Computation of Distance Based Connectivity Status Neighborhood Dakshayani Indices

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Abstract: *Connectivity indices are applied to measure the chemical characteristics of chemical compounds in Chemical Sciences. In this paper, we introduce the sum and product status neighborhood Dakshayani indices, modified first and second status neighborhood Dakshayani indices, general first and second status neighborhood Dakshayani indices, atom bond connectivity status neighborhood Dakshayani index, geometric-arithmetic status neighborhood Dakshayani index of a graph and compute exact formulas for complete, complete bipartite, wheel and friendship graphs.*

Keywords: *distance, status, connectivity status neighborhood Dakshayani indices, graph.*

Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*90.

I. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The distance $d(u, v)$ between any two vertices u and v is the length of shortest path connecting u and v. The status $\sigma(u)$ of a vertex u in a connected G is the sum of distances of all other vertices from u in G. Let $N(u)=N_G(u)=\{v{:}uv\in E(G)\}$.

Let $\quad \sigma_n(u)=\sum_{v\in N(u)} \sigma(v).$

Let $\quad \sigma_d(u)=\sigma(u)+\sum_{v\in N(u)} \sigma(v)=\sigma(u)+\sigma_n(u)=\sum_{v\in N[u]} \sigma(v)$

where $N[u] = N(u) \cup \{v\}$. Then $\sigma_d(u)$ is the status sum of closed neighborhood vertices of u. For graph theoretic technology, we refer [1].

Graph indices are very important on the development of Chemical Graph Theory, Mathematical Chemistry, see [2, 3]. Many graph indices were defined by using vertex degree concept, distance concept [4]. Several distance based indices of a graph have been appeared in the literature. Recently, some of the research works on the status indices can be found in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The first and second status neighborhood Dakshayani indices of a graph are introduced by Kulli in [16], and they are defined as

$$SD_1(G)=\sum_{uv\in E(G)} \left[\sigma_d(u)+\sigma_d(v)\right], \qquad SD_2(G)=\sum_{uv\in E(G)} \sigma_d(u)\sigma_d(v).$$

Motivated by the work on distance based graph indices, we introduce the following connectivity status neighborhood Dakshayani indices as follows:

The sum connectivity status neighborhood Dakshayani index of a graph G is defined as

$$SSD(G)=\sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma_d(u)+\sigma_d(v)}}.$$

The product connectivity status neighborhood Dakshayani index of a graph G is defined as

$$PSD(G)=\sum_{uv\in E(G)} \frac{1}{\sqrt{\sigma_d(u)\sigma_d(v)}}.$$

The reciprocal product connectivity status neighborhood Dakshayani index of a graph G is defined as

$$RPSD(G)=\sum_{uv\in E(G)} \sqrt{\sigma_d(u)\sigma_d(v)}.$$

The modified first and second status neighborhood Dakshayani indices of a graph G are defined as

$$ {}^{m}SD_1(G)=\sum_{uv\in E(G)}\frac{1}{\sigma_d(u)+\sigma_d(v)}, \qquad {}^{m}SD_2(G)=\sum_{uv\in E(G)}\frac{1}{\sigma_d(u)\sigma_d(v)}. $$

The general first and second status neighborhood Dakshayani indices of a graph G are defined as

$$ SD_1^a(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)\right]^a, \qquad SD_2^a(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)\sigma_d(v)\right]^a. $$

We introduce the atom bond connectivity status neighborhood Dakshayani index, geometric-arithmetic status neighborhood Dakshayani index, arithmetic-geometric status neighborhood index, augmented status neighborhood Dakshayani index of a graph as follows:

The atom bond connectivity status neighborhood Dakshayani index of a graph G is defined as

$$ ABCSD(G)=\sum_{uv\in E(G)}\sqrt{\frac{\sigma_d(u)+\sigma_d(v)-2}{\sigma_d(u)\sigma_d(v)}}. $$

The geometric-arithmetic status neighborhood Dakshayani index of a graph G is defined as

$$ GASD(G)=\sum_{uv\in E(G)}\frac{2\sqrt{\sigma_d(u)\sigma_d(v)}}{\sigma_d(u)+\sigma_d(v)}. $$

