The Problem Posing as a Key Tool for the Development of Students' Self-confidence in Mathematics

Dr. Charalabos Psaras[1], Konstantinos Simeonidis[2] Roza Vlachou[3]

[1]*Organizational Coordinator of Regional Educational Planning Center of South Aegean, Greek*

[2]*Director of Primary Education in Dodecanese, Greek , G. Mavrou 2, 85100 Rhodes, Greece*

[3]*Mathematics Education and Multimedia Laboratory, Department of Primary Education,*
University of the Aegean, 1 Demokratias av., 85100 Rhodes, Greece

Abstract — *International assessment programs indicate poor performance in mathematics of students of many countries worldwide with different educational systems, including Greece. On the occasion of the above, this paper aims to present instructional practices which supported in problem posing in order to help students develop the mathematical understanding and mathematical capacity. The research conducted on students of the first grade of elementary school in Greece for two school years. The results of the present research indicate that instructive interventions, with the use of problem posing, helped students to remove mathematics phobia and strengthen students' confidence in mathematics, developing their mathematical ability and mathematical attitude.*

Keywords — *Problem posing, students' self-confidence.*

I. INTRODUCTION

Many international assessment programmes have been developed to assess the effectiveness of the educational systems of the participated countries. PISA (Programme for International Student Assessment), a programme of OECD (Organization for Economic Co-operation and Development), the most widely known assessment programme, is a triennial international survey which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students [20]. As the findings are quite reliable, many countries, among which USA, Germany, Finland and Australia, have been led to reform their educational system.

The results of these programmes reveal negative trends of mathematical school education in different educational systems, including the Greek one. Therefore, many researchers worldwide have addressed students' difficulties in mathematics and have proposed methods to diminish them. Specifically, researches findings indicate the critical aspects of the way of teaching mathematics [8, 27, 24, 3, 22, 14 and 30], the use of representations of mathematical concepts [15, 16, 7, 25, 6 and 31] and teachers' and prospective teachers' attitudes and beliefs towards mathematics [17, 8, 1, 29, 23, 28].

In particular, according to reference [21], teachers can help students gain positive attitudes towards mathematics when they: 1) show enthusiasm and enjoy themselves in mathematics 2) use supervisory tools and active ways to approach mathematical concepts, 3) convincing students about the importance of mathematics, 4) preparing activities in which students can achieve, 5) assessing student attitudes and teaching improvements.

Thus, this paper presents instructional practices based on the problem posing and it aims to improve the mathematical perception and ability of students. The didactic interventions were implemented in two school years, 2012-2013 and 2015-2016 in the first grade of the elementary school during the mathematical subject and functioned supportively and complementary to the school books and to curriculum.

II. LITERATURE REVIEW

A. The Problem Posing

Problem posing is the creation of new problems and questions to investigate a given situation, but also the reshaping of a problem when it is solved [26]. Problem posing and solving have been identified as central issues in mathematical education [4], while it is proposed as a tool for the development of mathematical thinking [19]. Research on problem-solving has been shown to be linked to creativity and has a positive impact on students' ability to solve verbal problems [26]. Problem posing by students can lead to more flexible thinking, enhance problem solving skills, broaden the perception of mathematical concepts and consolidate key concepts [9].

On the other hand, perceptions play an important role in performance in mathematics, since they affect the way we learn and use mathematics [10]. In addition, students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics [13]. Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, the sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere [2].

B. Self-confidence in Mathematics

The importance of beliefs in mathematics education is in concordance with the constructivist understanding of teaching and learning. We understand beliefs as "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" [33:358]. Mathematical beliefs can be divided into four main components: beliefs on mathematics, beliefs on oneself as a mathematics learner/applier, beliefs on teaching mathematics, and beliefs on learning mathematics [32]. In this paper we will deal with beliefs on oneself as a mathematics learner/applier, in other words, with self-confidence.

