# A study of Jeffery Fluid Flow in a Vertical Channel with wall slip and Hall current 

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#### Abstract

This paper is devoted to the study of MHD Jeffrey fluid in a vertical channel in presence of wall slip and Hall current. The channel is caused due to peristaltic transport on the walls having different amplitudes and phase. The analytical solution has been carried out by using the long-wave approximation and low Reynolds number. Closed form expressions for velocity and temperature are developed. The expressions for the zeroth-order and the first order solutions are obtained and the results are presented graphically for different values of parameters entering into the problem


Keywords - MHD, Hall current, wall slip, Jeffery fluid, asymmetric channel.

## I. INTRODUCTION

The problem in mechanism of peristaltic transport has attracted the attention of many investigators and is still in demand due to its practical applications especially in physiology. Examples include urine transport from kidney to bladder through the ureter, chyme movement inside the gastro-intestinal tract, transport of spermatozoa in the ductus efferentes of the male reproductive tracts and so on. Since the first investigation of Latham [1], a number of analytical, numerical and experimental studies on peristaltic flow of different fluids have been reported under different conditions with reference to physiological and mechanical situations. In view of these applications, Hayat et al. [10] have investigated the series solutions for magnetohydrodynamic flow of a Jeffery fluid in a porous channel by using powerful analytic method namely the homotopy analysis method (HAM). Srinivas and Muthuraj [11] have analyzed the MHD mixed convective heat and mass transfer peristaltic flow through a vertical porous space in presence of a chemical reaction Srinivas and Muthuraj [12] have analyzed the problem of MHD peristaltic transport of a Jeffrey fluid in an inclined asymmetric channel under the influence of slip condition near the channel wall using long wavelength and low Reynolds number approximations.Muthuraj and Srinivas [13] have discussed MHD peristaltic flow of a Newtonian fluid through porous space in a vertical channel with compliant walls. Nadeem and Akram [14] have examined the peristaltic transport of couple-stress fluid in an asymmetric channel with an induced magnetic field under the assumptions of long wave length and low but finite Reynolds number. Eldabe et al. [15] have analyzed the effect of wall properties on the peristaltic transport of a dusty fluid with heat and mass transfer by using perturbation technique for small geometric parameter. Vajravelu et al. [16] have studied he peristaltic flow of a Jeffrey fluid
in a vertical porous stratum with heat transfer under long wavelength and low Reynolds number assumptions. The effect of heat and mass transfer in MHD peristaltic flow of a Maxwell fluid in a planar channel with compliant walls was studied by Hayat and Hina [17].

From the Literature survey, it is evident that, several authors have presented the fluid flow investigations when the flow field obeys the conventional no-slip condition. However, there are fluids, such as, polymeric materials that slip or stick-slip on solid boundaries. Such flow under an applied pressure gradient yields a sudden increase in the throughout at a critical pressure gradient causing "spurt". Ellahi et al. [18] have presented the analysis of steady flow of a third grade fluid between the concentric cylinders. Nadeem and Akram [19] have discussed the effects of partial slip on the peristaltic flow of a MHD Newtonian fluid in an asymmetric channel by using Adomian decomposition method. Akbar et al. [20] have investigated the peristaltic flow of a Williamson fluid in an inclined asymmetric channel in presence of velocity and thermal slip conditions. Ahmad et al. [21] have investigated the effects of Hall current on unsteady MHD flows of a second grade fluid. Ali et al.[22] have investigated the effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate. Srinivasacharya and Kaladhar [23] have presented the Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects.

To the best of the author's knowledge, no research work is carried out in peristaltic transport of Jeffrey fluid including the Hall effects and wall slip condition. Therefore, the main purpose of the present study is to investigate the influences of hall current and wall slip on peristaltic transport of a Jeffery fluid flow in a vertical channel. The study of such flows with Hall currents has an important role to play in many engineering problems of MHD generators and of Hall accelerators as well as in flight magnetohydrodynamics. In the present paper, the problem first formulated in wave frame of reference and then analytic solutions to the velocity and temperature are presented. The organization of the paper is as follows. The problem is formulated in section 2. Section 3 deals with the solution to the problem. Results and discussion are given in section 4. The conclusions have been summarized in section 5 .
II. Formulation of the problem: Consider an incompressible Jeffery fluid filling two-dimensional vertical channel (Fig.1) induced by sinusoidal wave trains propagating with constant speed c along the channel walls

$$
\begin{align*}
& \mathrm{H}_{1}=\mathrm{d}_{1}+\mathrm{a}_{1} \cos \left(\frac{2 \pi}{\lambda}(\mathrm{X}-\mathrm{ct})\right)  \tag{1}\\
& \mathrm{H}_{2}=-\mathrm{d}_{2}-\mathrm{b}_{1} \cos \left(\frac{2 \pi}{\lambda}(\mathrm{X}-\mathrm{ct})+\varphi\right) \tag{2}
\end{align*}
$$

where $\mathrm{a}_{1}, \mathrm{~b}_{1}$ are the amplitudes of the waves, $\lambda$ is the wavelength, $\mathrm{d}_{1}+\mathrm{d}_{2}$ is the width of the channel, the phase difference $\varphi$ varies in the range $0 \leq \varphi \leq \pi$. Further $a_{1}, b_{1}, d_{1}, d_{2}$ and $\varphi$ satisfies the condition

$$
\begin{equation*}
\mathrm{a}_{1}^{2}+\mathrm{b}_{1}^{2}+2 \mathrm{a}_{1} \mathrm{~b}_{1} \cos \varphi \leq\left(\mathrm{d}_{1}+\mathrm{d}_{2}\right)^{2} \tag{3}
\end{equation*}
$$

The temperature at right hand side wall is $\mathrm{T}_{1}^{\prime}$ and concentration is $\mathrm{C}_{1}^{\prime}$ while the temperature at the left hand side wall is $\mathrm{T}_{2}^{\prime}\left(\mathrm{T}_{2}^{\prime}>\mathrm{T}_{1}^{\prime}\right)$ and concentration is $\mathrm{C}_{2}^{\prime}$

The constitutive equations for an incompressible Jeffrey fluid are

$$
\begin{aligned}
& \overline{\mathrm{T}}=-\overline{\mathrm{p}} \overline{\mathrm{I}}+\overline{\mathrm{S}} \\
& \overline{\mathrm{~S}}=\frac{\mu}{1+\lambda_{1}}\left(\dot{\gamma}+\lambda_{2} \ddot{\gamma}\right)
\end{aligned}
$$

where $\overline{\mathrm{T}}$ and $\overline{\mathrm{S}}$ are Cauchy stress tensor and extra stress tensor respectively, $\overline{\mathrm{p}}$ is the pressure, $\overline{\mathrm{I}}$ is the identity tensor, $\lambda_{1}$ is the ratio of relaxation to retardation times, $\lambda_{2}$ is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities indicate differentiation with respect to time. Under the assumption of Boussinesq approximation, we shall investigate a coordinate system, moving with the wave speed c , in which the boundary shape is stationary. Defining in wave frame $(\mathrm{x}, \mathrm{y})$, the velocity components $(\mathrm{u}, \mathrm{v})$ and pressure p by

$$
\overline{\mathrm{x}}=\mathrm{X}-\mathrm{ct}, \mathrm{y}=\mathrm{Y}, \overline{\mathrm{u}}=\mathrm{U}-\mathrm{c}, \overline{\mathrm{v}}=\mathrm{V}, \mathrm{p}(\mathrm{x})=\mathrm{P}(\mathrm{X}, \mathrm{t})
$$

where $\overline{\mathrm{u}}, \overline{\mathrm{v}}$ are the velocity components in the wave frame $(\overline{\mathrm{x}}, \overline{\mathrm{y}}), \overline{\mathrm{p}}$ and $\overline{\mathrm{P}}$ are pressures in wave and fixed frame of references respectively.

$$
\begin{aligned}
& \overline{\mathrm{x}}=\frac{\mathrm{x}}{\lambda}, \overline{\mathrm{y}}=\frac{\mathrm{y}}{\mathrm{~d}_{1}}, \overline{\mathrm{u}}=\frac{\mathrm{u}}{\mathrm{c}}, \overline{\mathrm{v}}=\frac{\mathrm{v}}{\mathrm{c} \delta}, \mathrm{p}=\frac{\mathrm{d}_{1}^{2} \mathrm{p}}{\mu \mathrm{c} \lambda}, \overline{\mathrm{t}}=\frac{\mathrm{ct}}{\lambda}, \mathrm{~h}_{1}=\frac{\mathrm{H}_{1}}{\mathrm{~d}_{1}}, \mathrm{~h}_{2}=\frac{\mathrm{H}_{2}}{\mathrm{~d}_{1}}, \mathrm{~d}=\frac{\mathrm{d}_{2}}{\mathrm{~d}_{1}}, \\
& \delta=\frac{\mathrm{d}_{1}}{\lambda}, \mathrm{a}=\frac{\mathrm{a}_{1}}{\mathrm{~d}_{1}}, \theta=\frac{\mathrm{T}-\overline{\mathrm{T}}}{\mathrm{~T}_{2}^{\prime}-\overline{\mathrm{T}}}, \phi=\frac{\mathrm{C}-\overline{\mathrm{C}}}{\mathrm{C}_{2}^{\prime}-\overline{\mathrm{C}}}, \mathrm{~b}=\frac{\mathrm{b}_{1}}{\mathrm{~d}_{1}}
\end{aligned}
$$

Introducing the above non-dimensional variables, the non- dimensional form basic field equations can be expressed (See Refs. [11, 15])

$$
\begin{align*}
& \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0  \tag{4}\\
& \delta \operatorname{Re}\left(u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left[\delta \frac{\partial}{\partial x}\left(S_{x x}\right)+\frac{\partial}{\partial y}\left(S_{X Y}\right)\right]+\frac{M^{2}}{1+m_{1}^{2}}\left[\delta m_{1} v-(u+1)\right]+G_{t} \theta+G_{c} \phi \tag{5}
\end{align*}
$$