The arithmetic-geometric status neighborhood Dakshayani index of a graph G is defined as

$$ AGSD(G)=\sum_{uv\in E(G)}\frac{\sigma_d(u)+\sigma_d(v)}{2\sqrt{\sigma_d(u)\sigma_d(v)}}. $$

The augmented status neighborhood Dakshayani index of a graph G is defined as

$$ ASD(G)=\sum_{uv\in E(G)}\left(\frac{\sigma_d(u)\sigma_d(v)}{\sigma_d(u)+\sigma_d(v)-2}\right)^3. $$

The harmonic status neighborhood Dakshayani index of a graph G is defined as

$$ HSD(G)=\sum_{uv\in E(G)}\frac{2}{\sigma_d(u)+\sigma_d(v)}. $$

In this paper, some newly defined status neighborhood Dakshayani indices of complete, complete bipartite, wheel and friendship graphs are computed.

II. RESULTS FOR COMPLETE GRAPHS

Theorem 1. The general first status neighborhood Dakshayani index of a complete graph K_n is given by

$$ SD_1^a(K_n)=2^{a-1}\left[n(n-1)\right]^{a+1}. \qquad (1) $$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. Then $d_{K_n}(u)=n-1$ and σ(u)= n – 1 for any vertex u of K_n. Thus $\sigma_n(u)=(n-1)^2$ for any vertex u of K_n. Therefore $\sigma_d(u)=n(n-1)$ for any vertex u of K_n. Therefore

$$ SD_1^a(K_n)=\sum_{uv\in E(K_n)}\left[\sigma_n(u)+\sigma_n(v)\right]^a=\frac{n(n-1)}{2}\left[n(n-1)+n(n-1)\right]^a. $$

$$ =2^{a-1}\left[n(n-1)\right]^{a+1}. $$

We obtain the following results from Theorem 1.

Corollary 1.1. The sum connectivity status neighborhood Dakshayani index of a complete graph K_n is

$$ SSD(K_n)=\frac{\sqrt{n(n-1)}}{2\sqrt{2}}. $$

Corollary 1.2. The modified first status neighborhood Dakshayani index of a complete graph K_n is

$$^{m}SD_1(K_n) = \frac{1}{4}.$$

Theorem 2. The general second status neighborhood Dakshayani index of a complete graph K_n is given by

$$SN_2^a(K_n) = \frac{1}{2}[n(n-1)]^{2a+1}.$$

Proof: Let K_n be a complete graph. Then $\sigma_d(u) = n(n-1)$ for any vertex u of K_n. Thus

$$SN_2^a(K_n) = \sum_{uv\in E(K_n)} [\sigma_d(u)\sigma_d(v)]^a = \frac{n(n-1)}{2}[n(n-1)n(n-1)]^a.$$

$$= \frac{1}{2}[n(n-1)]^{2a+1}.$$

We establish the following results by using Theorem 2.

Corollary 2.1. The product connectivity status neighborhood Dakshayani index of a complete graph K_n is

$$PSD(K_n) = \frac{1}{2}.$$

Corollary 2.2. The reciprocal product connectivity status neighborhood Dakshayani index of a complete graph K_n is

$$RPSD(K_n) = \frac{1}{2}(n^4 - 2n^3 + n^2).$$

Corollary 2.3. The modified second status neighborhood Dakshayani index of a complete graph K_n is

$$^{m}SD_2(K_n) = \frac{1}{2n(n-1)}.$$

Theorem 3. Let K_n be a complete graph with n vertices. Then

(1) $\quad ABCSD(K_n) = \frac{1}{\sqrt{2}}(n^2 - n - 1)^{\frac{1}{2}}.$

(2) $\quad GASD(K_n) = \frac{n(n-1)}{2}.$

(3) $\quad AGSD(K_n) = \frac{n(n-1)}{2}.$

(4) $\quad ASD(K_n) = \frac{n(n-1)}{16}\left(\frac{n^4 - 2n^3 + n^2}{n^2 - n + 1}\right)^3.$

(5) $\quad HSD(K_n) = \frac{1}{2}.$

Proof: If K_n is a complete graph with n vertices, then it has $\frac{n(n-1)}{2}$ edges and for any vertex u of K_n, $\sigma_d(u) = n(n-1)$.