Several studies have shown that the learning of mathematics is influenced by a pupil's mathematics-related beliefs, especially self-confidence [34, 35]. So, self-confidence has a remarkable connection with success in mathematics. However, self-confidence and understanding self-esteem relate to the way of teaching. In the other words, the passage from "Knowledge" (academic) to "learned knowledge" (of the student) is the result of a long and delicate path leading first to the knowledge to be taught, then to the knowledge actually taught and finally to the knowledge learnt. In this sequence the first step of transforming "Knowledge" into "knowledge to teach" is called didactic transposition and constitutes a moment of great importance in which the professionalism and creativity of the teacher are of utmost importance. The present study investigates whether the problem posing in the teaching process can positively affects students' self-confidence in mathematics.

III. THE RESEARCH

A. Methods

The present research followed a qualitative, quantitative approach. Moreover, a content analysis and case study were carried out. Thus a triangulation was formed, which was methodological, temporal, topical and theoretical in order to achieve stabilization of findings [5].

The research has been implemented as teaching interventions in two first-grade classes (6-7 years-old pupils) of a Greek primary school, two hours per week for 6 months (for two school years, 2012-2013 and 2015-2016.). The teaching interventions have been designed and implemented by the research team.

First grade students have been chosen for the research because at this age they firstly address the formal teaching of mathematics. As a result, the effect of other factors such as previous negative experiences in mathematics, erroneous teaching methods or cognitive deficiencies and misunderstandings are minimal.

B. Aim of Research and Participants

The main purpose of the present study is to provide instructional practices over time implemented by the research team, which supported in problem posing in order to help students develop the mathematical understanding and mathematical capacity. That is, students play and learn through original, experiential and modern methods that enhance creativity and critical thinking, love for mathematics with the ultimate goal of developing mathematical ability and student perception.

The sample of the study involved 29 students of the first grade of elementary school in Greece, age 6-7 years. The sample selection was stratified and symptomatic according to the purposes and needs of the research.

C. Instrument

Research tools were the activities that were designed based on the problem posing. Altogether, 40 activities were designed and implemented, which were applied to 38 of the 63 sections of the mathematics schoolbook of the first grade of the elementary school throughout the school year. In addition, the effect of the interventions was measured by questionnaires on the attitudes of students and their cognitive level. The questionnaire was divided in two parts and was designed by the researchers. The first part consisted of 20 clauses in 3-point Likert-type scale regarding the attitudes and beliefs of pupils towards mathematics. There were questions regarding the preference (Bel), the satisfaction (BelEnj), the usefulness (BelUse), the avoidance (BelAvoid) and the motivation (BelMotiv) related to mathematics learning.

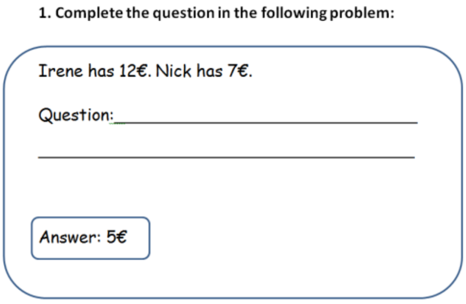

Fig.1 The first task of the second part of the questionnaire

The second part of the questionnaire had four tasks, regarding the problem posing. More specifically, in the first task (figure 1) the students are given the problem and they had to write the question of the problem. In the second task the students had to draw a representation based on a given mathematical proposition (figure 2). The third and fourth tasks were the construction of a problem posing based on a given image which was recorded, as the children of the first elementary school have not fully developed writing.

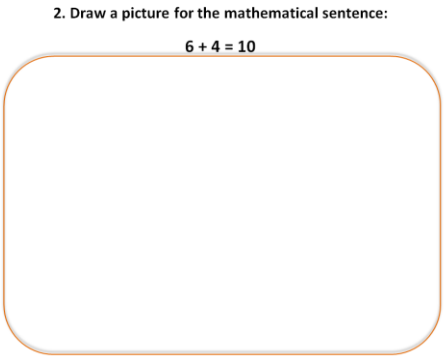

Fig.2 The second task of the second part of the questionnaire

D. Data Analysis

In order to analyze the survey data, and in addition to the descriptive analysis, the Statistical Implicative Analysis by Gras [11], using the CHIC (Cohesive Hierarchical Implicative Classification) software [12] and Microsoft Excel program were used. The implication analysis of data was performed through similarity tree, in which the variables were associated with each other depending on the similarity or non-similarity they present. Variables in whose solution the subjects behave similarly are grouped together.

