Einstein's field equations and non-locality

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Abstract

Objective. Reconciling locality and non-locality in accordance with Einstein's general theory of relativity appears to be more than only a hopeless endeavour. Theoretically this seems either not necessary or almost impossible. **Methods.**

The usual tensor calculus rules were used. The proof method modus ponens was used to proof the theorems derived.

Results.

A tensor of locality and a tensor of non-locality is derived from Riemannian curvature tensor. Einstein's field equations were reformulated in terms of the tensor of locality and the tensor of non-locality without any contradiction, without changing Einstein's field equations at all and without adding anything new to Einstein's field equations. Weyl's curvature tensor does not model non-locality sufficiently well under any circumstances. Under conditions of the general theory of relativity, it is more appropriate and straightforward to use the non-locality curvature tensor which is part of the Riemannian curvature tensor to describe non-locality completely. **Conclusions.**

The view is justified that there is no contradiction at all between Einstein's field equations and the concept of locality and non-locality.

Keywords — principium identitatis, principium contradictionis, theory of relativity, unified field theory, causality.

I. INTRODUCTION

What is gravity[1] doing over extremely short distances from the point of view of general theory of relativity? Contemporary elementary particle physics or Quantum Field Theory (QFT) as an extension of quantum mechanics (OM) dealing with particles taken seriously in its implications seems to contradict Einstein's general relativity theory especially on this point. To put it another way, it appears to be hard to reconcile gravitation with QFT because where energy[2], time[3] and space or gravitation[4]-[6] as such becomes significant QFT is most severely endangered and vice versa. Physicists have spent a lot of time trying to reconcile these two very different theories. To date, relativity and quantum mechanics are two fundamentally different theories with genuinely incompatible descriptions of reality. Nevertheless, authors proposed unified field theories [7] of all natural forces, "a generalization of the theory of the gravitational field" [8] including gravitation itself, to bridge the gap between QFT and general relativity theory (i. e. String theory) without a definite success. In this context, the Riemann curvature tensor or Riemann-Christoffel tensor (after Bernhard Riemann and Elwin Bruno Christoffel) is a central mathematical object in the theory of general relativity. However, Weyl curvature tensor, named after Hermann Weyl, is a measure of the curvature of spacetime too and of no less importance. However, the Weyl tensor as the traceless component of the Riemann tensor differs from the Riemann curvature tensor too. The Weyl curvature tensor convey some information on how the shape of the body is distorted i. e. by tidal forces but does not convey any information on how the volume of the same body changes. The Weyl curvature as a component of curvature for Ricci-flat manifolds exists in free space too and is related to the propagation of gravitational waves through regions of space devoid of any matter[6], [9]. Describing Einstein's field equations in terms of Weyl's curvature could be of help to reconcile relativity and quantum mechanics.

II. MATERIAL AND METHODS

The basic rules of tensor calculus can be found in secondary literature too.

A. Definitions

The Royal Society of London and the Royal Astronomical Society announced[10] the first empirical test of the confirmation of Einstein's general theory of relativity. In order to extend general relativity's geometrization of gravitational force to non-gravitational interactions it appears to be reasonable to assure that new basic physical concepts are in accordance with Einstein's general of relativity[5] too.

2.1 Definitions

The basic rules of tensor calculus can be found in secondary literature too.

Definition 2.1.1(Anti tensor).

Let $a_{\mu\nu}$ denote a certain co-variant tensor. Let $b_{\mu\nu}$, $c_{\mu\nu}$... denote other tensors. Let $S_{\mu\nu}$ denote another tensor. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + ... \equiv S_{\mu\nu}$ be given. The **anti-tensor** of a tensor $a_{\mu\nu}$, denoted in general as $\underline{a}_{\mu\nu}$, is defined as

$$\underline{a}_{\mu\nu} \equiv S_{\mu\nu} - a_{\mu\nu}$$

$$\equiv b_{\mu\nu} + c_{\mu\nu} + \cdots$$
(1)

There may exist circumstances were an **anti-tensor** is identical with an **anti-symmetrical tensor**. However, an anti-tensor and an anti-symmetrical tensor are not identical as such.