$\delta^{3} \operatorname{Re}\left(\mathrm{u} \frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\mathrm{v} \frac{\partial \mathrm{v}}{\partial \mathrm{y}}\right)=-\frac{\partial \mathrm{p}}{\partial \mathrm{y}}+\delta\left[\delta \frac{\partial}{\partial \mathrm{x}}\left(\mathrm{S}_{\mathrm{XY}}\right)+\frac{\partial}{\partial \mathrm{y}}\left(\mathrm{S}_{\mathrm{YY}}\right)\right]+\frac{\mathrm{M}^{2} \delta}{1+\mathrm{m}_{1}^{2}}\left[\mathrm{~m}_{1}(\mathrm{u}+1)+\mathrm{v} \delta\right]$

$$
\begin{align*}
& \delta \operatorname{ReP}_{\mathrm{r}}\left(\mathrm{u} \frac{\partial \theta}{\partial \mathrm{x}}+\mathrm{v} \frac{\partial \theta}{\partial \mathrm{y}}\right)=\left[\delta^{2} \frac{\partial^{2} \theta}{\partial \mathrm{x}^{2}}+\frac{\partial^{2} \theta}{\partial \mathrm{y}^{2}}\right]+2 \mathrm{~B}_{\mathrm{r}} \delta^{2}\left[\left(\frac{\partial \mathrm{u}}{\partial \mathrm{x}}\right)^{2}+\left(\frac{\partial \mathrm{v}}{\partial \mathrm{y}}\right)^{2}\right]+\mathrm{B}_{\mathrm{r}}\left[\frac{\partial \mathrm{u}}{\partial \mathrm{y}}+\delta^{2} \frac{\partial \mathrm{v}}{\partial \mathrm{x}}\right]^{2}+\alpha \theta  \tag{7}\\
& \delta \operatorname{Re}\left(\mathrm{u} \frac{\partial \phi}{\partial \mathrm{x}}+\mathrm{v} \frac{\partial \phi}{\partial \mathrm{y}}\right)=\frac{1}{\mathrm{~S}_{\mathrm{c}}}\left[\delta^{2} \frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}+\frac{\partial^{2} \phi}{\partial \mathrm{y}^{2}}\right]+\mathrm{S}_{\mathrm{r}}\left[\delta^{2} \frac{\partial^{2} \theta}{\partial \mathrm{x}^{2}}+\frac{\partial^{2} \theta}{\partial \mathrm{y}^{2}}\right] \tag{8}
\end{align*}
$$

where $\operatorname{Re}=\frac{\rho c d_{1}}{\mu}$ is the Reynolds number, $P_{r}=\frac{\mu c p}{k}$ is the Prandtl number, $S_{c}=\frac{\mu}{\rho D_{m}}$ is the Schmidt number, $\mathrm{S}_{\mathrm{r}}=\frac{\rho \mathrm{D}_{\mathrm{m}} \mathrm{k}_{\mathrm{T}}\left(\mathrm{T}_{1}^{\prime}-\overline{\mathrm{T}}\right)}{\overline{\mathrm{T}}\left(\mathrm{C}_{1}^{\prime}-\overline{\mathrm{C}}\right) \mu}$ is the Soret number, $\mathrm{G}_{\mathrm{t}}=\frac{\rho \mathrm{g} \beta_{\mathrm{t}} \mathrm{d}_{1}^{2}\left(\mathrm{~T}_{1}^{\prime}-\overline{\mathrm{T}}\right)}{\mu \mathrm{c}}$ is the local temperature Grashof number, $\mathrm{G}_{\mathrm{c}}=\frac{\rho \mathrm{g} \beta_{\mathrm{c}} \mathrm{d}_{1}^{2}\left(\mathrm{C}_{1}^{\prime}-\overline{\mathrm{C}}\right)}{\mu \mathrm{c}}$ is the local mass Grashof number, $\mathrm{M}^{2}=\frac{\sigma \mathrm{B}_{0}^{2} \mathrm{~d}_{1}^{2}}{\mu}$ is the Hartmann number, $E_{c}=\frac{c^{2}}{c_{p}\left(T_{1}^{\prime}-\bar{T}\right)}$ is the Eckert number, $B_{r}=P_{r} E_{c}$ is the Brinkman number, $m_{1}$ is the Hall effect parameter, $\delta=\frac{\mathrm{d}_{1}}{\lambda}$ is the dimensionless wave number and $\alpha=\frac{\mathrm{Qd}_{1}^{2}}{\mathrm{~K}}$ is the heat source parameter, $\lambda_{1}$ is the ratio of relaxation to retardation times, $\lambda_{2}$ is the retardation time, $g$ is the acceleration due to gravity, $B_{0}$ is the applied magnetic field, $\sigma$ is the electrical conductivity, K is thermal conductivity, T is the temperature of the fluid, C is the concentration of the fluid, $\overline{\mathrm{T}}$ is the mean value of wall temperatures, $\overline{\mathrm{C}}$ is the mean value of wall concentrations, $\rho$ is density of fluid, $\mu$ is dynamic viscosity of the fluid, Q is absorption coefficient of the fluid, $\beta_{\mathrm{t}}$ is the co-efficient of thermal expansion, $\beta_{\mathrm{c}}$ is the co-efficient of expansion with concentration, $\mathrm{c}_{\mathrm{p}}$ is the specific heat at constant pressure, $\mathrm{D}_{\mathrm{m}}$ is the co-efficient of mass diffusivity, $\mathrm{k}_{\mathrm{T}}$ is the thermal-diffusion ratio. Introducing the dimensionless stream function $\psi(\mathrm{x}, \mathrm{y})$ such that

$$
\begin{equation*}
u=\frac{\partial \psi}{\partial y} \text { and } v=-\frac{\partial \psi}{\partial x} \tag{9}
\end{equation*}
$$

and eliminating the pressure gradient, equations (5) - (8) becomes

$$
\begin{aligned}
\delta \operatorname{Re}\left[\left(\psi_{y} \psi_{x y y}-\psi_{x} \psi_{y y y}\right)+\delta^{2}\left(\psi_{y} \psi_{x x x}-\psi_{x} \psi_{x x y}\right)\right] & =\left[\delta \frac{\partial^{2}}{\partial x \partial y}\left(S_{x x}-S_{y y}\right)+\left(\frac{\partial^{2}}{\partial y^{2}}-\delta^{2} \frac{\partial^{2}}{\partial x^{2}}\right) S_{x y}\right]_{(10)} \\
& -\frac{\mathrm{M}^{2}}{1+\mathrm{m}_{1}^{2}}\left[\psi_{y y}+\delta^{2} \psi_{x x}\right]+G_{t} \theta_{y}+G_{c} \phi_{y}
\end{aligned}
$$

$\delta \operatorname{Re} P_{r}\left[\psi_{y} \theta_{x}-\psi_{x} \theta_{y}\right]=\left(\delta^{2} \theta_{x x}+\theta_{y y}\right)+B_{r}\left[4 \delta^{2}\left(\psi_{x y}\right)^{2}+\left(\psi_{y y}-\delta^{2} \psi_{x x}\right)^{2}\right]+\alpha \theta$
$\delta \operatorname{Re}\left[\psi_{y} \phi_{x}-\psi_{x} \phi_{y}\right]=\frac{1}{S_{c}}\left(\delta^{2} \phi_{x x}+\phi_{y y}\right)+S_{r}\left(\delta^{2} \theta_{x x}+\theta_{y y}\right)$
where

$$
\begin{align*}
& \mathrm{S}_{\mathrm{xx}}=\frac{2 \delta}{1+\lambda_{1}}\left[1+\frac{\delta \lambda_{2} \mathrm{c}}{\mathrm{~d}_{1}}\left(\frac{\partial \psi}{\partial \mathrm{y}} \frac{\partial}{\partial \mathrm{x}}-\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial}{\partial \mathrm{y}}\right)\right] \frac{\partial^{2} \psi}{\partial \mathrm{x} \partial \mathrm{y}}  \tag{13}\\
& \mathrm{~S}_{\mathrm{xy}}=\frac{1}{1+\lambda_{1}}\left[1+\frac{\delta \lambda_{2} \mathrm{c}}{\mathrm{~d}_{1}}\left(\frac{\partial \psi}{\partial \mathrm{y}} \frac{\partial}{\partial \mathrm{x}}-\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial}{\partial \mathrm{y}}\right)\right]\left(\frac{\partial^{2} \psi}{\partial \mathrm{y}^{2}}-\delta^{2} \frac{\partial^{2} \psi}{\partial \mathrm{x}^{2}}\right)  \tag{14}\\
& \mathrm{S}_{\mathrm{yy}}=-\frac{2 \delta}{1+\lambda_{1}}\left[1+\frac{\delta \lambda_{2} \mathrm{c}}{\mathrm{~d}_{1}}\left(\frac{\partial \psi}{\partial \mathrm{y}} \frac{\partial}{\partial \mathrm{x}}-\frac{\partial \psi}{\partial \mathrm{x}} \frac{\partial}{\partial \mathrm{y}}\right)\right] \frac{\partial^{2} \psi}{\partial \mathrm{x} \partial \mathrm{y}} \tag{15}
\end{align*}
$$

By using long wavelength approximation and neglecting the wave number along with low-Reynolds number, the equations (10) - (15) become

$$
\begin{align*}
& \frac{\partial^{4} \psi}{\partial \mathrm{y}^{4}}-\mathrm{H}\left(1+\lambda_{1}\right) \frac{\partial^{2} \psi}{\partial \mathrm{y}^{2}}+\mathrm{G}_{\mathrm{t}}\left(1+\lambda_{1}\right) \frac{\partial \theta}{\partial \mathrm{y}}+\mathrm{G}_{\mathrm{c}}\left(1+\lambda_{1}\right) \frac{\partial \phi}{\partial \mathrm{y}}=0  \tag{16}\\
& \frac{\partial^{2} \theta}{\partial \mathrm{y}^{2}}+\mathrm{B}_{\mathrm{r}}\left(\frac{\partial^{2} \psi}{\partial \mathrm{y}^{2}}\right)^{2}+\alpha \theta=0  \tag{17}\\
& \frac{\partial^{2} \phi}{\partial \mathrm{y}^{2}}+\mathrm{S}_{\mathrm{c}} \mathrm{~S}_{\mathrm{r}}\left(\frac{\partial^{2} \theta}{\partial \mathrm{y}^{2}}\right)=0 \tag{18}
\end{align*}
$$