E. Variables of Research

The variables were defined as a combination of letters and one number. The letters indicate the initial of concept which is examined. For example, the variable PPo1 is composed of the initial proposal "Problem Posing" because the posing problem is examined and number 1 indicates the question of questionnaire. According to the implicative analysis, equivalent to a value of 1 was assigned to every item if the answer is correct and 0 if the answer is wrong or missing.

IV. TEACHING INTERVENTIONS

The implementation of research was performed through teaching interventions with creative activities based on problem posing. 40 activities were designed and implemented, some of which are described below. The activities present the cognitive axis based on mathematical content and processes should students know and be able to use as they progress through school, according to the National Council of Teacher of Mathematics (NCTM), which suggests five content standards, number and operations, algebra, geometry, measurement, and data analysis and probability [19]. In addition, is presented the description of the activities.

A. *Indicative Activity1*

Content: Algebra

Description: Students are given the following figure 3. Students try to pose problems orally and write mathematical sentences in their notebook. One answer given is this: Ioanna has €6 and Saber €2. How many euros does Ioanna have to give to Saber to have the same amount?

Fig.3 Image for problem posing from indicative activity 1.

B. *Indicative Activity 2*

Content: Number and operations

Description: A student draws a picture of figure 4 and the other students try to pose problems. One answer given is this: All the flowers in the vase are 9. The 3 are with concentric circles. How many are the other flowers?

Fig.4 Image for problem posing drawing from student

C. *Indicative Activity 3*

Content: Number and operations

Description: In this activity, students, randomly, take three numbers and try to pose a problem orally and they write the solution in their notebook. For example of the figure 5 the following example was given: In a vase there were 6 flowers. Five of them withered. How many flowers were left in the vase?

Fig.5 Image for problem posing of the indicative activity 3

D. *Indicative Activity 4*

Content: Algebra

Description: Students have a paper target and 10 paper cards with numbers from 0 to 9 (figure 6). One group writes a number in the sum of the target and the other group tries to find out which numbers they can put in the empty circles so that the summation is vertically and horizontally for example 8. Students experiment with paper numbers for to find the solutions. They can use whatever numbers they want. A number cannot be repeated.

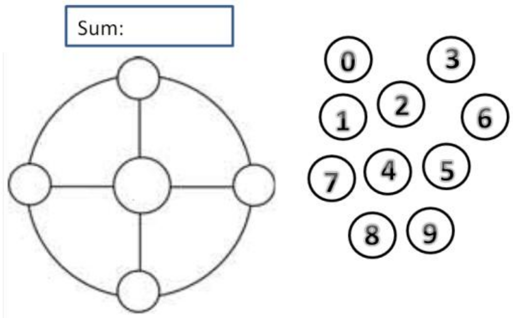

Fig.6 The paper target with numbers

E. *Indicative Activity 5*

Content: Number and operations

Description: Students are given the problem of Figure 7. They have to write two questions and two answers.

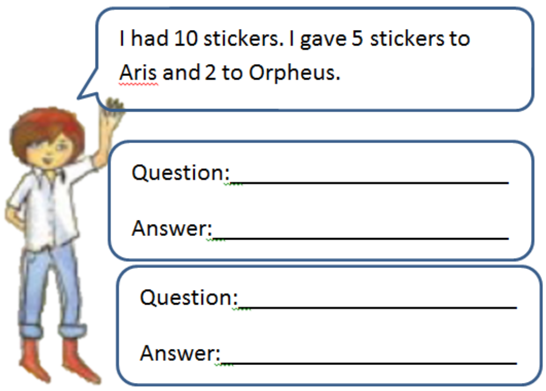

Fig.7 The problem of the indicative activity 5

F. Indicative Activity 6

Content: Numbers and operations, algebra

Description: Students dramatize images and etsh group have to pose problem, orally, and to write mathematical sentences (Figure 8). For example, they were two pigs and another came. How many pigs are there now?