Definition 2.1.2(Einstein's field equations).

Let $R_{\mu\nu}$ denote the **Ricci tensor**[11] of 'Einstein's general theory of relativity'[5], a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) which is able to encapsulate the degree to which a certain geometry of a given metric differs from the geometry of ordinary Euclidean space. Let R denote the **Ricci scalar**, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let Λ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "*anti-cosmological constant*"[12]. Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's[13] curvature tensor, the trace-reversed Ricci tensor. Let $\underline{G}_{\mu\nu}$ denote the "anti-tensor" [14] of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote the stress-energy tensor of energy / matter. Let $\underline{E}_{\mu\nu}$ denote the fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let γ denote Newton's gravitational "constant" [15]–[18]. Let π denote the number pi. Einstein's field equations, published by Albert Einstein[4] for the first time in 1915, and finally 1916[5] but later with the "cosmological constant" [6] term relate the curvature of spacetime to the energy and momentum as

$$(R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu}) \right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu}$$

$$\equiv \left(\frac{2 \times \pi \times \gamma}{c^4} \right) \times 4 \times T_{\mu\nu}$$

$$\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu})$$

$$\equiv R_{\mu\nu} - \underline{E}_{\mu\nu}$$

$$\equiv E_{\mu\nu}$$

$$\equiv a_{\mu\nu} + b_{\mu\nu}$$

$$EEFT SIDE = RIGHT SIDE$$

In point of fact, the left-hand side of the Einstein's field equations represents only one part (Ricci curvature) of the geometric structure. Formally, Weyl curvature is not included in this formalism.

Definition 2.1.3 (Weyl's curvature tensor).

Let $R_{kl\mu\nu}$ denote the Riemann curvature tensor. Let $W_{kl\mu\nu}$ denote Weyl's[19] curvature tensor, the traceless component of the Riemann tensor $R_{kl\mu\nu}$. Let $\underline{W}_{kl\mu\nu}$ denote anti Weyl's curvature tensor. In general, the Riemann tensor can be split[20] into a symmetric (massless) traceless Weyl's curvature tensor $W_{kl\mu\nu}$ and a trace-full massive part $\underline{W}_{kl\mu\nu}$ as

$$R_{kl\mu\nu} \equiv W_{kl\mu\nu} + \underline{W}_{kl\mu\nu}$$
(3)

nti Weyl's curvature tensor

The Weyl curvature tensor $W_{kl\mu\nu}$ differs from the Riemann curvature tensor $R_{kl\mu\nu}$. As mentioned before, the Weyl tensor $W_{kl\mu\nu}$ does not convey information on how the volume of a certain body changes, but rather how the shape of this body is distorted. In general relativity, the Weyl curvature exists in free space too and is related to the propagation of **gravitational waves**[20]^[21] through regions of space devoid of matter (vacuum solutions of Einstein's field equations). In point of fact, one of the many justified questions is whether there may exist circumstances where the three basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, (except gravitational waves $d_{\mu\nu}$) are determined by the following relationship:

$$\begin{array}{l} \left(a_{\mu\nu}\right) + \left(b_{\mu\nu}\right) + \left(c_{\mu\nu}\right) & \equiv & \underline{W}_{\mu\nu} \\ & \equiv & \left(R_{\mu\nu}\right) & - & \left(\underline{d}_{\mu\nu}\right) \\ & \equiv & \left(R_{\mu\nu}\right) & - & \left(W_{\mu\nu}\right) \end{array}$$

$$\tag{4}$$

or is it more likely that

The other theoretical possibility which has to be considered is the relationship

Questions like these can be solved by future publications. In particular, there is some evidence[22] that especially *Einstein's tensor* $G_{\mu\nu}$ is not completely identical with $\underline{W}_{\mu\nu}$. In this context, let $L_{kl\mu\nu}$ denote the locality curvature tensor. Let $\underline{L}_{kl\mu\nu}$ denote the non locality curvature tensor. In general, it is

$$\frac{\underline{L}_{kl\mu\nu}}{Non-locality curvature tensor} \equiv R_{kl\mu\nu} - \frac{L_{kl\mu\nu}}{Locality curvature tensor}$$
(7)

