Mathematical Model on the Function of Kidney in Reference to a Peristaltic Moment

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Abstract - *In the present paper, mathematical approach on kidney's function with a reference to physiological flow parameters is studied. The case of mass transfer due to peristaltic response is analyzed in the model so as to obtain mass average velocity, concentrations based on the wall moment of the ascending limb and descending limb. It is to compute the response of normal and abnormal flow corresponding to the mass average velocity. Equations of motion for biological viewpoint describe Henle's loop which acts as counter current exchanger. Analytical solutions are presented for obtaining the mass average velocity, concentration of metabolic mass transfer in the function of kidney. Numerical results show the response of the function of kidney which give the evidence of the transposed mass in the kidney measurable mathematically.*

Keywords : *Kidney, Peristaltic, flow, moment.*

I. Introduction

The kidneys are two bean shaped organs that are fastened outside the back wall of the abdominal cavity on the either side of the spinal column. The blood is flowing through the kidneys, they strain out waste matters. The urine passes from the kidneys through a tube that leads from each kidney down to the urinary bladder. When a person is in good health and drinking as much water as he should, the urine will be a very light yellow color, often times it will be almost as clear as water. If the urine voided is of red or brown colour, it shows that too little water is drunk. It is a part of the work of the kidneys to cast out of the body anything in the blood, the kidneys are themselves injured.

- Excretion of wastes principally urea, from the blood.
- They are supplied with blood by the renal arteries.
- The active units of the kidney are the nephrons,

Sodium in C_{SI} with molecular weight 23 and filterability as 1.0 will be one among the chemical species as mass transport in the kidney's function. The internal chemical environment of the body results from a multitude of complex interacting chemical and physical phenomena. The maintenance of relatively constant internal environment is essential to the maintenance of life. With these factors into consideration it is necessary to study the functions of the kidney which control the concentration of various chemical species in the blood and hence throughout the body. Sodiumchloride (NaCl) and $NaHCO_2$ determine the amount of water within the cellular regions through an osmotic balance. The metabolic reactions produce several nitrogenous waste products such as urea and creatinine for which the body has a low level of tolerance. Mathematical modeling through the differential equations will be of great importance and adequate for controlling the chemistry of a multicomponent fluid.

The manner in which the nephron performs this task is the subject of this section.

I. Nearly all of the glomerular filtrate is reabsorbed back into the blood from nephron of 125ml/min of filtrate (both kidneys) only 1 ml/min of urine is formed.
II. Substances of nutritional value are selectively reabsorbed. The glomerular filtrate carries about 125 mg/min of glucose. Under normal conditions no glucose appears in urine.
III. Metabolic products are poorly reabsorbed. The concentration of urea in urine is 70 times its concentration in the glomerular filtrate.

To model the function of the kidney it appears necessary to account for the geometry that is peculiar to the organ. Most researchers agree that Henle'sloop acts as counter current exchanger. It is to note that interstitium is progressively hyperosmatic to fluid in the collecting duct as one moves towards the tip of Henle's loop. This causes the concentration of urine. Few observations can be made to construct the mathematical modeling.

I. In the descending limb water passes from the limb into the interstituim by filtration, principally through osmotic pressure.
II. In the ascending limb water cannot cross into the interstitium.
III. Na^+ can diffuse across the membrane of the descending limb.
IV. In the ascending limb, Na^+ is actively transported into the interstitium.

Because NaCl is the principle osmotically active solute, it will be assumed that a model which considers only water and Na^+should be sufficient to model the gross features of Henle's loop.

Over the past few years analytical and numerical studies have been carried out to estimate the mass average velocity and the concentration metabolic waste products in descending and ascending limb. David C. Weber [1] presented the evaluation of a fiber- reinforced cellophane membrane.G. Radhakrishnamacharya et. al. [2] investigated by considering a narrow tube of varying cross section with reabsorption at the wall. Carlas Enrich et. al. [3] observed the specific changes in the plasma membrane. Ruth Osterbyet. al. [4] showed a correlation between the parameters of glomerular structural lesions.Aurelie Edwards et. al. [5] characterized the permeability properties of the GBM necessary for understanding normal barrier function. Matthew J. Lazzaraet. al. [7] developed a mathematical model which assumes saturable endocytosis kinetics with a maximum reabsorptive capacity, Vmax, and which includes the effects of flow and diffusion in the lumen. Anita T. Layton [8] described modeling efforts that have sought to better understand kidney physiology. Brown et. al. [9] recommended for the management of canine glomerular disease. Benjamas Wichapoonet. al. [10] investigated immunohistochemically using tight junction-associated protein, ZO-1. Ioannis Sgouralis et. ai.[11]utilized a mathematical model of a single nephron to investigate the extent to which changes in systemic oxygenation during the surgical procedures performed on CPB might alter the medullary oxygenation. Adel Almarashi et.al.[12] constructed by taking samples from Jazanhospitals (positive samples and negative samples) to train the system and then test it on different samples. Marta Marulli et. al.[13] presented a mathematical model describing the transport of sodium in a fluid circulating in a counter-current tubular architecture, which constitutes a simplified model of Henle's loop in a kidney nephron.