Fig.8 dramatize images for problem posing of the indicative activity 6

G. Indicative Activity 7

Content: geometry

Description: In this activity students have the triangles (Figure 9) and try to pose problem. One problem, that students posed, is the following: Can you construct a rectangle with the three triangles (figure 10)?

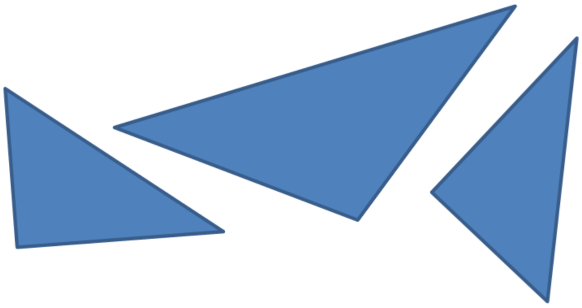

Fig.9 The triangles of the indicative activity 6

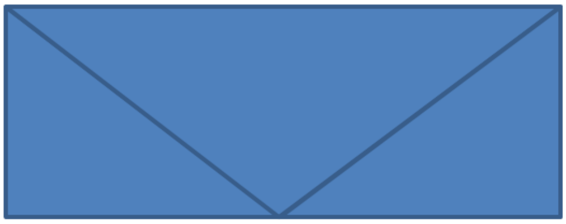

Fig.10 The solution of the problem which students posed

V. RESULTS AND FINDINGS

After implementing of the problem posing activities, 89% of the students said that their favorite lesson was mathematics. Also, 85% of the students said that they feel pleased when they do mathematics at school and 80% of them feel happy when they read mathematics at home. In addition, 96% of students consider mathematics to be a good lesson, and 100% of students said that they like mathematics. 93% of students said that they did not scare mathematics, and 100% said that they wanted to be good at mathematics, because they liked it. Also, 98% of the students said that when they pose problems they felt pleasant. Finally, 80% of students responded correctly to pose problem activities.

According to reference [10], perceptions play an important role in performance in mathematics, since they affect the way we learn and use mathematics. We observe, therefore, that the students who were the sample of our research had positive attitudes and perceptions regarding the mathematics developed through the activities of the problem posing.

In addition, students' perceptions of the usefulness of mathematics can largely explain the dispersion in performance in mathematics and fear of mathematics [13]. Phobia for Mathematics has as a primary cause the students' perception that they cannot improve their math results. This impression affects not only the cognitive domain but also the emotional one, creating a dislike that results in a lack of motivation and interest. Also, reference [2] argues that sources of influencing students' perceptions are the effects of the teacher himself and the classroom atmosphere. In our sample, we observe that 100% of students said that math is a useful lesson, while 93% are not afraid of mathematics, as opposed to research that highlights the aversion and phobia of students about mathematics. In addition, the playful form of problem posing has created a positive atmosphere in the classroom and the students' perception that they can do it, as it is a game and not a lesson.

A. Similarity Analysis

To further understand the connections between problem posing and students' mathematical ability and attitude variables, the following similarity analysis was performed. According to similarity tree (Figure 11), in which the variables were associated with each other depending on the similarity or non-similarity they present – variables in whose solution the subjects behave similarly are grouped together- it is possible to distinguish observe four groups.