Let $L_{\mu\nu}$ denote *the tensor of locality*, the contraction of the locality curvature tensor $L_{k\mu\nu}$. Let $\underline{L}_{\mu\nu}$ denote *the tensor of non locality*, the contraction of the non locality curvature tensor $\underline{L}_{k\mu\nu}$. In general, it is

$$\underline{L}_{\mu\nu} \equiv g^{kl}R_{kl\mu\nu} - g^{kl}L_{kl\mu\nu}
\equiv R_{\mu\nu} - L_{\mu\nu}$$
(8)

and

$$L_{\mu\nu} \equiv g^{kl}R_{kl\mu\nu} - g^{kl}\underline{L}_{kl\mu\nu}$$

$$\equiv R_{\mu\nu} - \underline{L}_{\mu\nu}$$
(9)

The tensor $Z_{kl\mu\nu}$ is determined as

$$Z_{kl\mu\nu} \equiv \underline{W}_{kl\mu\nu} - L_{kl\mu\nu}$$
(10)

Definition 2.1.4 (The Einstein's curvature tensor).

The physics of gravitation as described by general relativity demands that the curvature of spacetime is determined by the presence of energy-momentum to manifest itself as gravity and vice versa. In turn, matter or energy and momentum acts on spacetime to create curvature. Under conditions of Einstein's general theory of relativity, Einstein's tensor of curvature denoted as $G_{\mu\nu}$ is defined/determined as

$$G_{\mu\nu} \equiv (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right)$$

$$\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} - b_{\mu\nu}$$

$$\equiv a_{\mu\nu} + d_{\mu\nu} + c_{\mu\nu} - d_{\mu\nu}$$

$$\equiv a_{\mu\nu} + c_{\mu\nu}$$

$$\equiv G \times g_{\mu\nu}$$

$$G \equiv S - \left(\frac{R}{2}\right)$$

$$\equiv E + t - \left(\frac{R}{2}\right)$$

$$\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) + \left(\frac{R}{2} - \Lambda\right) - \left(\frac{R}{2}\right)$$

$$\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) - \Lambda$$
(11)
(12)

where G is equal to

Definition 2.1.5 (The tensor of non-energy).

Under conditions of Einstein's general theory of relativity, the tensor of non-energy, denoted as $\underline{E}_{\mu\nu}$, or the antitensor of the stress energy tensor of energy is defined/derived/determined as follows:

$$\underline{E}_{\mu\nu} \equiv (R_{\mu\nu}) - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
\equiv \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - (\Lambda \times g_{\mu\nu}) \\
\equiv \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \\
\equiv c_{\mu\nu} + d_{\mu\nu} \\
\equiv c_{\mu\nu} + d_{\mu\nu} \\
\equiv t \times g_{\mu\nu} \\
\equiv t \times g_{\mu\nu} \\
t \equiv \underline{E} \equiv \left(\frac{R}{2} - \Lambda\right) \quad (14)$$

where

Definition 2.1.6 (The anti-Einstein's curvature tensor $\underline{G}_{\mu\nu}$ or the tensor or non-curvature).

Under conditions of Einstein's general theory of relativity, the anti-Einstein tensor, the tensor of non-curvature, denoted as $\underline{G}_{\mu\nu}$, is defined/determined

$$\underline{G}_{\mu\nu} \equiv (R_{\mu\nu}) - (G_{\mu\nu}) \\
\equiv (R_{\mu\nu}) - (\overline{R}_{\mu\nu} - (\frac{R}{2}) \times (g_{\mu\nu})) \\
\equiv (\frac{R}{2}) \times (g_{\mu\nu}) \\
\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\
\equiv b_{\mu\nu} + d_{\mu\nu} \\
\equiv \underline{G} \times g_{\mu\nu}$$
(15)

where $\underline{\mathbf{G}} = (\mathbf{R}/2)$.