(1) $\quad ABCSD(K_n) = \sum_{uv\in E(K_n)} \sqrt{\frac{\sigma_d(u)+\sigma_d(v)-2}{\sigma_d(u)\sigma_d(v)}}$

$$= \frac{n(n-1)}{2}\left(\frac{n(n-1)+n(n-1)-2}{n(n-1)n(n-1)}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}(n^2 - n - 1)^{\frac{1}{2}}.$$

(2) $\quad GASD(K_n) = \sum_{uv\in E(K_n)} \frac{2\sqrt{\sigma_d(u)\sigma_d(v)}}{\sigma_d(u)+\sigma_d(v)}$

$$=\frac{n(n-1)}{2}\left(\frac{2\sqrt{n(n-1)n(n-1)}}{n(n-1)+n(n-1)}\right)=\frac{n(n-1)}{2}.$$

(3) $$AGSD(K_n)=\sum_{uv\in E(K_n)}\frac{\sigma_d(u)+\sigma_d(v)}{2\sqrt{\sigma_d(u)\sigma_d(v)}}$$

$$=\frac{n(n-1)}{2}\left(\frac{n(n-1)+n(n-1)}{2\sqrt{n(n-1)n(n-1)}}\right)=\frac{n(n-1)}{2}.$$

(4) $$ASD(K_n)=\sum_{uv\in E(K_n)}\left(\frac{\sigma_d(u)\sigma_d(v)}{\sigma_d(u)+\sigma_d(v)-2}\right)^3$$

$$=\frac{n(n-1)}{2}\left(\frac{n(n-1)n(n-1)}{n(n-1)+n(n-1)-2}\right)^3=\frac{n(n-1)}{16}\left(\frac{n^4-2n^3+n^2}{n^2-n-1}\right)^3.$$

(5) $$HSD(K_n)=\sum_{uv\in E(K_n)}\frac{2}{\sigma_d(u)+\sigma_d(v)}=\frac{n(n-1)}{2}\left(\frac{2}{n(n-1)+n(n-1)}\right)=\frac{1}{2}.$$

III. RESULTS FOR COMPLETE BIPARTITE GRAPHS

Theorem 4. The general first status neighborhood Dakshayani index of a complete bipartite graph $K_{p,q}$ is

$$SD_1^a(K_{p,q})=pq\left[2(p^2+q^2)+(p+q)+2pq-4\right]^a. \qquad (3)$$

Proof: The vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that $u \in V_1$ and $v \in V_2$ for every edge uv in $K_{p,q}$. Let $K=K_{p,q}$. We have $d_K(u)=q$, $d_K(v)=p$. Then $\sigma(u)= q + 2p - 2$ and $\sigma(v)= p + 2q - 2$. Hence $\sigma_n(u)= p(q + 2p - 2)$ and $\sigma_n(v)= q(p + 2q - 2)$. Thus $\sigma_d(u)= (1+p)(q + 2p - 2)$ and $\sigma_n(v)= (1+q) (p + 2q - 2)$. Therefore

$$SD_1^a(K_{p,q})=\sum_{uv\in E(K)}\left[\sigma_d(u)+\sigma_d(v)\right]^a=pq\left[(1+p)(q+2p-2)+(1+q)(p+2q-2)\right]^a$$

$$=pq\left[2(p^2+q^2)+(p+q)+2pq-4\right]^a.$$

We establish the following results from Theorem 4.

Corollary 4.1. The sum connectivity status neighborhood Dakshayani index of $K_{p,q}$ is

$$SSD(K_{p,q})=\frac{pq}{\sqrt{2(p^2+q^2)+(p+q)+2pq-4}}.$$

Corollary 4.2. The modified first status neighborhood Dakshayani index of $K_{p,q}$ is

$${}^mSD_1(K_{p,q})=\frac{pq}{2(p^2+q^2)+(p+q)+2pq-4}.$$

Theorem 5. The general second status neighborhood Dakshayani index of $K_{p,q}$ is given by

$$SN_2^a(K_{p,q})=pq(1+p)^a(1+q)^a\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^a.$$

Proof: Let $K_{p,q}$ be a complete bipartite graph with pq edges. Then $\sigma_d(u)= (1+p)(q + 2p - 2)$ and $\sigma_d(v)= (1+q)(p + 2q - 2)$. Thus

$$SN_2^a(K_{p,q})=\sum_{uv\in E(K)}\left[\sigma_n(u)\sigma_n(v)\right]^a=pq\left[(1+p)(q+2p-2)(1+q)(p+2q-2)\right]^a$$

$$=pq(1+p)^2(1+q)^2\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^a.$$

We obtain the following results by using Theorem 5.