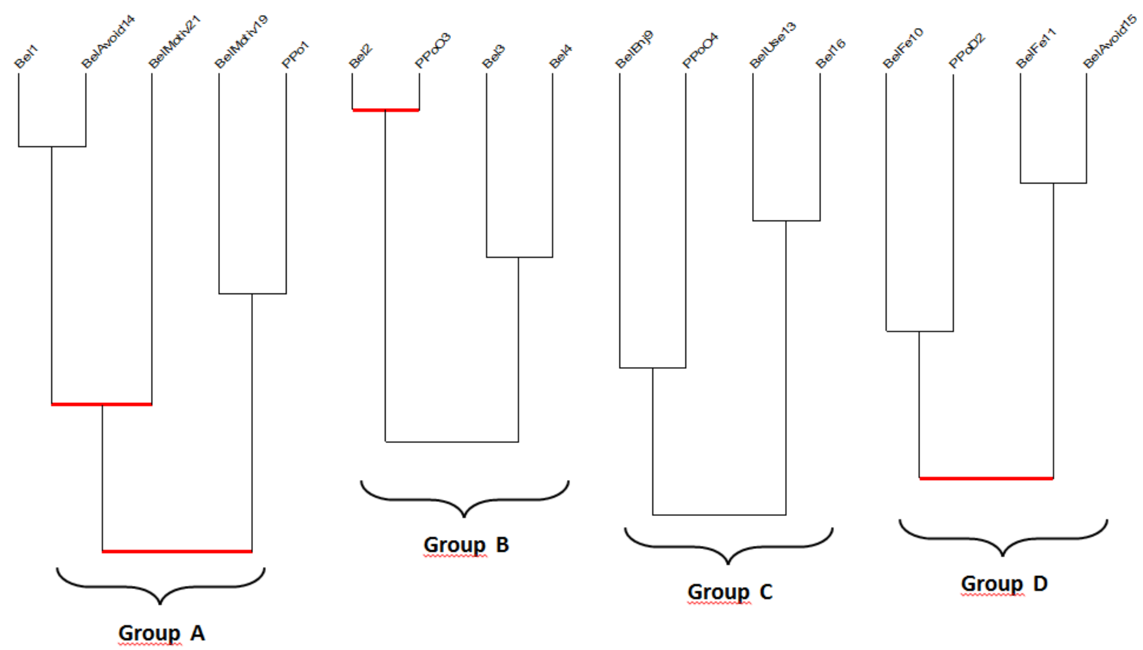

Fig.11 Similarity tree of the solution of the problem which students posed

Based on the similarity tree (Figure 11), we notice that a significant relationship has been created in group B between the variables (Bel2, PPoO3). The students who correctly solved the problem posing (PPoO2) were the ones who stated that they choose to read mathematics as a first home lesson (Bel2). These two variables (Bel2, PPoO3) of group B are then linked to the variables (Bel3 Bel4). In these variables, students stated that when they do mathematics at school and at home they feel comfortable (Bel3 and Bel4, respectively).

Another important relationship created in the similarity tree is that of the A group between the variables ((((Bel1 BelAvoid14) BelMotiv21) (BelMotiv19 PPo1)). The students who were able to formulate the question of a problem with the given answer (PPo1) are those who say they want to be good at mathematics, not to please their parents (BelMotiv19). This student profile states that he loves mathematics (Bel1) and that he wants to be one of the best students in mathematics (BelMotiv21).

In Group D, between the variables ((BelFe10 PPoD2) (BelFe11 BelAvoid15)) an important relationship has been created. In particular, students who were able to pose a problem by representing from a mathematical proposition (PPoD2) stated that they were not afraid of mathematics (BelFe10), they felt confident about themselves (BelFe11) and did not avoid doing mathematics (BelAvoid15).

In addition, in group C we have the relationship of the variables (BelEnj9 PPoO4) (BelUse13 Bel16)). That is, students who were able to pose a problem from a given image (PPoO4) stated that they would like to do more math hours at school (BelEnj9). This student profile recognizes that people who do not know mathematics, will have difficulty in their lives (BelUse13) and state that they like to solve math exercises.

V. CONCLUSIONS

In the present research a part of the activities implemented in the 1st grade of the elementary school was presented in the course of the mathematics, course based on the problem posing. These activities, which designed by the researchers themselves, were carried out for two school years in elementary school pupils in order to complement the mathematics course and to develop students' mathematical ability and perception, as both the international literature and international reviews show disappointing evidence about our pupils' performance in mathematics and highlight the phobia and aversion that students feel about them [13, 2].

The results of this research showed that students who participated in the activities of problem posing said they were happy to study mathematics at school and at home, that they loved mathematics and wanted to be one of the best students in them. It was also shown that the problem position helped students to remove mathematics phobia and strengthen students' confidence in mathematics. Finally, it helped them recognize the importance of mathematics in people's lives by stating that people who don't know mathematics, will have a hard time in their lives.

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