Definition 2.1.7 (The tensor $d_{\mu\nu}$ (neither curvature nor momentum)).

Under conditions of Einstein's general theory of relativity, the tensor of neither curvature nor momentum, denoted as $d_{\mu\nu}$, is defined/derived/determined as follows:

$$d_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times (g_{\mu\nu}) - (b_{\mu\nu})$$
$$\equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (\Lambda \times (g_{\mu\nu})) - (c_{\mu\nu})$$
$$\equiv d \times g_{\mu\nu}$$
(16)

where

$$d \equiv \left(\frac{R}{2}\right) - b$$

$$\equiv \left(\frac{R}{2}\right) - \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right)\right)$$
(17)
$$(17)$$

$$(17)$$

while F is called the (traceless) Faraday/electromagnetic/field strength tensor. Until proof is provided otherwise, it is presumed that the tensor $d_{\mu\nu}$ is the tensor of gravitational waves. It remains to be seen in the further course of research whether there are conditions under which is it given that

$$W_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times (g_{\mu\nu}) - (b_{\mu\nu})$$
$$\equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (\Lambda \times (g_{\mu\nu})) - (c_{\mu\nu})$$
$$\equiv d \times g_{\mu\nu}$$
(18)

Definition 2.1.8 (The tensor $c_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the tensor of non-momentum and curvature, denoted as $c_{\mu\nu}$, is defined/determined as follows:

$$c_{\mu\nu} \equiv (G_{\mu\nu}) - (a_{\mu\nu})$$

$$\equiv (a_{\mu\nu}) + (c_{\mu\nu}) - (a_{\mu\nu})$$

$$\equiv (\frac{R}{2} \times (g_{\mu\nu})) - (\Lambda \times (g_{\mu\nu})) - (d_{\mu\nu})$$

$$\equiv (c_{\mu\nu}) + (d_{\mu\nu}) - (d_{\mu\nu})$$

$$\equiv (b_{\mu\nu}) - (\Lambda \times (g_{\mu\nu}))$$

$$\equiv c \times g_{\mu\nu}$$
(19)

while c is equal to

$$c \equiv \left(\left(\frac{1}{4 \times 4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F^{\mu c} \right) - \left(F_{de} \times F^{de} \right) \right) \right) - \Lambda$$
(20)

Proof.

In general, it is

Q. e. d.

Furthermore, the following relationships are to be considered. **Proof.**

$$(unknown_{\mu\nu}) + (d_{\mu\nu}) \equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (\Lambda \times (g_{\mu\nu}))$$

$$(unknown_{\mu\nu}) + (d_{\mu\nu}) \equiv (c_{\mu\nu}) + (d_{\mu\nu})$$

$$(unknown_{\mu\nu}) = (c_{\mu\nu})$$

$$(22)$$

Q. e. d.

From these relationships we must accept the following. **Proof.**

In general, it is

$$(unknown_{\mu\nu}) \equiv (unknown_{\mu\nu}) (b_{\mu\nu}) - (\Lambda \times (g_{\mu\nu})) \equiv (c_{\mu\nu})$$
(23)

Q. e. d.

Einstein's tensor is determined as follows. **Proof.** In general, it is

$$(G_{\mu\nu}) \equiv (a_{\mu\nu}) + (unknown_{\mu\nu})$$

$$\equiv (a_{\mu\nu}) + (c_{\mu\nu})$$
(24)

Q. e. d.

Definition 2.1.9 (The tensor $b_{\mu\nu}$).