The corresponding boundary conditions will be
$\psi=\frac{\mathrm{q}}{2}, \quad \psi_{\mathrm{y}}+\mathrm{L} \psi_{\mathrm{yy}}=-1, \quad \theta=1, \quad \phi=1 \quad$ at $\quad \mathrm{y}=\mathrm{h}_{1}=1+\mathrm{a} \cos 2 \pi \mathrm{x}$
$\psi=-\frac{\mathrm{q}}{2}, \quad \psi_{\mathrm{y}}-\mathrm{L} \psi_{\mathrm{yy}}=-1, \quad \theta=-1, \quad \phi=-1 \quad$ at $\quad \mathrm{y}=\mathrm{h}_{2}=-\mathrm{d}-\mathrm{b} \cos (2 \pi \mathrm{x}+\varphi)$
where $L\left(=\frac{\beta}{1+\lambda_{1}}\right)$ is the dimensional slip parameter.
In which q is the flux in the wave frame, $\beta$ is the slip parameter and $\mathrm{a}, \mathrm{b}, \varphi$ and d satisfy the relation

$$
\begin{equation*}
\mathrm{a}^{2}+\mathrm{b}^{2}+2 \mathrm{ab} \cos \varphi \leq(1+\mathrm{d})^{2} \tag{21}
\end{equation*}
$$

The flux at any axial station in the fixed frame is

$$
\begin{equation*}
\mathrm{Q}=\int_{\mathrm{h}_{2}}^{\mathrm{h}_{1}}\left(\frac{\partial \psi}{\partial \mathrm{y}}+1\right) \mathrm{dy}=\mathrm{h}_{1}-\mathrm{h}_{2}+\mathrm{q} \tag{22}
\end{equation*}
$$

The average volume flow rate over one period of the peristaltic wave is defined as

$$
\begin{equation*}
\Phi=\frac{1}{\mathrm{~T}} \int_{0}^{\mathrm{T}} \mathrm{Qdt}=\frac{1}{\mathrm{~T}} \int_{0}^{\mathrm{T}}\left(\mathrm{q}+\mathrm{h}_{1}-\mathrm{h}_{2}\right) \mathrm{dt}=\mathrm{q}+1+\mathrm{d} . \tag{23}
\end{equation*}
$$

The pressure gradient is obtained from the dimensionless momentum equation for the axial velocity as

$$
\begin{equation*}
\frac{\partial \mathrm{p}}{\partial \mathrm{x}}=\frac{1}{1+\lambda_{1}} \frac{\partial^{3} \psi}{\partial \mathrm{y}^{3}}-\mathrm{H}(\mathrm{u}+1)+\mathrm{G}_{\mathrm{t}} \theta+\mathrm{G}_{\mathrm{c}} \phi \tag{24}
\end{equation*}
$$

The non-dimensional expression for pressure rise per wavelength $\Delta P_{\lambda}$ is given as follows

$$
\begin{equation*}
\Delta \mathrm{p}_{\lambda}=\int_{0}^{2 \pi}\left(\frac{\partial \mathrm{p}}{\partial \mathrm{x}}\right) \mathrm{dx} \tag{25}
\end{equation*}
$$

The non-dimensional shear stress (14) at the upper wall of the channel is reduced to

$$
\begin{equation*}
S_{x y}=\frac{1}{1+\lambda_{1}} \frac{\partial^{2} \psi}{\partial y^{2}} \tag{26}
\end{equation*}
$$

The Coefficient of heat transfer at the right wall is

$$
\begin{equation*}
\mathrm{Z}=\frac{\partial \mathrm{h}_{1}}{\partial \mathrm{x}} \frac{\partial \theta}{\partial \mathrm{y}} \tag{27}
\end{equation*}
$$

## III. Method of solution

To have a solution for a system of equations (16-18) subjected to the boundary conditions (19-20), we assume the following perturbation method for small geometric parameter (i.e., $\mathrm{B}_{\mathrm{r}} \ll 1$ ) as:
$\psi=\psi_{0}+\mathrm{B}_{\mathrm{r}} \psi_{1}+\mathrm{B}_{\mathrm{r}}^{2} \psi_{2}+\ldots$
$\phi=\phi_{0}+\mathrm{B}_{\mathrm{r}} \phi_{1}+\mathrm{B}_{\mathrm{r}}^{2} \phi_{2}+\ldots$
$\theta=\theta_{0}+B_{r} \theta_{1}+B_{r}^{2} \theta_{2}+\ldots$
Substituting equations (28-30) in equations (16-20) and collecting the coefficient of various powers
of $B_{r}$ on both sides, we obtain the following sets of equations:

## Zeroth order equation:

$$
\begin{align*}
& \frac{\partial^{4} \psi_{0}}{\partial \mathrm{y}^{4}}-\mathrm{H}\left(1+\lambda_{1}\right) \frac{\partial^{2} \psi_{0}}{\partial \mathrm{y}^{2}}=-\mathrm{G}_{\mathrm{t}}\left(1+\lambda_{1}\right) \frac{\partial \theta_{0}}{\partial \mathrm{y}}-\mathrm{G}_{\mathrm{c}}\left(1+\lambda_{1}\right) \frac{\partial \phi_{0}}{\partial \mathrm{y}}  \tag{31}\\
& \frac{\partial^{2} \theta_{0}}{\partial \mathrm{y}^{2}}+\alpha \theta_{0}=0 \tag{32}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial^{2} \phi_{0}}{\partial \mathrm{y}^{2}}+\mathrm{S}_{\mathrm{c}} \mathrm{~S}_{\mathrm{r}}\left(\frac{\partial^{2} \theta_{0}}{\partial \mathrm{y}^{2}}\right)=0  \tag{33}\\
& \Psi_{0}=\frac{\mathrm{q}}{2}, \frac{\partial \psi_{0}}{\partial \mathrm{y}}+\mathrm{L} \frac{\partial^{2} \psi_{0}}{\partial \mathrm{y}^{2}}=-1, \quad \theta_{0}=1, \quad \phi_{0}=1 \quad \text { at } \mathrm{y}=\mathrm{h}_{1}  \tag{34}\\
& \Psi_{0}=-\frac{\mathrm{q}}{2}, \frac{\partial \psi_{0}}{\partial \mathrm{y}}-\mathrm{L} \frac{\partial^{2} \psi_{0}}{\partial \mathrm{y}^{2}}=-1, \quad \theta_{0}=-1, \phi_{0}=-1 \quad \text { at } \mathrm{y}=\mathrm{h}_{2} \tag{35}
\end{align*}
$$

## First order equation:

$$
\begin{align*}
& \frac{\partial^{4} \psi_{1}}{\partial \mathrm{y}^{4}}-\mathrm{H}\left(1+\lambda_{1}\right) \frac{\partial^{2} \psi_{1}}{\partial \mathrm{y}^{2}}=-\mathrm{G}_{\mathrm{t}}\left(1+\lambda_{1}\right) \frac{\partial \theta_{1}}{\partial \mathrm{y}}-\mathrm{G}_{\mathrm{c}}\left(1+\lambda_{1}\right) \frac{\partial \phi_{1}}{\partial \mathrm{y}}  \tag{36}\\
& \frac{\partial^{2} \theta_{1}}{\partial \mathrm{y}^{2}}+\mathrm{B}_{\mathrm{r}}\left(\frac{\partial^{2} \psi_{0}}{\partial \mathrm{y}^{2}}\right)^{2}+\alpha \theta_{1}=0  \tag{37}\\
& \frac{\partial^{2} \phi_{1}}{\partial \mathrm{y}^{2}}+\mathrm{S}_{\mathrm{c}} \mathrm{~S}_{\mathrm{r}}\left(\frac{\partial^{2} \theta_{1}}{\partial \mathrm{y}^{2}}\right)=0  \tag{38}\\
& \Psi_{1}=0, \frac{\partial \psi_{1}}{\partial \mathrm{y}}+\mathrm{L} \frac{\partial^{2} \psi_{1}}{\partial \mathrm{y}^{2}}=0, \quad \theta_{1}=0, \phi_{1}=0 \quad \text { at } \mathrm{y}=\mathrm{h}_{1}  \tag{39}\\
& \Psi_{1}=0, \frac{\partial \psi_{1}}{\partial \mathrm{y}}-\mathrm{L} \frac{\partial^{2} \psi_{1}}{\partial \mathrm{y}^{2}}=0, \quad \theta_{1}=0, \phi_{1}=0 \quad \text { at } \mathrm{y}=\mathrm{h}_{2} \tag{40}
\end{align*}
$$