Corollary 5.1. The product connectivity status neighborhood Dakshayani index of $K_{p,q}$ is

$$PSD\left(K_{p,q}\right)=\frac{pq}{\sqrt{(1+p)(1+q)\left[2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right]}}.$$

Corollary 5.2. The reciprocal product connectivity status neighborhood Dakshayani index of $K_{p,q}$ is

$$RPSD\left(K_{p,q}\right)=pq\left[(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}\right]^{\frac{1}{2}}.$$

Corollary 5.3. The modified second status neighborhood Dakshayani index of $K_{p,q}$ is

$$^{m}SD_2\left(K_{p,q}\right)=\frac{pq}{(1+p)(1+q)\left[2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right]}.$$

Theorem 6. Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. Then

(1) $$ABCSD\left(K_{p,q}\right)=pq\left(\frac{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-2}{(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}}\right)^{\frac{1}{2}}.$$

(2) $$GASD\left(K_{p,q}\right)=pq\frac{2\left[(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}\right]^{\frac{1}{2}}}{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-4}.$$

(3) $$AGSD\left(K_{p,q}\right)=pq\frac{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-4}{2\left[(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}\right]^{\frac{1}{2}}}.$$

(4) $$ASD\left(K_{p,q}\right)=pq\left(\frac{(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}}{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-2}\right)^{3}.$$

(5) $$HSD\left(K_{p,q}\right)=\frac{2pq}{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-4}.$$

Proof: Let $K_{p,q}$ be a complete bipartite graph. Then $\sigma_d(u)= (1+p)(q + 2p – 2)$ and $\sigma_d(v)= (1+q)(p + 2q – 2)$ for every uv of $K_{p,q}$.

(1) $$ABCSD\left(K_{p,q}\right)=\sum_{uv\in E(K)}\sqrt{\frac{\sigma_d(u)+\sigma_d(v)-2}{\sigma_d(u)\sigma_d(v)}}$$

$$=pq\left(\frac{(1+p)(q+2p-2)+(1+q)(p+2q-2)-2}{(1+p)(q+2p-2)(1+q)(p+2q-2)}\right)^{\frac{1}{2}}$$

$$=pq\left(\frac{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-2}{(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}}\right)^{\frac{1}{2}}$$

(2) $$GASD\left(K_{p,q}\right)=\sum_{uv\in E(K)}\frac{2\sqrt{\sigma_d(u)\sigma_d(v)}}{\sigma_d(u)+\sigma_d(v)}$$

$$=pq\frac{2\left[(1+p)(q+2p-2)(1+q)(p+2q-2)\right]^{\frac{1}{2}}}{(1+p)(q+2p-2)+(1+q)(p+2q-2)}$$

$$=pq\frac{2\left[(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6\left(p+q\right)+5pq+4\right\}\right]^{\frac{1}{2}}}{2\left(p^2+q^2\right)+\left(p+q\right)+2pq-4}$$

(3) $$AGSD\left(K_{p,q}\right)=\sum_{uv\in E(K)}\frac{\sigma_d(u)+\sigma_d(v)}{2\sqrt{\sigma_d(u)\sigma_d(v)}}$$
$$=pq\times\frac{(1+p)(q+2p-2)+(1+q)(p+2q-2)}{2\left[(1+p)(q+2p-2)(1+q)(p+2q-2)\right]^{\frac{1}{2}}}$$
$$=pq\frac{2\left(p^2+q^2\right)+(p+q)+2pq-4}{2\left[(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}\right]^{\frac{1}{2}}}$$