The co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two while its components can be displayed by a 4×4 matrix too. Under conditions of Einstein's general theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field expressed more compactly and, in a coordinate independent form as

$$b_{\mu\nu} \equiv (E_{\mu\nu}) - (a_{\mu\nu})$$

$$\equiv (\overline{a_{\mu\nu}} + (b_{\mu\nu})) - (a_{\mu\nu})$$

$$\equiv (\frac{R}{2} \times (g_{\mu\nu})) - (d_{\mu\nu})$$

$$\equiv (\overline{b_{\mu\nu}} + (d_{\mu\nu})) - (d_{\mu\nu})$$

$$\equiv (\frac{1}{4 \times 4 \times \pi})$$

$$\times ((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de})) \times g_{\mu\nu}$$

$$\equiv b \times g_{\mu\nu}$$
(25)

where (as can be seen in the following of this paper)

$$b \equiv \left(\left(\frac{1}{4 \times 4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F^{\mu c} \right) - \left(F_{de} \times F^{de} \right) \right) \right)$$
(26)

while F is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.1.10 (The stress-energy tensor of ordinary matter $a_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the stress-energy tensor of ordinary matter, denoted as $a_{\mu\nu}$, is defined/derived/determined as follows:

$$a_{\mu\nu} \equiv a_{\mu\nu} + b_{\mu\nu} - b_{\mu\nu}$$

$$\equiv a_{\mu\nu} + c_{\mu\nu} - c_{\mu\nu}$$

$$\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$

$$\stackrel{-}{(}{\left(\frac{1}{4 \times 4 \times \pi}\right)} \times (27)$$

$$\times g_{\mu\nu}$$

$$\stackrel{-}{(}{\left(F_{\mu c} \times F^{\mu c}\right) - (F_{de} \times F^{de})\right)}$$

$$\stackrel{-}{\equiv} a_{\mu\nu} - b_{\mu\nu} - c_{\mu\nu} - d_{\mu\nu}$$

$$\equiv a_{\mu\nu} + \left(\Lambda \times (g_{\mu\nu})\right) - b_{\mu\nu}$$

$$\equiv a \times g_{\mu\nu}$$

$$a \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)$$

$$\stackrel{-}{(}{\left(\frac{1}{4 \times 4 \times \pi}\right)} \times (28)$$

$$\times ((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de}))$$

$$\stackrel{-}{)}$$

while

The question is justified whether the tensor
$$a_{\mu\nu}$$
 determines somehow locality like

$$a_{\mu\nu} \equiv L_{\mu\nu}$$

(29)

(28)

and therefore, need to be treated as **the tensor of locality** ($L_{\mu\nu}$) from the point of view of Einstein's theory of general relativity.

Definition 2.1.11 (The stress-energy momentum tensor $E_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the stress-energy momentum tensor, denoted as $E_{\mu\nu}$, is defined/derived/determined as follows:

$$(E_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^{4}}\right) \times T_{\mu\nu}$$

$$\equiv \left(\frac{2 \times \pi \times \gamma}{c^{4}}\right) \times 4 \times T_{\mu\nu}$$

$$\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^{4}}\right) \times g_{\mu\nu}$$

$$\equiv a_{\mu\nu} + b_{\mu\nu}$$

$$\equiv a_{\mu\nu} + b_{\mu\nu}$$

$$\equiv \underbrace{(R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right)}_{G_{\mu\nu}} + \Lambda \times g_{\mu\nu}$$

$$\equiv \underbrace{G_{\mu\nu}}_{G_{\mu\nu}} + \Lambda \times g_{\mu\nu}$$

$$\equiv a_{\mu\nu} + c_{\mu\nu} + (\Lambda \times g_{\mu\nu})$$

$$\equiv R_{\mu\nu} - \underbrace{E}_{\mu\nu}$$

$$\equiv E \times g_{\mu\nu}$$

$$(30)$$

while E is determined a

$$E \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \tag{31}$$

Definition 2.1.12 (The Ricci tensor $R_{\mu\nu}$).