Solving equations (31-33) and (36-38) together with the boundary conditions (34-35) and (39-40), we get the stream functions, velocity, temperature and concentration of fluid as:
$\psi_{0}=\mathrm{E}+\mathrm{Fy}+\mathrm{A}_{1} \cosh \beta_{2} \mathrm{y}+\mathrm{B}_{1} \sinh \beta_{2} \mathrm{y}+\mathrm{T}_{8} \sinh \beta_{1} \mathrm{y}+\mathrm{T}_{9} \cosh \beta_{1} \mathrm{y}+\mathrm{T}_{10} \mathrm{y}^{2}$
$u_{0}=F+A_{1} \beta_{2} \sinh \beta_{2} y+B_{1} \beta_{2} \cosh \beta_{2} y+T_{8} \beta_{1} \cosh \beta_{1} y+T_{9} \beta_{1} \sinh \beta_{1} y+2 T_{10} y$
$\theta_{0}=A \cosh \beta_{1} y+B \sinh \beta_{1} y$
$\phi_{0}=C y+D+T_{3} \cosh \beta_{1} y+T_{4} \sinh \beta_{1} y$
$\psi_{1}=\mathrm{A}_{4}+\mathrm{B}_{4} \mathrm{y}+\cosh \beta_{2} \mathrm{y}\left(\mathrm{A}_{5}+\mathrm{T}_{91} \mathrm{y}\right)+\sinh \beta_{2} \mathrm{y}\left(\mathrm{B}_{5}+\mathrm{T}_{92} \mathrm{y}\right)+\cosh \beta_{1} \mathrm{y}\left(\mathrm{T}_{83}+\mathrm{T}_{89} \mathrm{y}\right)+$
$\mathrm{T}_{84} \sinh 2 \beta_{2} \mathrm{y}+\mathrm{T}_{85} \cosh 2 \beta_{2} \mathrm{y}+\mathrm{T}_{86} \cosh 2 \beta_{1} \mathrm{y}+\mathrm{T}_{87} \sinh \left(\beta_{1}+\beta_{2}\right) \mathrm{y}+\mathrm{T}_{88} \cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{y}$ $+\mathrm{T}_{93} \mathrm{y}^{2}+\mathrm{T}_{94} \sinh \beta_{1} \mathrm{y}+\mathrm{S}_{6} \sinh 2 \beta_{1} \mathrm{y}$

$$
\begin{align*}
& \mathrm{u}_{1}=\mathrm{B}_{4}+\sinh \beta_{2} \mathrm{y}\left(\mathrm{~A}_{5} \beta_{2}+\mathrm{T}_{92}+\mathrm{T}_{91} \beta_{2} \mathrm{y}\right)+\cosh \beta_{2} \mathrm{y}\left(\mathrm{~B}_{5} \beta_{2}+\mathrm{T}_{91}+\mathrm{T}_{92} \beta_{2} \mathrm{y}\right)+ \\
& \sinh \beta_{1} y\left(T_{83} \beta_{1}+\mathrm{T}_{89} \beta_{1} y\right)+2 \mathrm{~T}_{84} \beta_{2} \cosh 2 \beta_{2} y+2 \mathrm{~T}_{85} \beta_{2} \sinh 2 \beta_{2} y+2 \mathrm{~T}_{86} \beta_{1} \sinh 2 \beta_{1} y+ \\
& \mathrm{T}_{87}\left(\beta_{1}+\beta_{2}\right) \cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{y}+\mathrm{T}_{88}\left(\beta_{1}-\beta_{2}\right) \sinh \left(\beta_{1}-\beta_{2}\right) \mathrm{y}+\cosh \beta_{1} \mathrm{y}\left(\mathrm{~T}_{89}+\mathrm{T}_{94} \beta_{1}\right)  \tag{46}\\
& +2 \mathrm{~T}_{93} \mathrm{y}+2 \mathrm{~S}_{6} \beta_{1} \cosh 2 \beta_{1} \mathrm{y} \\
& \theta_{1}=A_{2} \cosh \beta_{1} y+\sinh \beta_{1} y\left(B_{2}+T_{45} y\right)+T_{40} \cosh 2 \beta_{2} y+T_{41} \sinh 2 \beta_{2} y+ \\
& \mathrm{T}_{42} \sinh 2 \beta_{1} \mathrm{y}+\mathrm{T}_{43} \cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{y}+\mathrm{T}_{44} \cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{y}+  \tag{47}\\
& \mathrm{T}_{46} \cosh \beta_{2} \mathrm{y}+\mathrm{T}_{47}+\mathrm{S}_{2} \cosh 2 \beta_{1} \mathrm{y} \\
& \phi_{1}=\mathrm{A}_{3}+\mathrm{B}_{3} \mathrm{y}+\cosh \beta_{1} \mathrm{y}\left(\mathrm{~T}_{59}+\mathrm{T}_{67}\right)+\sinh \beta_{1} \mathrm{y}\left(\mathrm{~T}_{60}+\mathrm{T}_{66} \mathrm{y}\right)+\mathrm{T}_{61} \cosh 2 \beta_{2} \mathrm{y}+ \\
& \mathrm{T}_{62} \sinh 2 \beta_{2} \mathrm{y}+\mathrm{T}_{63} \sinh 2 \beta_{1} \mathrm{y}+\mathrm{T}_{64} \cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{y}+\mathrm{T}_{65} \sinh \left(\beta_{1}-\beta_{2}\right) \mathrm{y}+  \tag{48}\\
& \mathrm{T}_{68} \cosh \beta_{2} \mathrm{y}+\mathrm{S}_{4} \cosh 2 \beta_{1} \mathrm{y} \\
& \text { where } \beta_{1}=\sqrt{-\alpha}, \beta_{2}=\sqrt{H\left(1+\lambda_{1}\right)}, \mathrm{H}=\frac{\mathrm{M}^{2}}{1+\mathrm{m}_{1}^{2}}
\end{align*}
$$

Results and discussion: In order to identify the quantitative variations of emerging parameters the graphical results are presented. The effects of magnetic parameter $(M)$, slip parameter $(\beta)$, Hall parameter $\left(m_{1}\right)$, geometric parameter (a), Material parameter $\left(\lambda_{1}\right)$, phase angle $(\varphi)$ and Grashof number $\left(G_{t}\right)$ on the axial velocity are plotted in Fig.2. In order to show the effects of slip parameter and the magnetic parameter M, we have prepared Fig. 2a. It reveals that the fluid velocity decreases with an increase in magnetic parameter M, which means that, when M increases, it creates the Lorentz force, which opposes the flow and leads to enhanced deceleration of the flow (as noted in Ref.[10]). From the same figure, we observe that an increase in slip velocity tends to decrease fluid velocity because the slip condition at the boundary slows down the fluid velocity. Fig. 2 b depicts the variation of fluid velocity with Hall parameter $\left(\mathrm{m}_{1}\right)$ in presence of a wall slip. It shows that velocity profile increases with increase in Hall parameter. The reason behind such discrepancy in results may be due to increase in Hall parameter, which reduces the magnetic damping force on the velocity. The effect of geometric parameter ' $a$ ' on velocity distribution is depicted in Fig. 2c. It is shown that as ' $a$ ' increases the velocity profiles become perfect parabolic and also the profiles move slowly from left to the centre of the channel whereas there is no significant variation in magnitude of the velocity with increase in the parameter ' $a$ '. Such an effect may be expected, because an increase in Grashof number physically means increase in buoyancy force, which supports the flow.

Fig. 3 describes the influences of $\mathrm{m}_{1}, \lambda_{1}, \mathrm{M}$ on dimensionless pressure gradient $\mathrm{dp} / \mathrm{dx}$ over one wavelength $x \in[0,1]$. Through Figs. 3(a)-3(c), we can see that $d p / d x$ is decreasing function on $m_{1}, \lambda_{1}$ with fixed values of all other parameters. Further, from the same figure, we can notice that $\mathrm{dp} / \mathrm{dx}$ increases with increasing M significantly. Moreover, in the wider part of the channel $0 \leq x \leq 0.2$ and $0.8 \leq x \leq 1$, the pressure gradient is relatively smaller i.e. flow can easily pass without the imposition of large pressure gradient where the reverse trend can be seen in narrower part of channel $0.2 \leq x \leq 0.8$.


Fig. 1 Flow Geometry of the Problem


Fig. 2 Velocity distribution
$\left(\mathrm{b}=0.4, \mathrm{~d}=1, \operatorname{Re}=1, \mathrm{~S}_{\mathrm{c}}=1, \mathrm{q}=1, \varphi=\pi / 2 \alpha=2, \mathrm{P}_{\mathrm{r}}=0.71, \mathrm{~B}_{\mathrm{r}}=0.01\right)$
(a) (-) $\mathrm{M}=0.1,\left(^{*}\right) \mathrm{M}=2$, (о) $\mathrm{M}=4,\left({ }^{\wedge}\right) \mathrm{M}=6, \mathrm{~m}_{1}=0.5, \mathrm{a}=0.3, \mathrm{G}_{\mathrm{t}}=5, \lambda_{1}=0.5$
(b) (-) $\mathrm{m}_{1}=0,\left({ }^{*}\right) \mathrm{m}_{1}=1,(\mathrm{o}) \mathrm{m}_{1}=2,\left(^{\wedge}\right) \mathrm{m}_{1}=5, \mathrm{M}=2, \mathrm{G}_{\mathrm{t}}=5, \mathrm{a}=0.3, \lambda_{1}=0.5$
(c) (-) $\mathrm{a}=0.3$, ( $^{*}$ ) $\mathrm{a}=0.4$, (o) $\mathrm{a}=0.5,(\wedge) \mathrm{a}=0.6, \mathrm{~m}_{1}=0.5, \mathrm{M}=2, \mathrm{G}_{\mathrm{t}}=5, \lambda_{1}=0.5$


Fig. 3 Pressure gradient
$\left(a=0.3, b=0.5, d=1.2, P_{r}=0.71, S_{c}=1, G_{t}=1, q=-3, \operatorname{Re}=1, B_{r}=0.001, \alpha=1\right)$
(a) (-) $\mathrm{m}_{1}=0$, (*) $\mathrm{m}_{1}=0.5$, (о) $\mathrm{m}_{1}=1$, (^) $\mathrm{m}_{1}=1.5, \lambda_{1}=0.5, \mathrm{M}=2, \beta=0.1, \varphi=0$
(b) (-) $\lambda_{1}=0$, (*) $^{*} \lambda_{1}=0.5$, (o) $\lambda_{1}=1$, (^) $\lambda_{1}=1.5, \mathrm{~m}_{1}=0.5, \mathrm{M}=2, \beta=0.1, \varphi=0$
(c) (-) $\mathrm{M}=0.1,\left({ }^{*}\right) \mathrm{M}=2,(\mathrm{o}) \mathrm{M}=4,\left(^{\wedge}\right) \mathrm{M}=6, \lambda_{1}=0.5, \mathrm{~m}_{1}=0.5, \beta=0.1, \varphi=0$

CONCLUSIONS: In this paper, we have analyzed MHD heat and mass transfer peristaltic flow of a Jeffery fluid in a vertical channel in the presence of Hall current and wall slip condition. The momentum and energy equations have been linearized under long-wavelength approximation. Expressions for velocity and temperature have been obtained. The influences of pertinent parameters on flow, heat and mass transfer characteristics are analyzed through graphs and discussed in detail. The main findings are summarized as follows:

- The fluid velocity decreases with an increase in magnetic parameter M, which means that, when M increases, it creates the Lorentz force, which opposes the flow and leads to enhanced deceleration of the flow.
- The influences of $m_{1}, \lambda_{1}, M$ on dimensionless pressure gradient $d p / d x$ over one wavelength $x \in[0,1]$.