(4) $$ASD\left(K_{p,q}\right)=\sum_{uv\in E(K)}\left(\frac{\sigma_d(u)\sigma_d(v)}{\sigma_d(u)+\sigma_d(v)-2}\right)^3$$
$$=pq\left(\frac{(1+p)(q+2p-2)(1+q)(p+2q-2)}{(1+p)(q+2p-2)+(1+q)(p+2q-2)-2}\right)^3$$
$$=pq\left(\frac{(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}}{2\left(p^2+q^2\right)+(p+q)+2pq-2}\right)^3$$

(5) $$HSD\left(K_{p,q}\right)=\sum_{uv\in E(K)}\frac{2}{\sigma_d(u)+\sigma_d(v)}=\frac{2pq}{(1+p)(q+2p-2)+(1+q)(p+2q-2)}$$
$$=\frac{2pq}{2\left(p^2+q^2\right)+(p+q)+2pq-4}$$

IV. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1. A graph W_4 is presented in Figure 1.

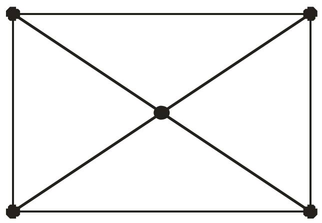

Figure 1. Wheel graph W_4

A wheel graph W_n has n+1 vertices and $2n$ edges. In W_n, by calculation, we obtain that there are two types of status neighborhood Dakshayani edges as given in Table 1.

$\sigma_d(u),\sigma_d(v)\setminus uv\in E(W_n)$	$(7n-9,\ 7n-9)$	$(7n-9,\ 2n^2-2n)$
Number of edges	n	n

Table 1. Status neighborhood Dakshayani edge partition of W_n

Theorem 7. The general first status neighborhood Dakshayani index of a wheel graph W_n is given by

$$SD_1^a(W_n)=n(14n-18)^a+n\left(2n^2+5n-9\right)^a.$$

Proof: From definition and by using Table 1, we deduce

$$SD_1^a(W_n)=\sum_{uv\in E(W_n)}\left[\sigma_d(u)+\sigma_d(v)\right]^a=n(7n-9+7n-9)^a+n\left(7n-9+2n^2-2n\right)^a$$
$$=n(14n-18)^a+n\left(2n^2+5n-9\right)^a.$$

We obtain the following results by using Theorem 7.

Corollary 7.1. The sum connectivity status neighborhood Dakshayani index of W_n is

$$SSD(W_n) = \frac{n}{\sqrt{14n-18}} + \frac{n}{\sqrt{2n^2+5n-9}}.$$

Corollary 7.2. The modified first status neighborhood Dakshayani index of W_n is

$$^{m}SD_1(W_n) = \frac{n}{10n-18} + \frac{n}{2n^2+5n-9}.$$

Theorem 8. The general second status neighborhood Dakshayani index of a wheel graph W_n is given by

$$SD_2^a(W_n) = n(7n-9)^{2a} + n(7n-9)^a(2n^2-2n)^a. \tag{6}$$

Proof: Using definition and Table 1, we derive

$$SD_2^a(W_n) = \sum_{uv\in E(W_n)} \left[\sigma_d(u)\sigma_d(v)\right]^a = n[(7n-9)(7n-9)]^a + n\left[(7n-9)(2n^2-3n)\right]^a$$

$$= n(7n-9)^{2a} + n(7n-9)^a(2n^2-2n)^a.$$

From Theorem 8, we obtain the following results.

Corollary 8.1. The product connectivity status neighborhood Dakshayani index of W_n is

$$PSD(W_n) = \frac{n}{7n-9} + \frac{n}{\sqrt{(7n-9)(2n^2-2n)}}.$$

Corollary 8.2. The reciprocal product connectivity status neighborhood Dakshayani index of W_n is

$$RPSD(W_n) = n(7n-9) + n\left[(7n-9)(2n^2-2n)\right]^{\frac{1}{2}}.$$

Corollary 8.3. The modified second status neighborhood Dakshayani index of W_n is

$$^{m}SD_2(W_n) = \frac{n}{(7n-9)^2} + \frac{n}{(7n-9)(2n^2-2n)}.$$

Theorem 9. Let W_n be a wheel graph with $n+1$ vertices and $2n$ edges. Then

(1) $$ABCSD(W_n) = \frac{n\sqrt{14n-20}}{7n-9} + n\left(\frac{2n^2+5n-11}{14n^3-32n^2+18n}\right)^{\frac{1}{2}}.$$

