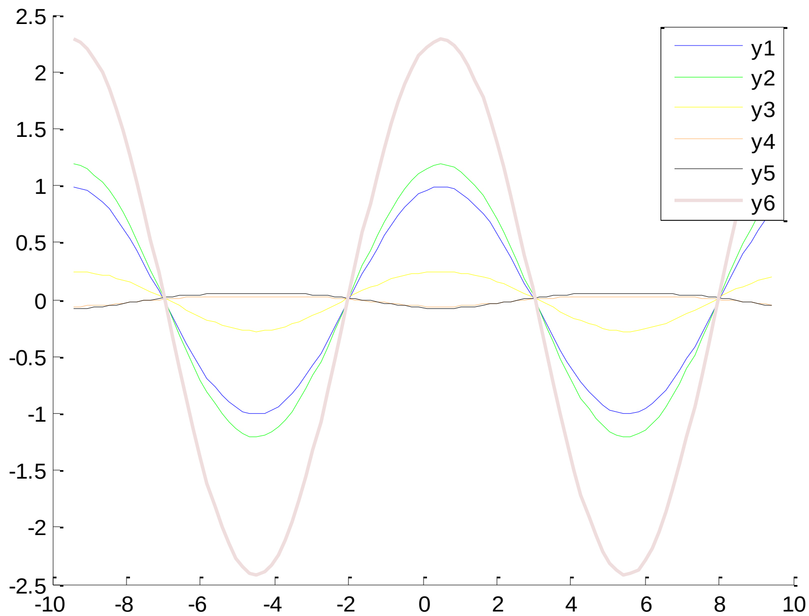

Fig. 3 This is when the convergence control parameter h=-2, y1 = $\eta^1(x,t)$, y2 = $\eta^2(x,t)$, y3 = $\eta^3(x,t)$, y4 = $\eta^4(x,t)$, y5 = $\eta^5(x,t)$ and y6 = $\eta^n(x,t)$.

V. CONCLUSION

Our interest in this investigation is to extend this expansion to higher order and observe the behaviour of surface profile. Firstly we start with zero order term of this solution of expression to the high order using HAM. Our aim is to study it's effects in relation to observed wave height for rogue wave event. The deduction of higher order terms are not the interesting part of it, but it's geophysical impart it will made on the development of rogue wave phenomenon.

The usual appearance of waves with extreme high crest and deep trough can be initiated by the local intercrossing of a large number of quasi-monochromatic wave group with differing phases, wave numbers and the wave current interaction are effective mechanism capable of initiating rogue wave event. In this consideration and at the initial state of wave development in deep water short wave group are at the front of long wave group (wave packet). As time involves, the longer wave components with higher group velocity will overtake the shorter one. Consequently, the longer wave components will extract energy from the shorter components, thus it will grow in size. We have already stated the rogue wave destructive effects on the ocean going vessels and also offshore and on shore engineering structures, more especially onshore structures such as oil platform. This study will eventually lead to significant understanding of the Evolution of this rogue wave event.In summary, based on the concept of homotopy topology, the HAM is a novel analytic approximation method for highly nonlinear problems, with great freedom and flexibility to choose equation-type and solution expression of high-order approximation equations and also with a convenient way to guarantee the convergence, so that it might overcome restrictions of perturbation techniques and other non-perturbation methods.

REFERENCES

[1] Dysthe, K., Krogstad, H.E., and Muller, P.,Oceanic Rogue Waves; Annu. Rev. Fluid Mech. (2008)Vol. 40:287-310.

[2] Lawton, G.,; *Monsters of deep sea.* New scientist (2001)170,(2297); 28-32.

[3] Lavrenov, I.V., and Porubor, A.V., *Three reasons for freak wave generation in the non uniform current*. Eur J. Mech B/fluids, vol.25 pp574-585.

[4] White, B.S., Fornberg B.,(1998); *On the chance of freak waves at the sea,* J. Fluid Mech. (2006) Vol. 255 PP 113-138

[5] Brown, M.G.,); *The Meslov integral representation of slowly varying dispersive wavetrains in inhomogeneous moving media*, Waves motion, (2000 Vol.32; 247-266.

[6] Brown, M.G.,;*Space-time surface gravity waves caustics; structurally stable extreme wave events,* Wave motion (2001) Vol. 33; 117-143.

[7] Lavrenov, I.V.,; *The wave energy concentration at the Agulhas current of South Africa*. Hazard (1998) Vol. 17; 171-177.

[8] Arena, F, and Fedele, F *"Nonlinear Space-Time Evolution of wave Groups with a Hight Crest* J. of Offshore Mech. And Arctic Eng(2005).Vol 127 pp 46-51

[9] Fedele, F., and Arena, F., *"Weakly Nonlinear Statistics of High random waves"* Phys fluids (2005).Vol 17 pp 0266 07-1610

[10] Fedele, F. *"Extreme Events in Nonlinear Random Seas"*. J.A Offshore Mech. And Arctic Eng (2006). Vol 128 pp 11-16

[11] Ejinkonye, I. O. and Ifidon, E.O., *The Higher Order effects of the Rogue wave events*. ABACUS (2013)pp 141-155, Vol. 40 No. 2

[12] Ejinkonye I.O. *The effect of interaction of Large Amplitude wave on sea with its Application*.ScientiaAfricana (2019) Vol. 18 No.1 pp 59-69

[13] Liao, S. J. *Proposed homotopy analysis techniques for the solution of nonlinear problems.*PhD thesis, Shanghai Jiao Tong University. 1992

[14] Liao, S. J. *An approximate solution technique which does not depend upon small parameters (2): an application in fluid mechanics.* Intl J. Non-Linear Mech. 1997 Vol. 32, 815–822.

[15] Liao, S. J.. *Beyond Perturbation: Introduction to the Homotopy Analysis Method*. Chapman & Hall, CRC, Boca Raton. (2003)

[16] Liao, S. J. *On the homotopy analysis method for nonlinear problems*.Appl. MathsComput.(2004) Vol.147, 499–513.

[17] Liao, S. J. *On the homotopy multiple-variable method and its applications in the interactions of nonlinear gravity waves.*Commun.Nonlinear Sci. Numer.Simul.(2011) Vol.16 (3), 1274–1303.

[18] Liao, S. J. *Homotopy Analysis Method in Nonlinear Differential Equations.* Springer & Higher Education.(2012)