## APPENDIX

$\mathrm{T}_{1}=-\mathrm{S}_{\mathrm{c}} \beta_{1}^{2} \mathrm{~A} ; \mathrm{T}_{2}=-\mathrm{S}_{\mathrm{c}} \beta_{1}^{2} \mathrm{~B} ; \mathrm{T}_{3}=\left(\frac{\mathrm{T}_{1}}{\beta_{1}^{2}}\right) ; \mathrm{T}_{4}=\left(\frac{\mathrm{T}_{2}}{\beta_{1}^{2}}\right) ; \mathrm{T}_{5}=\left[-\mathrm{G}_{\mathrm{t}} \mathrm{A}\right]\left(1+\lambda_{1}\right) \beta_{1} ;$
$\mathrm{T}_{6}=\left[-\mathrm{G}_{\mathrm{t}} \mathrm{B}\right]\left(1+\lambda_{\mathrm{r}}\right) \beta_{1} ; \mathrm{T}_{8}=\left(\frac{\mathrm{T}_{5}}{\beta_{1}^{4}-\beta_{2}^{2} \beta_{1}^{2}}\right) ;$
$T_{9}=\left(\frac{T_{6}}{\beta_{1}^{4}-\beta_{2}^{2} \beta_{1}^{2}}\right) ; T_{11}=\frac{q}{2}-T_{8} \sinh \beta_{1} h_{1}-T_{9} \cosh \beta_{1} h_{1} ;$
$\mathrm{T}_{12}=-\frac{\mathrm{q}}{2}-\mathrm{T}_{8} \sinh \beta_{1} \mathrm{~h}_{2}-\mathrm{T}_{9} \cosh \beta_{1} \mathrm{~h}_{2} ;$