(2) $$GASD(W_n) = n + \frac{2n\sqrt{14n^3-32n+18n}}{2n^2+5n-9}.$$

(3) $$AGSD(W_n) = n + \frac{n(2n^2+5n-9)}{2\sqrt{14n^3-32n+18n}}.$$

(4) $$ASD(W_n) = n\left(\frac{(7n-9)^2}{14n-20}\right)^3 + n\left(\frac{14n^3-32n^2+18n}{n^2+5n-11}\right)^3.$$

(5) $$HSD(W_n) = \frac{n}{7n-9} + \frac{2n}{2n^2+5n-9}.$$

Proof: Let W_n be a wheel graph with $n+1$ vertices and $2n$ edges.

(1) Using definition and Table 1, we deduce

$$ABCSD(W_n) = \sum_{uv\in E(W_n)} \sqrt{\frac{\sigma_d(u)+\sigma_d(v)-2}{\sigma_d(u)\sigma_d(v)}}$$

$$=\left(\frac{7n-9+7n-9-2}{(7n-9)(7n-9)}\right)^{\frac{1}{2}}+n\left(\frac{7n-9+2n^2-2n-2}{(7n-9)(2n^2-2n)}\right)^{\frac{1}{2}}$$

$$=\frac{n\sqrt{14n-20}}{7n-9}+n\left(\frac{2n^2+5n-11}{14n^3-32n^2+18n}\right)^{\frac{1}{2}}$$

(2) From equation and by using Table 1, we derive

$$GASD(W_n)=\sum_{uv\in E(W_n)}\frac{2\sqrt{\sigma_d(u)\sigma_d(v)}}{\sigma_d(u)+\sigma_d(v)}$$

$$=n\frac{2\sqrt{(7n-9)(7n-9)}}{7n-9+7n-9}+n\frac{2\sqrt{(7n-9)(2n^2-2n)}}{7n-9+2n^2-2n}$$

$$=n+\frac{2n\sqrt{14n^3-32n+18n}}{2n^2+5n-9}$$

(3) Using definition and Table 1, we obtain

$$AGSD(W_n)=\sum_{uv\in E(W_n)}\frac{\sigma_d(u)+\sigma_d(v)}{2\sqrt{\sigma_d(u)\sigma_d(v)}}$$

$$=n\frac{7n-9+7n-9}{2\sqrt{(7n-9)(7n-9)}}+n\frac{7n-9+2n^2-2n}{2\sqrt{(7n-9)(2n^2-2n)}}$$

$$=n+\frac{n(2n^2+5n-9)}{2\sqrt{14n^3-32n+18n}}$$

(4) From definition and by using Table 1, we deduce

$$ASD(W_n)=\sum_{uv\in E(W_n)}\left(\frac{\sigma_d(u)\sigma_d(v)}{\sigma_d(u)+\sigma_d(v)-2}\right)^3$$

$$=n\left(\frac{(7n-9)(7n-9)}{7n-9+7n-9-2}\right)^3+n\left(\frac{(7n-9)(2n^2-2n)}{7n-9+2n^2-2n-2}\right)^3.$$

$$=n\left(\frac{(7n-2)^2}{14n-20}\right)^3+n\left(\frac{14n^3-32n^2+18n}{n^2+5n-11}\right)^3.$$

(5) $$HSD(W_n)=\sum_{uv\in E(W_n)}\frac{2}{\sigma_d(u)+\sigma_d(v)}=\frac{2n}{7n-9+7n-9}+\frac{2n}{7n-9+2n^2-2n}$$

$$=\frac{n}{7n-9}+\frac{2n}{2n^2+5n-9}.$$

V. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_n is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. A graph F_4 shown in Figure 2.

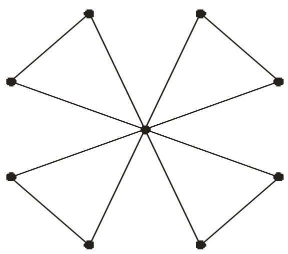

Figure 2. Friendship graph F_4

A graph F_n has $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of status neighborhood Dakshayani edges as given in Table 2.