 $R_{\mu\nu}$

Under conditions of Einstein's general theory of relativity, the Ricci tensor, denoted as $R_{\mu\nu}$, is defined/detrived/determined as follows:

$$= a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}$$

$$= \underbrace{\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}}_{E^4} + \underbrace{\left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - (\Lambda \times g_{\mu\nu})}_{E^2}$$

$$= E_{\mu\nu} + \underline{E}_{\mu\nu}$$

$$= G_{\mu\nu} + \underline{G}_{\mu\nu}$$

$$= \underbrace{(E+t)}_{S} \times g_{\mu\nu}$$

$$= S \times g_{\mu\nu}$$

$$(32)$$

while S is defined/derived/determined as follows:

$$\equiv \begin{array}{c} E & + & t \\ \equiv & \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) & + & \left(\frac{R}{2} - \Lambda\right) \end{array}$$
(33)

Proof. In general, it is

$$S \times g_{\mu\nu} \equiv R_{\mu\nu}$$
 (34)

and therefore

$$S \times g_{\mu\nu} \times g^{\mu\nu} \equiv R_{\mu\nu} \times g^{\mu\nu} \equiv R \equiv 4 \times S$$
(35)

Q. e. d.

Definition 2.1.13 (The Ricci scalar R).

Under conditions of Einstein's general theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric tensor $g_{\mu\nu}$ is determined at each point in space-time by lamda Λ and *anti-lamda* $\underline{\Lambda}$ [12] as

$$R \equiv g^{\mu\nu} \times R_{\mu\nu}$$

$$\equiv g^{\mu\nu} \times g_{\mu\nu} \times S$$

$$\equiv \underbrace{g^{\mu\nu} \times g_{\mu\nu}}_{4} \times S$$

$$\equiv \Lambda + \underline{\Lambda}$$

ationship S = (R/4) or that
(36)

From this definition follows the relationship S = (R/4) or that

$$\frac{R}{S} \equiv +4 \equiv g^{\mu\nu} \times g_{\mu\nu}$$
(37)

A Ricci scalar curvature *R* which is *positive* at a certain point indicates that the volume of a small ball about a certain point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a *Ricci* scalar curvature R which is *negative* at a certain point indicates that the volume of a small ball is larger than it

would be in Euclidean space. The Ricci tensor is determined completely by the cosmological constant and the anti-cosmological constant.

Proof.

In general, multiplying the relationship before by the metric tensor $g_{\mu\nu}$, we obtain

$$R \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu})$$
(38)

or

$$(4 \times S) \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu})$$
(39)

or

$$S \times g_{\mu\nu} \equiv \left(\frac{\Lambda}{4} \times g_{\mu\nu}\right) + \left(\frac{\Lambda}{4} \times g_{\mu\nu}\right) \tag{40}$$

In other words, the Ricci tensor is completely determined by the cosmological constant and the anticosmological constant.

$$R_{\mu\nu} \equiv S \times g_{\mu\nu} \equiv \left(\frac{\Lambda}{4} \times g_{\mu\nu}\right) + \left(\frac{\Lambda}{4} \times g_{\mu\nu}\right)$$
(41)

Q. e. d.

However, this implies the fundamental possibility, that the cosmological constant is not a constant but may vary as such.

The value of the anti-cosmological constant can be calculated very precisely.

Proof.

In this context, Einstein's field equations are determined as

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times \left(g_{\mu\nu} \right) \right) + \Lambda \times g_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4} \right) \times g_{\mu\nu}$$
(42)

Rearranging again, we obtain

$$R_{\mu\nu} - \left(\left(\frac{\mathbf{R}}{2}\right) \times (\mathbf{g}_{\mu\nu})\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) + \Lambda \times g_{\mu\nu} \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{\mathbf{R}}{2}\right) \times (\mathbf{g}_{\mu\nu})\right)$$
(43)

or

$$R_{\mu\nu} - \left(R \times (g_{\mu\nu}) - \Lambda \times g_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right)$$
(44)

or

$$R_{\mu\nu} + \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv \left(R \times (g_{\mu\nu}) - \Lambda \times g_{\mu\nu}\right)$$
(45)