$$
\begin{aligned}
& T_{13}=\left(h_{1}+h_{2}\right)\left[\cosh \beta_{2} h_{1}-\cosh \beta_{2} h_{2}\right]-\left(h_{1}-h_{2}\right)\left[\cosh \beta_{2} h_{1}+\cosh \beta_{2} h_{2}\right] ; \\
& T_{14}=\left(h_{1}+h_{2}\right)\left[\sinh \beta_{2} \mathrm{~h}_{1}-\sinh \beta_{2} \mathrm{~h}_{2}\right]-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left[\sinh \beta_{2} \mathrm{~h}_{1}+\sinh \beta_{2} \mathrm{~h}_{2}\right] \text {; } \\
& \mathrm{T}_{15}=\left(\mathrm{T}_{11}-\mathrm{T}_{12}\right)\left(\mathrm{h}_{1}+\mathrm{h}_{2}\right)-\left(\mathrm{T}_{11}+\mathrm{T}_{12}\right)\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right) \text {; } \\
& \mathrm{T}_{16}=\mathrm{L} \beta_{2}^{2} ; \mathrm{T}_{17}=\left(-\mathrm{LT}_{8} \beta_{1}^{2}-\mathrm{T}_{9} \beta_{1}\right) ; \mathrm{T}_{18}=\left(-\mathrm{LT}_{9} \beta_{1}^{2}-\mathrm{T}_{8} \beta_{1}\right) ; \mathrm{T}_{19}=-1 ; \\
& \mathrm{T}_{20}=\left(-\mathrm{LT}_{8} \beta_{1}^{2}+\mathrm{T}_{9} \beta_{1}\right) ; \mathrm{T}_{21}=\left(-\mathrm{LT}_{9} \beta_{1}^{2}+\mathrm{T}_{8} \beta_{1}\right) ; \mathrm{T}_{22}=1 ; \\
& \mathrm{T}_{23}=\mathrm{T}_{17} \sinh \beta_{1} \mathrm{~h}_{1}+\mathrm{T}_{18} \cosh \beta_{1} \mathrm{~h}_{1}+\mathrm{T}_{19} ; \mathrm{T}_{24}=\mathrm{T}_{20} \sinh \beta_{1} \mathrm{~h}_{2}+\mathrm{T}_{21} \cosh \beta_{1} \mathrm{~h}_{2}+\mathrm{T}_{22} ; \\
& \mathrm{T}_{25}=\mathrm{T}_{16}\left(\cosh \beta_{2} \mathrm{~h}_{1}-\cosh \beta_{2} \mathrm{~h}_{2}\right)+\beta_{2}\left(\sinh \beta_{2} \mathrm{~h}_{1}+\sinh \beta_{2} \mathrm{~h}_{2}\right) \text {; } \\
& \mathrm{T}_{26}=\mathrm{T}_{16}\left(\sinh \beta_{2} \mathrm{~h}_{1}-\sinh \beta_{2} \mathrm{~h}_{2}\right)+\beta_{2}\left(\cosh \beta_{2} \mathrm{~h}_{1}+\cosh \beta_{2} \mathrm{~h}_{2}\right) \text {; } \\
& T_{27}=T_{16}\left(\cosh \beta_{2} h_{1}+\cosh \beta_{2} h_{2}\right)+\beta_{2}\left(\sinh \beta_{2} h_{1}-\sinh \beta_{2} h_{2}\right) \text {; } \\
& T_{28}=T_{16}\left(\sinh \beta_{2} h_{1}+\sinh \beta_{2} h_{2}\right)+\beta_{2}\left(\cosh \beta_{2} h_{1}-\cosh \beta_{2} h_{2}\right) \text {; } \\
& \mathrm{T}_{29}=2\left(\cosh \beta_{2} \mathrm{~h}_{1}-\cosh \beta_{2} \mathrm{~h}_{2}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right) \mathrm{T}_{25} \text {; } \\
& \mathrm{T}_{30}=2\left(\sinh \beta_{2} \mathrm{~h}_{1}-\sinh \beta_{2} \mathrm{~h}_{2}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right) \mathrm{T}_{26} \text {; } \\
& \mathrm{T}_{31}=2\left(\mathrm{~T}_{11}-\mathrm{T}_{12}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left(\mathrm{T}_{23}-\mathrm{T}_{24}\right) ; \quad \mathrm{T}_{32}=\left[-\mathrm{B}_{\mathrm{r}}\left(\frac{\mathrm{~A}_{1}^{2} \beta_{2}^{4}}{2}+\frac{\mathrm{B}_{1}^{2} \beta_{2}^{4}}{2}\right)\right] ; \\
& \mathrm{T}_{33}=\left[-\mathrm{B}_{\mathrm{r}}\left(\frac{2 \mathrm{~A}_{1} \mathrm{~B}_{1} \beta_{2}^{4}}{2}\right)\right] ; \mathrm{T}_{34}=\left[-\mathrm{B}_{\mathrm{r}} \mathrm{~T}_{8} \mathrm{~T}_{9} \beta_{1}^{4}\right] ; \mathrm{T}_{35}=\left[-\mathrm{B}_{\mathrm{r}} \mathrm{~B}_{1} \mathrm{~T}_{8} \beta_{2}^{2} \beta_{1}^{2}\right] \text {; } \\
& \mathrm{T}_{36}=\left[\mathrm{B}_{\mathrm{r}} \mathrm{~B}_{1} \mathrm{~T}_{8} \beta_{2}^{2} \beta_{1}^{2}\right] ; \quad \mathrm{T}_{37}=\left[-\mathrm{B}_{\mathrm{r}}\left(4 \mathrm{~T}_{9} \mathrm{~T}_{10} \beta_{1}^{2}\right)\right] ; \quad \mathrm{T}_{38}=\left[-\mathrm{B}_{\mathrm{r}}\left(4 \mathrm{~A}_{1} \mathrm{~T}_{10} \beta_{2}^{2}\right)\right] ; \\
& \mathrm{T}_{39}=\left[\mathrm{B}_{\mathrm{r}}\left(\frac{-\mathrm{A}_{1}^{2} \beta_{2}^{4}}{2}+\frac{\mathrm{B}_{1}^{2} \beta_{2}^{4}}{2}+\frac{\mathrm{T}_{8}^{2} \beta_{1}^{4}}{2}-\frac{\mathrm{T}_{9}^{2} \beta_{1}^{4}}{2}-4 \mathrm{~T}_{10}^{2}\right)\right] ; \\
& S_{1}=\left[-B_{r}\left(\frac{T_{8}^{2} \beta_{1}^{4}}{2}+\frac{T_{9}^{2} \beta_{1}^{4}}{2}\right)\right] ; \mathrm{T}_{40}=\left(\frac{\mathrm{T}_{32}}{4 \beta_{2}^{2}-\beta_{1}^{2}}\right) ; \mathrm{T}_{41}=\left(\frac{\mathrm{T}_{33}}{4 \beta_{2}^{2}-\beta_{1}^{2}}\right) ; \\
& \mathrm{T}_{42}=\left(\frac{\mathrm{T}_{34}}{3 \beta_{1}^{2}}\right) ; \mathrm{T}_{43}=\left(\frac{\mathrm{T}_{35}}{\left(\beta_{1}+\beta_{2}\right)^{2}-\beta_{1}^{2}}\right) ; \mathrm{T}_{44}=\left(\frac{\mathrm{T}_{36}}{\left(\beta_{1}-\beta_{2}\right)^{2}-\beta_{1}^{2}}\right) ; \\
& \mathrm{T}_{45}=\left(\frac{-\mathrm{T}_{37}}{2}\right) ; \mathrm{T}_{46}=\left(\frac{\mathrm{T}_{38}}{\beta_{2}^{2}-\beta_{1}^{2}}\right) ; \mathrm{T}_{47}=\left(\frac{-\mathrm{T}_{39}}{\beta_{1}^{2}}\right) ; \mathrm{S}_{2}=\left(\frac{\mathrm{S}_{1}}{3 \beta_{1}^{2}}\right) \text {; } \\
& \mathrm{T}_{48}=-\left[\begin{array}{l}
\mathrm{T}_{40} \cosh 2 \beta_{2} \mathrm{~h}_{1}+\mathrm{T}_{41} \sinh 2 \beta_{2} \mathrm{~h}_{1}+\mathrm{T}_{42} \sinh \beta_{1} \mathrm{~h}_{1}+\mathrm{T}_{43} \cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{1}+ \\
\mathrm{T}_{44} \cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{1}+\mathrm{T}_{45} \mathrm{~h}_{1} \sinh \beta_{1} \mathrm{~h}_{1}+\mathrm{T}_{46} \cosh \beta_{2} \mathrm{~h}_{1}+\mathrm{T}_{47}+\mathrm{S}_{2} \cosh 2 \beta_{1} \mathrm{~h}_{1}
\end{array}\right] \text {; } \\
& \mathrm{T}_{49}=-\left[\begin{array}{l}
\mathrm{T}_{40} \cosh 2 \beta_{2} \mathrm{~h}_{2}+\mathrm{T}_{41} \sinh 2 \beta_{2} \mathrm{~h}_{2}+\mathrm{T}_{42} \sinh \beta_{1} \mathrm{~h}_{2}+\mathrm{T}_{43} \cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{2}+ \\
\mathrm{T}_{44} \cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{2}+\mathrm{T}_{45} \mathrm{~h}_{2} \sinh \beta_{1} \mathrm{~h}_{2}+\mathrm{T}_{46} \cosh \beta_{2} \mathrm{~h}_{2}+\mathrm{T}_{47}+\mathrm{S}_{2} \cosh 2 \beta_{1} \mathrm{~h}_{2}
\end{array}\right] ; \\
& \mathrm{T}_{50}=-\mathrm{S}_{\mathrm{c}}\left(\beta_{1}^{2} \mathrm{~A}_{2}+2 \mathrm{~T}_{45} \beta_{1}\right) ; \mathrm{T}_{51}=-\mathrm{S}_{\mathrm{c}}\left(\beta_{1}^{2} \mathrm{~B}_{2}\right) ; \mathrm{T}_{52}=-\mathrm{S}_{\mathrm{c}}\left(4 \beta_{2}^{2} \mathrm{~T}_{40}\right) ; \mathrm{T}_{53}=-\mathrm{S}_{\mathrm{c}}\left(4 \beta_{2}^{2} \mathrm{~T}_{41}\right) ;
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{T}_{54}=-\mathrm{S}_{\mathrm{c}}\left(4 \beta_{1}^{2} \mathrm{~T}_{42}\right) ; \mathrm{T}_{55}=-\mathrm{S}_{\mathrm{c}} \mathrm{~T}_{43}\left(\beta_{1}+\beta_{2}\right)^{2} ; \mathrm{T}_{56}=-\mathrm{S}_{\mathrm{c}} \mathrm{~T}_{44}\left(\beta_{1}-\beta_{2}\right)^{2} ; \\
& \mathrm{T}_{57}=-\mathrm{S}_{\mathrm{c}} \mathrm{~T}_{45} \beta_{1}^{2} ; \mathrm{T}_{58}=-\mathrm{S}_{\mathrm{c}} \mathrm{~T}_{46} \beta_{2}^{2} ; \mathrm{S}_{3}=-\mathrm{S}_{\mathrm{c}} \mathrm{~S}_{\mathrm{r}}\left(4 \beta_{1}^{2} \mathrm{~S}_{2}\right) ; \\
& \mathrm{T}_{59}=\left(\frac{\mathrm{T}_{50}}{\beta_{1}^{2}}\right) ; \mathrm{T}_{60}=\left(\frac{\mathrm{T}_{51}}{\beta_{1}^{2}}\right) ; \mathrm{T}_{61}=\left(\frac{\mathrm{T}_{52}}{4 \beta_{2}^{2}}\right) ; \mathrm{T}_{62}=\left(\frac{\mathrm{T}_{53}}{4 \beta_{2}^{2}}\right) ; \\
& \mathrm{T}_{63}=\left(\frac{\mathrm{T}_{54}}{4 \beta_{1}^{2}}\right) ; \mathrm{T}_{64}=\left(\frac{\mathrm{T}_{55}}{\left(\beta_{1}+\beta_{2}\right)^{2}}\right) ; \mathrm{T}_{65}=\left(\frac{\mathrm{T}_{56}}{\left(\beta_{1}-\beta_{2}\right)^{2}}\right) \text {; } \\
& \mathrm{T}_{66}=\left(\frac{\mathrm{T}_{57}}{\beta_{1}^{2}}\right) ; \mathrm{T}_{67}=\left(\frac{-2 \mathrm{~T}_{57}}{\beta_{1}^{3}}\right) ; \mathrm{T}_{68}=\left(\frac{\mathrm{T}_{58}}{\beta_{2}^{2}}\right) ; \mathrm{S}_{4}=\left(\frac{\mathrm{S}_{3}}{4 \beta_{1}^{2}}\right) \text {; } \\
& T_{69}=-\left[\begin{array}{l}
T_{59} \cosh \beta_{1} h_{1}+T_{60} \sinh \beta_{1} h_{1}+T_{61} \cosh 2 \beta_{2} h_{1}+T_{62} \sinh 2 \beta_{2} h_{1}+ \\
T_{63} \sinh 2 \beta_{1} h_{1}+T_{64} \cosh \left(\beta_{1}+\beta_{2}\right) h_{1}+T_{65} \sinh \left(\beta_{1}-\beta_{2}\right) h_{1}+ \\
T_{66} h_{1} \sinh \beta_{1} h_{1}+T_{67} \cosh \beta_{1} h_{1}+T_{68} \cosh \beta_{2} h_{1}+S_{4} \cosh 2 \beta_{1} h_{1}
\end{array}\right] ; \\
& T_{70}=-\left[\begin{array}{l}
T_{59} \cosh \beta_{1} h_{2}+T_{60} \sinh \beta_{1} h_{2}+T_{61} \cosh 2 \beta_{2} h_{2}+T_{62} \sinh 2 \beta_{2} h_{2}+ \\
T_{63} \sinh 2 \beta_{1} h_{2}+T_{64} \cosh \left(\beta_{1}+\beta_{2}\right) h_{2}+T_{65} \sinh \left(\beta_{1}-\beta_{2}\right) h_{2}+ \\
T_{66} h_{2} \sinh \beta_{1} h_{2}+T_{67} \cosh \beta_{1} h_{2}+T_{68} \cosh \beta_{2} h_{2}+S_{4} \cosh 2 \beta_{1} h_{2}
\end{array}\right] ; \\
& \mathrm{T}_{71}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~A}_{2} \beta_{1}+\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{45}\right)\right] \text {; } \\
& \mathrm{T}_{72}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~B}_{2} \beta_{1}\right)\right] ; \mathrm{T}_{73}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{40} 2 \beta_{2}\right)\right] \text {; } \\
& \mathrm{T}_{74}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{41} 2 \beta_{2}\right)\right] ; \mathrm{T}_{75}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{42} 2 \beta_{1}\right)\right] \text {; } \\
& \mathrm{T}_{76}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{43}\left(\beta_{1}+\beta_{2}\right)\right)\right] \text {; } \\
& \mathrm{T}_{77}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{44}\left(\beta_{1}-\beta_{2}\right)\right)\right] ; \mathrm{T}_{78}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{45} \beta_{1}\right)\right] \text {; } \\
& \mathrm{T}_{79}=\left[-\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~T}_{46} \beta_{2}\right)\right] ; \mathrm{S}_{5}=\left[-2\left(1+\lambda_{1}\right)\left(\mathrm{G}_{\mathrm{t}} \mathrm{~S}_{2} \beta_{1}\right)\right] \text {; } \\
& \mathrm{T}_{82}=\left(\frac{\mathrm{T}_{71}}{\beta_{1}^{4}-\beta_{2}^{2} \beta_{1}^{2}}\right) ; \mathrm{T}_{83}=\left(\frac{\mathrm{T}_{72}}{\beta_{1}^{4}-\beta_{2}^{2} \beta_{1}^{2}}\right) ; \\
& \mathrm{T}_{84}=\left(\frac{\mathrm{T}_{73}}{12 \beta_{2}^{4}}\right) ; \mathrm{T}_{85}=\left(\frac{\mathrm{T}_{74}}{12 \beta_{2}^{4}}\right) ; \mathrm{T}_{86}=\left(\frac{\mathrm{T}_{75}}{16 \beta_{1}^{4}-4 \beta_{2}^{2} \beta_{1}^{2}}\right) \text {; } \\
& \mathrm{T}_{87}=\left(\frac{\mathrm{T}_{76}}{\left(\beta_{1}+\beta_{2}\right)^{2}\left[\left(\beta_{1}+\beta_{2}\right)^{2}-\beta_{2}^{2}\right]}\right) ; \mathrm{T}_{88}=\left(\frac{\mathrm{T}_{77}}{\left(\beta_{1}-\beta_{2}\right)^{2}\left[\left(\beta_{1}-\beta_{2}\right)^{2}-\beta_{2}^{2}\right]}\right) ; \\
& \mathrm{T}_{89}=\left(\frac{\mathrm{T}_{78}}{\beta_{1}^{2}\left(\beta_{1}^{2}-\beta_{2}^{2}\right)}\right) ; \mathrm{T}_{90}=\left(\frac{-2 \mathrm{~T}_{78}}{\beta_{1}^{3}\left(\beta_{1}^{2}-\beta_{2}^{2}\right)}-\frac{2 \mathrm{~T}_{78}}{\beta_{1}\left(\beta_{1}^{2}-\beta_{2}^{2}\right)^{2}}\right) ; \mathrm{T}_{91}=\left(\frac{\beta_{2} \mathrm{~T}_{79}}{2 \beta_{2}^{3}}\right) ;
\end{aligned}
$$