In the present study, mass average velocity, concentration of metabolic waste products in descending and ascending limb are estimated using Runge – Kutta fourth order method. It is to note that interstitium is progressively hyperosmatic to fluid in the collecting duct as one moves towards the tip of Henle's loop. This causes the concentration of urine.

II. Formulation

With a reference to function of kidney the peristaltic wave propagation is analyzed using the differential equation for concentration of sodium ion with respect to displacement of the wave such that, C represents the concentration of sodium ion, with c the concentration in the descending limb, C_i – concentration in interstitium(or Ca-concentration in ascending limb).

A balance on sodium ion in the descending limb,

$$\frac{d(uC_d)}{dx} = -h(C_d - C_i) \tag{1}$$

On differentiating equation (1) we get,

$$u\frac{dC_d}{dx} + C_d\frac{du}{dx} = -h(C_d - C_i) \tag{2}$$

where u-mass average velocity of flow of solution down the descending limb, h- the coefficient which includes the permeability of the limb wall to sodium ion and any geometrical factors that affect the area available for transfer.

A water balance in the descending limb will be written in the form,

$$\frac{du}{dx} = -h(C_i - C_d) \tag{3}$$

Above equation gives the water flux proportional to the concentration difference between the descending limb and the ascending limb. The osmotic pressure is taken as proportional to the concentration of sodium ion.

For solute flow we can assume that the active transport of sodium ion into the interstitium proceeds at a rate linearly dependent on sodium concentration. The solute balance on C_a takes the form,

$$U_a\frac{dC_a}{dx} = K_aC_a \tag{4}$$

Rearranging equation (4) we get

$$\frac{dC_a}{dx} = \frac{K_aC_a}{U_a} \tag{5}$$

Further

$$K_a C_a = -h(C_d - C_i) \tag{6}$$

Boundary conditions are,

$U = U_0$, $x = 0$where $0 \leq x \leq 1.0$. $C_d = C_{a=i}$, $x = L$Where $0 \leq C_d \leq 8$, and $C_d = C_{d_0}$, $x = 0$ where $0 \leq C_i \leq 8$ (7)

III. Analysis

Solving equations (3), (5) and (6) using Runge – Kutta method for simultaneous equations, we obtained the following:

Mass average velocity of waste products in kidney increases in the ranges from 1.03030 to 1.030885, concentration for waste products in descending limb decreases in the ranges from 0.94110 to 0.940579, concentration for waste products in ascending limb decreases in the ranges from 0.47010 to 0.4695807 at different displacements.

Further to obtain the validation of the findings, using shooting method, these iterations are carried out until $U_1$is sufficiently close to U_0, we obtain the values for the mass average velocity, concentration of metabolic waste products in descending limb and ascending limb. Mass average velocity of metabolic waste products in kidney increases in the ranges from 1.03030 to 1.030885, concentration for metabolic waste products in descending limb decreases in the ranges from 0.94110 to 0.940579, concentration for metabolic waste products in ascending limb decreases in the ranges from 0.47010 to 0.4695807 at different displacements.

IV. Results And Discussion

Mass average velocity, concentration of metabolic waste products in descending and ascending limb are estimated using Runge – Kutta fourth order method. We observe that the mass average velocity of metabolic waste products in kidney increases in the ranges from 1.03031 to 1.2417171, concentration of metabolic waste products in descending limb decreases in the ranges from 0.94111 to 0.58504, concentration of metabolic waste products in ascending limb decreases in the ranges from 0.47011 to 0.279809, at different displacement.