The anti lamda field can be derived in detail from Einstein's field equation as

$$\underline{\Lambda} \times \boldsymbol{g}_{\mu\nu} \equiv \boldsymbol{R}_{\mu\nu} + \left(\left(\frac{\boldsymbol{R}}{2}\right) \times \left(\boldsymbol{g}_{\mu\nu}\right)\right) - \left(\frac{2 \times \boldsymbol{\pi} \times \boldsymbol{\gamma} \times \boldsymbol{T}}{c^4}\right) \times \boldsymbol{g}_{\mu\nu}$$
(46)
In other words, the anti lamda field can be simplified as

$$\underline{\Lambda} \times g_{\mu\nu} \equiv \left(S + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right) \times g_{\mu\nu}$$

However, doubts are more than justified whether there can be identified any kind of a particle which can be ascribed to such a more or less purely artificial and unrealistic field without any practical significance. Mathematics' as one product of human mind and imagination can reach us far while reality need not to follow at all. Nonetheless, as found before, it is S = (R/4) and *anti lambda* $\underline{\Lambda}$ can be calculated as

$$\underline{\Lambda} \equiv \left(S + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right)$$
$$\equiv \left(\left(\frac{R}{4}\right) + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right)$$
(48)

or as

$$\underline{\Lambda} \equiv \left(\left(\frac{3 \times R}{4} \right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4} \right) \right)$$
(49)

Q. e. d.

The cosmological constant Λ can also be written algebraically as part of the stress-energy tensor of energy/matter, a second order tensor as the source of gravity (energy density). In this context, the cosmological constant Λ can be determined as

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4} \right) + \left(\frac{R}{4} \right) \right)$$
(50)

which forces us to accept that

(47)

$$\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \equiv \left(\Lambda - \left(\frac{R}{4}\right)\right) \tag{51}$$

Table 1 may provide a preliminary overview of the definitions of the four basic fields of nature.

		Curvature		
		Yes	No	
Momentum	Yes	a _{µv}	b _{µv}	E×g _{µν}
	No	c _{µv}	d _{µv}	$t \times \underline{g}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$S \times g_{\mu\nu}$

Table 1. Four basic fields of nature	Table	1. Four	basic	fields	of nature
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Einstein's field equations simplify as

$$(S \times g_{\mu\nu}) - (t \times g_{\mu\nu}) \equiv (E \times g_{\mu\nu})$$
(52)

Under conditions where the Hamiltonian operator H is equal to H=E and the wave function ψ is equal[23] to ψ =t, Einstein's field equations becomes

$$(S \times g_{\mu\nu}) - (\psi \times g_{\mu\nu}) \equiv (H \times g_{\mu\nu})$$
(53)

Definition 2.1.14 (The inverse metric tensor $g^{\mu\nu}$ **and the metric tensor** $g_{\mu\nu}$). Under conditions of Einstein's general[5] theory of relativity, it is

$$g^{\mu\nu} \times g_{\mu\nu} \equiv +4 \tag{54}$$

or

$$\frac{1}{g^{\mu\nu}} \times \frac{1}{g_{\mu\nu}} \equiv \frac{1}{+4}$$
(55)

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other. Einstein replaced the scalar Newtonian gravitational potential from Poisson's equation for gravity in general relativity by the metric tensor $g_{\mu\nu}$. Einstein's emphasizes this issue using the following statement "... *in the general theory of relativity* ... *must be* ... *the tensor* $g_{\mu\nu}$ *of the gravitational potential*"[24]^(p. 88)

Definition 2.1.15 (Laue's scalar T).

Max von Laue (1879-1960) proposed a scalar[25] as the contraction or as the trace of the stress-energy momentum tensor $T_{\mu\nu}$, denoted as T and written without subscripts or arguments. However, Einstein himself criticized Laue's scalar[26] T. Under conditions of Einstein's general[5] theory of relativity, it is

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{56}$$

where $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter" [24] ^(p. 88). Einstein explained further in the text: "Considered phenomenologically, this **energy tensor** is composed of that of the **electromagnetic field** and of **matter in the narrower sense**." [24] ^(p. 93)

Definition 2.1.16 (Index raising).