$$
\begin{aligned}
& S_{6}=\left(\frac{S_{5}}{16 \beta_{1}^{4}-4 \beta_{1}^{2} \beta_{2}^{2}}\right) ; T_{94}=\left[T_{82}+T_{90}\right] ; \\
& T_{95}=-\left[\begin{array}{l}
T_{83} \cosh \beta_{1} h_{1}+T_{84} \sinh 2 \beta_{2} h_{1}+T_{85} \cosh 2 \beta_{2} h_{1}+T_{86} \cosh 2 \beta_{1} h_{1}+ \\
T_{87} \sinh \left(\beta_{1}+\beta_{2}\right) h_{1}+T_{88} \cosh \left(\beta_{1}-\beta_{2}\right) h_{1}+T_{89} h_{1} \cosh \beta_{1} h_{1}+ \\
T_{91} h_{1} \cosh \beta_{2} h_{1}+T_{92} h_{1} \sinh \beta_{2} h_{1}+T_{93} h_{1}^{2}+T_{94} \sinh \beta_{1} h_{1}+S_{6} \sinh 2 \beta_{1} h_{1}
\end{array}\right] ; \\
& T_{96}=-\left[\begin{array}{l}
T_{83} \cosh \beta_{1} h_{2}+T_{84} \sinh 2 \beta_{2} h_{2}+T_{85} \cosh 2 \beta_{2} h_{2}+T_{86} \cosh 2 \beta_{1} h_{2}+ \\
T_{87} \sinh \left(\beta_{1}+\beta_{2}\right) h_{2}+T_{88} \cosh \left(\beta_{1}-\beta_{2}\right) h_{2}+T_{89} h_{2} \cosh \beta_{1} h_{2}+ \\
T_{91} h_{2} \cosh \beta_{2} h_{2}+T_{92} h_{2} \sinh \beta_{2} h_{2}+T_{93} h_{2}^{2}+T_{94} \sinh \beta_{1} h_{2}++S_{6} \sinh 2 \beta_{1} h_{2}
\end{array}\right] ; \\
& \mathrm{T}_{97}=\left(\mathrm{h}_{1}+\mathrm{h}_{2}\right)\left(\mathrm{T}_{95}-\mathrm{T}_{96}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left(\mathrm{T}_{95}+\mathrm{T}_{96}\right) ; \mathrm{T}_{98}=\left(\mathrm{L} \beta_{2}^{2} \cosh \beta_{2} \mathrm{~h}_{1}+\beta_{2} \sinh \beta_{2} \mathrm{~h}_{1}\right) \text {; } \\
& T_{99}=\left(L \beta_{2}^{2} \sinh \beta_{2} h_{1}+\beta_{2} \cosh \beta_{2} h_{1}\right) \text {; } \\
& \mathrm{T}_{100}=\cosh \beta_{1} \mathrm{~h}_{1}\left(\mathrm{LT}_{83} \beta_{1}^{2}+\mathrm{T}_{89}+\mathrm{T}_{94} \beta_{1}\right)+\sinh 2 \beta_{2} \mathrm{~h}_{1}\left(4 \mathrm{LT}_{84} \beta_{2}^{2}+2 \mathrm{~T}_{85} \beta_{2}\right) \text {; } \\
& \mathrm{T}_{101}=\cosh 2 \beta_{2} \mathrm{~h}_{1}\left(4 \mathrm{LT}_{85} \beta_{2}^{2}+2 \mathrm{~T}_{84} \beta_{2}\right)+\cosh 2 \beta_{1} \mathrm{~h}_{1}\left(4 \mathrm{LT}_{86} \beta_{1}^{2}+2 \mathrm{~S}_{6} \beta_{1}\right) \text {; } \\
& \mathrm{T}_{102}=\sinh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{1}\left(\operatorname{LT}_{87}\left(\beta_{1}+\beta_{2}\right)^{2}\right)+\cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{1}\left(\operatorname{LT}_{88}\left(\beta_{1}-\beta_{2}\right)^{2}\right) ; \\
& \mathrm{T}_{103}=\cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{1}\left(\mathrm{~T}_{87}\left(\beta_{1}+\beta_{2}\right)\right)+\sinh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{1}\left(\mathrm{~T}_{88}\left(\beta_{1}-\beta_{2}\right)\right) \text {; } \\
& \mathrm{T}_{104}=\mathrm{h}_{1} \cosh \beta_{1} \mathrm{~h}_{1}\left(\mathrm{LT}_{89} \beta_{1}^{2}\right)+\sinh \beta_{1} \mathrm{~h}_{1}\left(2 \mathrm{LT}_{89} \beta_{1}+\mathrm{LT}_{94} \beta_{1}^{2}+\mathrm{T}_{83} \beta_{1}\right) \text {; } \\
& \mathrm{T}_{105}=\mathrm{h}_{1} \cosh \beta_{2} \mathrm{~h}_{1}\left(\mathrm{LT}_{91} \beta_{2}^{2}+\mathrm{T}_{92} \beta_{2}\right)+\sinh \beta_{2} \mathrm{~h}_{1}\left(2 \mathrm{LT}_{91} \beta_{2}+\mathrm{T}_{92}\right) \text {; } \\
& \mathrm{T}_{106}=\mathrm{h}_{1} \sinh \beta_{2} \mathrm{~h}_{1}\left(\mathrm{~T}_{91} \beta_{2}\right)+\cosh \beta_{2} \mathrm{~h}_{1}\left(\mathrm{~T}_{91}\right) \text {; } \\
& \mathrm{T}_{107}=\sinh 2 \beta_{1} \mathrm{~h}_{1}\left(2 \mathrm{~T}_{86} \beta_{1}+4 L S_{6} \beta_{1}^{2}\right)+\mathrm{h}_{1} \sinh \beta_{1} \mathrm{~h}_{1}\left(\mathrm{~T}_{89} \beta_{1}\right) \text {; } \\
& T_{108}=\left(L \beta_{2}^{2} \cosh \beta_{2} h_{2}-\beta_{2} \sinh \beta_{2} h_{2}\right) ; T_{109}=\left(L \beta_{2}^{2} \sinh \beta_{2} h_{2}-\beta_{2} \cosh \beta_{2} h_{2}\right) \text {; } \\
& \mathrm{T}_{110}=\cosh \beta_{1} \mathrm{~h}_{2}\left(\mathrm{~L} \beta_{1}^{2} \mathrm{~T}_{83}-\mathrm{T}_{89}-\mathrm{T}_{94} \beta_{1}\right)+\sinh 2 \beta_{2} \mathrm{~h}_{2}\left(4 \mathrm{~L} \beta_{2}^{2} \mathrm{~T}_{84}-2 \mathrm{~T}_{85} \beta_{2}\right) \text {; } \\
& \mathrm{T}_{111}=\cosh 2 \beta_{2} \mathrm{~h}_{2}\left(4 \mathrm{~L} \beta_{2}^{2} \mathrm{~T}_{85}-2 \mathrm{~T}_{84} \beta_{2}\right)+\cosh 2 \beta_{1} \mathrm{~h}_{2}\left(4 \mathrm{~L} \beta_{1}^{2} \mathrm{~T}_{86}-2 \mathrm{~S}_{6} \beta_{1}\right) \text {; } \\
& \mathrm{T}_{112}=\sinh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{2}\left(\operatorname{LT}_{87}\left(\beta_{1}+\beta_{2}\right)^{2}\right)+\cosh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{2}\left(\operatorname{LT}_{88}\left(\beta_{1}-\beta_{2}\right)^{2}\right) ; \\
& \mathrm{T}_{113}=\cosh \left(\beta_{1}+\beta_{2}\right) \mathrm{h}_{2}\left(-\mathrm{T}_{87}\left(\beta_{1}+\beta_{2}\right)\right)+\sinh \left(\beta_{1}-\beta_{2}\right) \mathrm{h}_{1}\left(-\mathrm{T}_{88}\left(\beta_{1}-\beta_{2}\right)\right) \text {; } \\
& \mathrm{T}_{114}=\mathrm{h}_{2} \cosh \beta_{1} \mathrm{~h}_{2}\left(\operatorname{LT}_{89} \beta_{1}^{2}\right)+\sinh \beta_{1} \mathrm{~h}_{2}\left(2 \mathrm{LT}_{89} \beta_{1}+\mathrm{LT}_{94} \beta_{1}^{2}-\mathrm{T}_{83} \beta_{1}\right) \text {; } \\
& \mathrm{T}_{115}=\mathrm{h}_{2} \cosh \beta_{2} \mathrm{~h}_{2}\left(\mathrm{LT}_{91} \beta_{2}^{2}-\mathrm{T}_{92} \beta_{2}\right)+\sinh \beta_{2} \mathrm{~h}_{2}\left(2 \mathrm{LT}_{91} \beta_{2}-\mathrm{T}_{92}\right) \text {; } \\
& \mathrm{T}_{116}=\mathrm{h}_{2} \sinh \beta_{2} \mathrm{~h}_{2}\left(\mathrm{LT}_{92} \beta_{2}^{2}-\mathrm{T}_{91} \beta_{2}\right)+\cosh \beta_{2} \mathrm{~h}_{2}\left(2 \mathrm{LT}_{92} \beta_{2}-\mathrm{T}_{91}\right) \text {; } \\
& \mathrm{T}_{117}=\sinh 2 \beta_{1} \mathrm{~h}_{2}\left(4 \mathrm{LS}_{6} \beta_{1}^{2}-2 \mathrm{~T}_{86} \beta_{1}\right)+\mathrm{h}_{2} \sinh \beta_{1} \mathrm{~h}_{2}\left(-\mathrm{T}_{89} \beta_{1}\right) \text {; } \\
& \mathrm{T}_{118}=-\left[\mathrm{T}_{100}+\mathrm{T}_{101}+\mathrm{T}_{102}+\mathrm{T}_{103}+\mathrm{T}_{104}+\mathrm{T}_{105}+\mathrm{T}_{106}+\mathrm{T}_{107}\right] \text {; }
\end{aligned}
$$