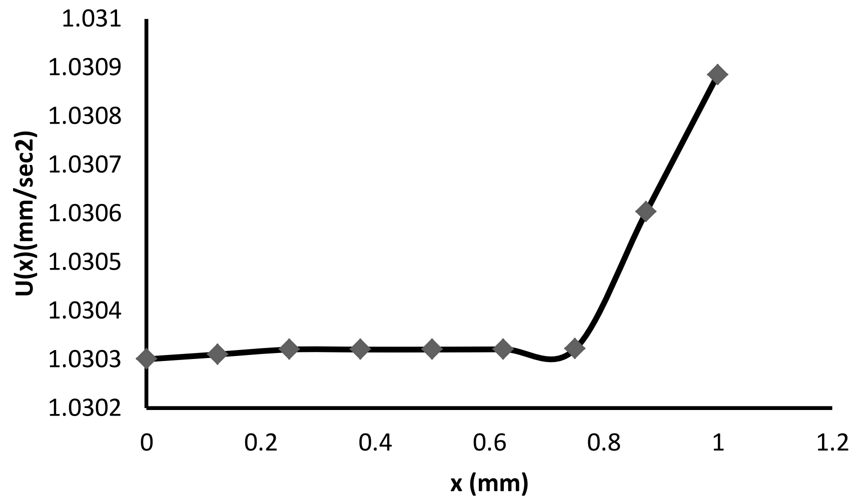

Fig.1: mass average velocity v/s displacement

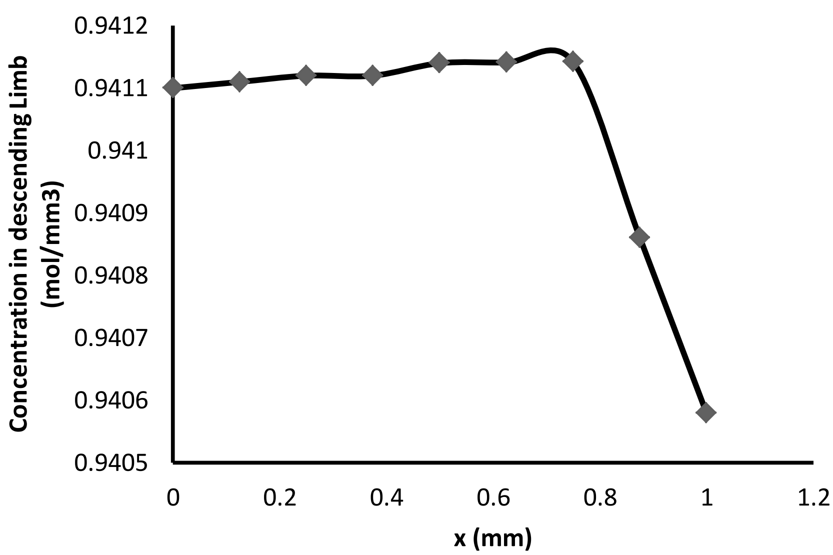

Fig.2: concentration in descending limb v/s displacement

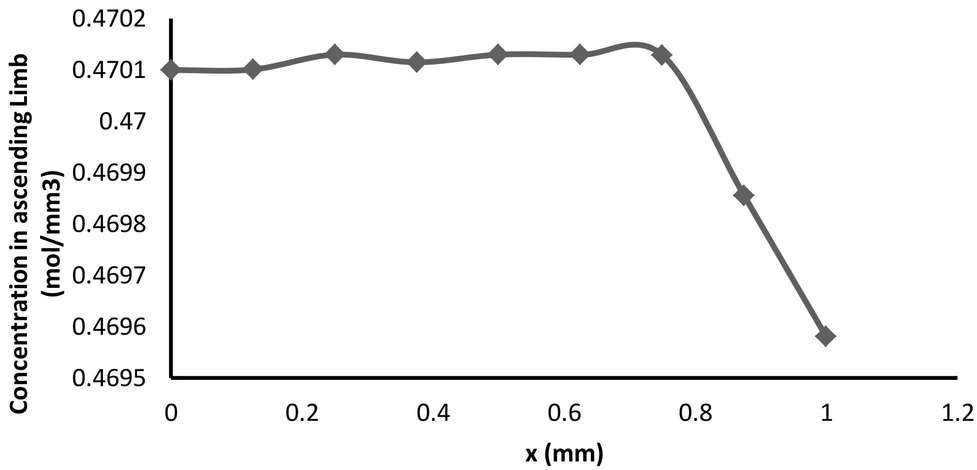

Fig.3: concentration in ascending limb v/s displacement

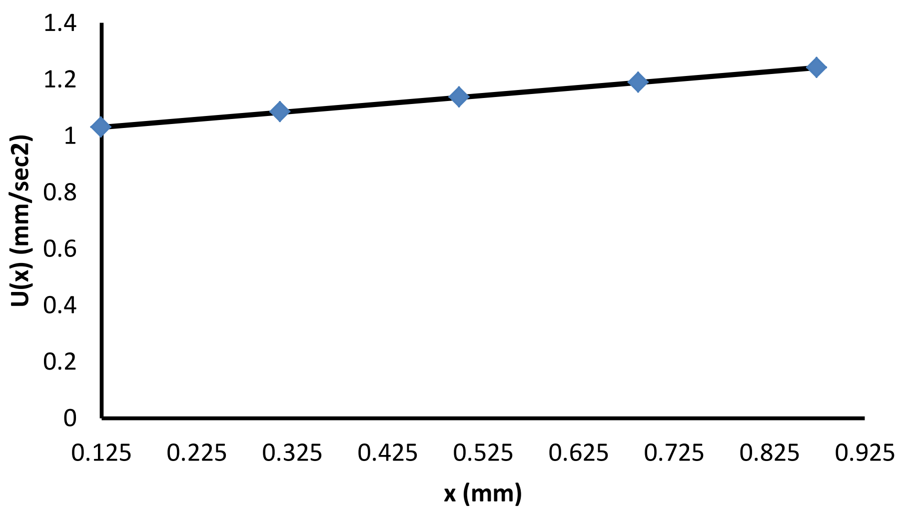