$\sigma_d(u),\sigma_d(v)\setminus uv\in E(F_n)$	$(10n-4,\ 10n-4)$	$(10n-4,\ 2n(4n-1))$
Number of edges	n	$2n$

Table 2. Status neighborhood Dakshayani edge partition of F_n

Theorem 10. The general first status neighborhood Dakshayani index of a friendship graph F_n is given by

$$SD_1^a(F_n)=n(20n-8)^a+2n(8n^2+8n-4)^a.$$

Proof: Using definition and Table 2, we deduce

$$SD_1^a(F_n)=\sum_{uv\in E(F_n)}\left[\sigma_d(u)+\sigma_d(v)\right]^a=n(10n-4+10n-4)^a+2n(10n-4+8n^2-2n)^a$$

$$=n(20n-8)^a+2n(8n^2+8n-4)^a.$$

We establish the following results from Theorem 10.

Corollary 10.1. The sum connectivity status neighborhood Dakshayani index of F_n is

$$SSD(F_n)=\frac{n}{\sqrt{20n-8}}+\frac{n}{\sqrt{2n^2+2n-1}}.$$

Corollary 10.2. The modified first status neighborhood Dakshayani index of F_n is

$${}^mSD_1(F_n)=\frac{n}{20n-8}+\frac{n}{2(2n^2+2n-1)}.$$

Theorem 11. The general second status neighborhood Dakshayani index of a friendship graph F_n is given by

$$SD_2^a(F_n)=n(10n-4)^{2a}+2n(80n^3-52n^2+8n)^a. \tag{8}$$

Proof: From definition and by using Table 2, we obtain

$$SD_2^a(F_n)=\sum_{uv\in E(F_n)}\left[\sigma_d(u)\sigma_d(v)\right]^a=n[(10n-4)(10n-4)]^a+2n\left[(10n-4)(8n^2-2n)\right]^a$$

$$=n(10n-4)^{2a}+2n(80n^3-52n^2+8n)^a.$$

Using Theorem 11, we obtain the following results.

Corollary 11.1. The product connectivity by status neighborhood Dakshayani index of F_n is

$$PSD(F_n)=\frac{n}{10n-4}+\frac{n}{\sqrt{20n^3-13n^2+2n}}.$$

Corollary 11.2. The reciprocal product connectivity by status neighborhood Dakshayani index of F_n is

$$RPSD(F_n)=n(10n-4)+4n\sqrt{20n^3-13n^2+2n}$$

Corollary 11.3. The modified second status neighborhood Dakshayani index of F_n is

$${}^mSD_2(F_n)=\frac{n}{(10n-4)^2}+\frac{1}{40n^2-26n+2}.$$

Theorem 12. Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(1) $$ABCSD(F_n)=\frac{n\sqrt{20n-10}}{10n-4}+2n\left(\frac{4n^2+4n-3}{40n^3-26n^2+4n}\right)^{\frac{1}{2}}.$$

(2) $$GASD(F_n)=n+\frac{2n\sqrt{20n^3-13n^2+2n}}{2n^2+2n-1}.$$

(3) $$AGSD(F_n)=n+\frac{2n(2n^2+2n-1)}{\sqrt{20n^3-13n^2+2n}}.$$

(4) $$ASD(F_n)=n\left(\frac{(10n-4)^2}{20n-10}\right)^3+2n\left(\frac{40n^3-26n^2+4n}{4n^2+4n-3}\right)^3.$$

(5) $$HSD(F_n)=\frac{n}{10n-4}+\frac{n}{2n^2+2n-1}.$$

Proof: (1) From definition and Table 2, we deduce

$$ABCSD(F_n)=\sum_{uv\in E(F_n)}\left(\frac{\sigma_d(u)+\sigma_d(v)-2}{\sigma_d(u)\sigma_d(v)}\right)^{\frac{1}{2}}$$

$$=n\left(\frac{10n-4+10n-4-2}{(10n-4)(10n-4)}\right)^{\frac{1}{2}}+2n\left(\frac{10n-4+8n^2-2n-2}{(10n-4)(8n^2-2n)}\right)^{\frac{1}{2}}$$

$$=\frac{n\sqrt{20n-10}}{10n-4}+2n\left(\frac{4n^2+4n-3}{40n^3-26n^2+4n}\right)^{\frac{1}{2}}$$