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\begin{aligned}
& \mathrm{T}_{119}=-\left[\mathrm{T}_{110}+\mathrm{T}_{111}+\mathrm{T}_{112}+\mathrm{T}_{113}+\mathrm{T}_{114}+\mathrm{T}_{115}+\mathrm{T}_{116}+\mathrm{T}_{117}\right] \text {; } \\
& \mathrm{T}_{120}=2\left(\cosh \beta_{2} \mathrm{~h}_{1}-\cosh \beta_{2} \mathrm{~h}_{2}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left(\mathrm{T}_{98}-\mathrm{T}_{108}\right) \text {; } \\
& \mathrm{T}_{121}=2\left(\sinh \beta_{2} \mathrm{~h}_{1}-\sinh \beta_{2} \mathrm{~h}_{2}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left(\mathrm{T}_{99}-\mathrm{T}_{109}\right) \text {; } \\
& \mathrm{T}_{122}=2\left(\mathrm{~T}_{95}-\mathrm{T}_{96}\right)-\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)\left(\mathrm{T}_{118}-\mathrm{T}_{119}\right) ; \mathrm{A}=\frac{1-\mathrm{B} \sinh \beta_{1} \mathrm{~h}_{1}}{\cosh \beta_{1} \mathrm{~h}_{1}} ; \\
& A_{5}=\frac{\left(T_{118}+T_{119}\right)-B_{5}\left(T_{99}+T_{109}\right)}{\left(\mathrm{T}_{98}+\mathrm{T}_{108}\right)} ; B=\frac{\cosh \beta_{1} \mathrm{~h}_{2}+\cosh \beta_{1} \mathrm{~h}_{1}}{\sinh \beta_{1}\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)} ; \\
& C=\frac{2-T_{3}\left(\cosh \beta_{1} \mathrm{~h}_{1}-\cosh \beta_{1} \mathrm{~h}_{2}\right)-\mathrm{T}_{4}\left(\sinh \beta_{1} \mathrm{~h}_{1}-\sinh \beta_{1} \mathrm{~h}_{2}\right)}{\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)} ; \\
& D=1-C h_{1}-T_{3} \cosh \beta_{1} h_{1}-T_{4} \sinh \beta_{1} h_{1} ; A_{1}=\frac{\left(T_{23}+T_{24}\right)-B_{1} T_{28}}{T_{27}} ; B_{1}=\frac{T_{29}\left(T_{23}+T_{24}\right)-T_{27} T_{31}}{T_{28} T_{29}-T_{27} T_{30}} ; \\
& A_{2}=\frac{T_{48}-B_{2} \sinh \beta_{1} h_{1}}{\cosh \beta_{1} h_{1}} ; E=T_{11}-F h_{1}-A_{1} \cosh \beta_{2} h_{1}-B_{1} \sinh \beta_{2} h_{1} ; \\
& \mathrm{F}=\frac{\left(\mathrm{T}_{11}-\mathrm{T}_{12}\right)-\mathrm{A}_{1}\left(\cosh \beta_{2} \mathrm{~h}_{1}-\cosh \beta_{2} \mathrm{~h}_{2}\right)-\mathrm{B}_{1}\left(\sinh \beta_{2} \mathrm{~h}_{1}-\sinh \beta_{2} \mathrm{~h}_{2}\right)}{\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)} ; \\
& \mathrm{B}_{2}=\frac{\mathrm{T}_{48} \cosh \beta_{1} \mathrm{~h}_{2}-\mathrm{T}_{49} \cosh \beta_{1} \mathrm{~h}_{1}}{\sinh \beta_{1}\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)} ; \mathrm{A}_{4}=\frac{\mathrm{A}_{5} \mathrm{~T}_{13}+\mathrm{B}_{5} \mathrm{~T}_{14}-\mathrm{T}_{97}}{2\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)} ; \mathrm{B}_{3}=\frac{\mathrm{T}_{69}-\mathrm{T}_{70}}{\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)} ; \\
& \mathrm{A}_{3}=\mathrm{T}_{69}-\mathrm{B}_{3} \mathrm{~h}_{1} ; \mathrm{B}_{5}=\frac{\mathrm{T}_{120}\left(\mathrm{~T}_{118}+\mathrm{T}_{119}\right)-\mathrm{T}_{122}\left(\mathrm{~T}_{98}+\mathrm{T}_{108}\right)}{\mathrm{T}_{120}\left(\mathrm{~T}_{99}+\mathrm{T}_{109}\right)-\mathrm{T}_{121}\left(\mathrm{~T}_{98}+\mathrm{T}_{108}\right)} \text {; } \\
& \mathrm{B}_{4}=\frac{\mathrm{T}_{95}-\mathrm{A}_{4}-\mathrm{A}_{5} \cosh \beta_{2} \mathrm{~h}_{1}-\mathrm{B}_{5} \sinh \beta_{2} \mathrm{~h}_{1}}{\mathrm{~h}_{1}}
\end{aligned}
$$

## REFERENCES

[1] T. W. Latham, Fluid motion in Peristaltic Pump, M.S. Thesis, MIII, Cambridge, Mass, 1966.
[2] T. Hayat, Y. Wang, A. M. Siddiqui, K. Hutter and S. Asghar, Peristaltic transport of a third-order fluid in a circular cylindrical tube, Math. Models and methods in Appl. Sci., 12 (2002)1691-1706.
[3] M. Mishra and A. Ramachandra Rao, Peristaltic transport of a Newtonian fluid in an asymmetric channel, ZAMP, 54 (2003) 532 550.
[4] T. Hayat, N. Ali, S. Asghar and A.M. Siddiqui, Exact peristaltic flow in tubes with an endoscope, Applied Math. Computation, 182 (2006) $359-368$.
[5] M. Kothandapani and S. Srinivas, Non-linear peristaltic transport of Newtonian fluid in an inclined asymmetric channel through a porous medium, Phys. Lett. A, 372 (2008) 1265-1276.
[6] Kh. S. Mekheimer and Y. Abd elmaboud, The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus: Application of an endoscope, Phys. Lett. A, 372 (2008) 1657-1665.
[7] T. Hayat and N. Ali, Peristaltically induced motion of a MHD third grade fluid in a deformable tube, Physica A, 370 (2006) 225-239.
[8] T. Hayat, N. Ali and Z. Abbas, Peristaltic flow of a micropolar fluid in a channel with different wave forms, Phys. Lett. A, 370 (2008) 331-344.
[9] S. Srinivas and R.Gayathri, Peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium, Appl. Math. Comput. 215 (2009) 185-196.
[10] T. Hayat, R. Sajjad, S. Asghar, Series solution for MHD channel flow of a Jeffery fluid, Commun Nonlinear Sci Numer Simulat., 15 (2010) 2400-2406.
[11] S. Srinivas and R. Muthuraj, Effects of chemical reaction and space porosity on MHD mixed convective flow in a vertical asymmetric channel with peristalsis, Math. Comput. Model., 54 (2011) 1213-1227.
[12] S. Srinivas and R. Muthuraj, Peristaltic transport of a Jeffrey fluid under the effect of slip in an inclined asymmetric channel, Int. J Appl. Mech. 2 (2010) 437-455.
[13] R. Muthuraj and S. Srinivas, Effects of compliant wall properties and space porosity in the MHD peristaltic transport with Heat and Mass Transfer in a vertical channel, Int. J. of Appl. Math and Mech., 7 (2011) 97-112.
[14] S. Nadeem and S. Akram, Peristaltic flow of a couple stress fluid under the effect of induced magnetic field in an asymmetric channel, Arch. Appl. Mech., 81 (2011) 97-109.
[15] N. T. Eldabe, K. A. Kamel, G. M. Abd- Allah and S.F. Ramadan, Peristaltic motion with Heat and Mass transfer of a Dusty fluid through a Horizontal porous channel under the effect of wall properties, Int. J Research Reviews in Appl. Sciences., 15 (2013) 300311.
[16] K. Vajravelu S. Sreenadh, P. Lakshminarayana, The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum, Commun Nonlinear Sci Numer Simulat., 16 (2011) 3107-3125.
[17] S. Hina, T. Hayat and A. Alsaedi, Heat and mass transfer effects on the peristaltic flow of Johnson-Segalman fluid in a curved channel with compliant walls, Int. J Heat Mass Transfer., 55 (2012) 3511-3521.
[18] R. Ellahi, T. Hayat, F. M. Mahomed and S. Asghar, Effects of slip on the non-linear flows of a third grade fluid, Nonlinear anal-Real world App., 11(2010) 139-146.
[19] S. Nadeem and S.Akram, Heat transfer in a peristaltic flow of MHD fluid with partial slip, Commun Nonlinear Sci Numer Simulat., 15 (2010) 312-321.
[20] N. S. Akbar, T. Hayat, S. Nadeem and S. Obaidat, Peristaltic flow of a Williamson fluid in an inclined asymmetric channel with partial slip and heat transfer, Int. J. Heat Mass Transfer, 55 (2012) 1855-1862.
[21] M.Ahmad, H. Zaman and N. Rehman, Effects of Hall current on unsteady MHD flows of a second grade fluid, Cent. Eur. J. Phys., 8 (2010) 422-431.
[22] F.M. Ali, R. Nazar, N.M. Arifin and I. Pop, Effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate, Meccanica 46 (2011) 1103-1112.
[23] D. Srinivasacharya and K. Kaladhar, Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ionslip effects, Commun Nonlinear Sci Numer Simulat., 17 (2012) 2447-2462.