Fig 4: mass average velocity v/s displacement

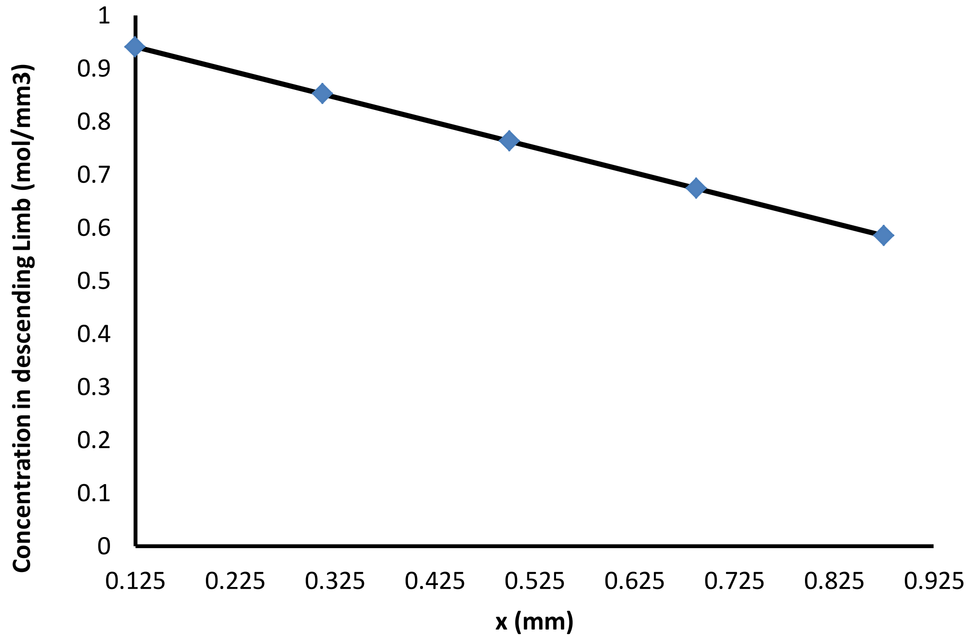

Fig.5: concentration in descending limb v/s displacement

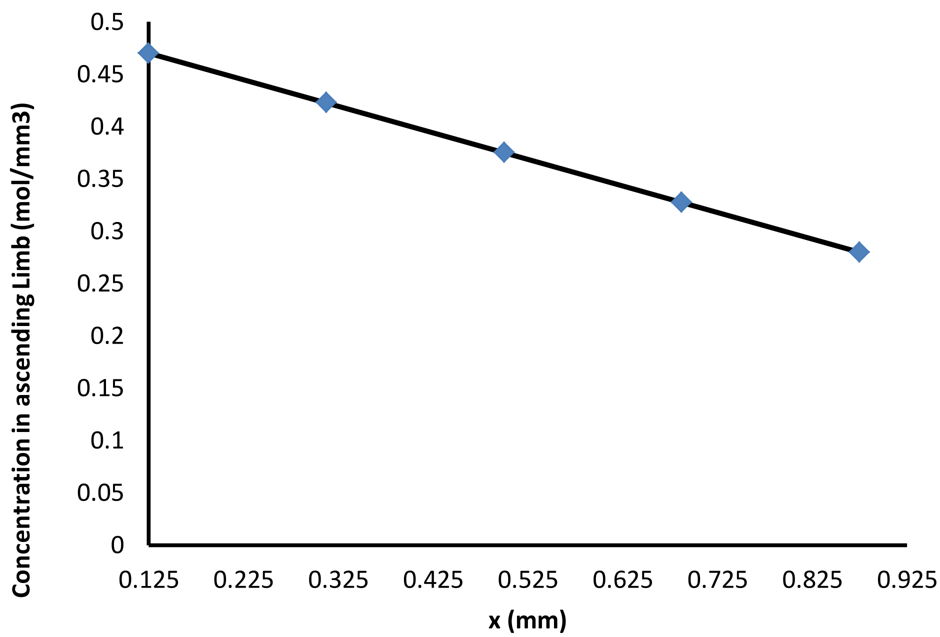

Fig.6: concentration in ascending limb v/s displacement

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