(2) Using definition and Table 2, we derive

$$GASD(F_n)=\sum_{uv\in E(F_n)}\frac{2\sqrt{\sigma_d(u)\sigma_d(v)}}{\sigma_d(u)+\sigma_d(v)}=n\frac{2\sqrt{(10n-4)(10n-4)}}{10n-4+10n-4}+2n\frac{2\sqrt{(10n-4)(8n^2-2n)}}{10n-4+8n^2-2n}$$

$$=n+\frac{2n\sqrt{20n^3-13n^2+2n}}{2n^2+2n-1}$$

(3) From definition and by using Table 2, we obtain

$$AGSD(F_n)=\sum_{uv\in E(F_n)}\frac{\sigma_d(u)+\sigma_d(v)}{2\sqrt{\sigma_d(u)\sigma_d(v)}}=n\frac{10n-4+10n-4}{2\sqrt{(10n-4)(10n-4)}}+2n\frac{10n-4+8n^2-2n}{2\sqrt{(10n-4)(8n^2-2n)}}$$

$$=n+\frac{2n(2n^2+2n-1)}{\sqrt{20n^3-13n^2+2n}}$$

(4) From definition and by using Table 2, we obtain

$$ASD(F_n)=\sum_{uv\in E(F_n)}\left(\frac{\sigma_d(u)\sigma_d(v)}{\sigma_d(u)+\sigma_d(v)-2}\right)^3$$

$$=n\left(\frac{(10n-4)(10n-4)}{10n-4+10n-4-2}\right)^3+2n\left(\frac{(10n-4)(8n^2-2n)}{10n-4+8n^2-2n-2}\right)^3.$$

$$=n\left(\frac{(10n-4)^2}{20n-10}\right)^3+2n\left(\frac{40n^3-26n^2+4n}{4n^2+4n-3}\right)^3.$$

(5) Using definition and by using Table 2, we deduce

$$HSD(F_n)=\sum_{uv\in E(F_n)}\frac{2}{\sigma_d(u)+\sigma_d(v)}=\frac{2n}{10n-4+10n-4}+\frac{4n}{10n-4+8n^2-2n}$$

$$=\frac{n}{10n-4}+\frac{n}{2n^2+2n-1}.$$

REFERENCES

[1] V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
[2] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).
[3] V.R.Kulli, *Multiplicative Connectivity Indices of Nanostructures*, LAP LEMBERT Academic Publishing, (2018).
[4] V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2020) 66-91.
[5] V.R.Kulli, Some new status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(10) (2019) 70-76.
[6] V.R.Kulli, Some new multiplicative status indices of graphs, *International Journal of Recent Scientific Research*, 10, 10(F) (2019) 35568-35573.
[7] V.R.Kulli, Computation of status indices of graphs, *International Journal of Mathematics Trends and Technology*, 65(12) (2019) 54-61.
[8] V.R.Kulli, The (a, b)-status index of graphs, *Annals of Pure and Applied Mathematics* 21(2) (2020) 103-108.
[9] V.R.Kulli, Computation of ABC, AG and augmented status indices of graphs, *International Journal of Mathematical Trends and Technology*, 66(1) (2020) 1-7.
[10] V.R.Kulli, Status Gourava indices of graphs, *International Journal of Recent Scientific Research*, 11, 1(A) (2020) 36770-36773.
[11] V.R.Kulli, Multiplicative ABC, GA, AG, augmented and harmonic status indices of graphs, *International Journal of Mathematical Archive*, 11(1) (2020) 32-40.
[12] V.R.Kulli, Computation of status neighborhood indices of graphs, *International Journal of Recent Scientific Research*, 11(4) (2020) 38079-38085.
[13] V.R.Kulli, Computation of multiplicative status indices of graphs, *International Journal of Mathematical Archive*, 11(4) (2020) 1-6.
[14] V.R.Kulli, Distance based connectivity status neighborhood indices of certain graphs, *International Journal of Mathematical Archive*, 11(6) (2020) 17-23.
[15] V.R.Kulli, Computation of some new status neighborhood indices of graphs, *International Research Journal of Pure Algebra*, to appear.
[16] V.R.Kulli, Status neighborhood Dakshayani indices, submitted.