For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[27] raises each index. In simple words, it is

or more professionally

$$F^{\mu c} \equiv g^{\mu v} \times g^{cd} \times F_{vd}$$
(58)

2.2 Axioms

2.2.1 Axioms in general

Making errors is indeed only human. However, it is important to be able to recognize erroneous thinking as such. Axioms[28] and rules chosen carefully can be of use to recognize and to avoid logical inconsistency and equally may be of help to prevent science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms[29] too. In this context, Einstein himself reiterates again the importance of implementing axioms in science. "Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden."[30] ^(p. 17) Einstein's previous position now been translated into English: The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction. It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from such axioms as a main

logical foundation of any 'theory'. "*Grundgesetz (Axiome) und Folgerungen zusammen bilden das was man eine* '*Theorie' nennt.*" [30] ^(p. 17) Albert Einstein's (1879-1955) message translated into English as: "*Basic law (axioms) and conclusions together form what is called a 'theory'*" has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited *the law of excluded middle* and *the law of contradiction* as examples of axioms. However, *lex identitatis* is an axiom too, which possess the potential to serve as the most basic and equally as the simplest axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716): "*Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B.*" [31] ^(p. 327) or A = A, B = B or +1 = +1. In this context, lex contradictionis, the negative of lex identitatis, or +0 = +1 is of no minor importance too.

2.2.2 Axiom I. Lex identitatis

To say that +1 is identical[32], [33] to +1 is to say that both are the same. AXIOM1. LEX IDENTITATIS.

$$+1 \equiv +1 \tag{59}$$

or

$$g^{\mu\nu} \times g_{\mu\nu} - 3 \equiv +1 \tag{60}$$

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and equally be different from itself [34]? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

2.2.3 Axiom II. Lex contradictionis

AXIOM2. LEX CONTRADICTIONIS.

$$+0 \equiv +1 \tag{61}$$

or

$$\left(\left(g^{\mu\nu} \times g_{\mu\nu}\right) - 3\right) - \left(\left(g^{\mu\nu} \times g_{\mu\nu}\right) - 3\right) \equiv +1$$
(62)

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (a path is a straight line from the standpoint of a co-moving observer at a certain point in space-time) and the other of itself, its own opposition (the same path is not a straight line, the same path is curved, from the standpoint of a stationary observer at a certain point in space-time)[34]. We may simply deny the existence of objective or of any other contradictions. However, even if it remains justified especially according to Einstein's special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is not secured, Einstein's special theory of relativity insist that contradictions are objective and real[34]. This fact highlights and implies the question too, why should and how can an identical be a contradictory too?

2.2.4 Axiom III. Lex negationis

AXIOM3. LEX NEGATIONIS.

$$\neg \times 0 \equiv +1 \tag{63}$$

where ¬ denotes the (natural "determinatio negatio est" [35] / logical [36]) process of negation [33].

3. Results

Theorem 3.1 (THE GEOMETRICAL FORM OF THE STRESS-ENERGY TENSOR OF THE ELECTROMAGNETIC FIELD $b_{\mu\nu}$).

All mater considered as some deformation of space-time would enable a combined theory of electromagnetism and gravitation in terms of geometry too. However, it is of course quite natural to ask whether and why nature's all (long range) fields (of force) have a common origin at all? **In particular, is the electromagnetic field a property of spacetime or merely something being embedded in spacetime**? Einstein's mathematics of the theory of general relativity related the gravitational field to the curvature of spacetime, while the electromagnetic field remains completely unrelated to the spacetime geometry. Unfortunately, the geometrization of the electromagnetic fields within the frame of Einstein's theory of general relativity has been left behind by Einstein as an unsolved problem. In recent years, the work on the geometrization of electromagnetism has begun to increase. Many different trials proposed its own way to extend the geometry of general relativity that would, so it seemed, serve as a geometrization of the electromagnetic field as well. However, the most of these trials[37]–[39] were thus far not widely accepted or are totally ignored by scientific community. While wrong papers don't make it through history on the long run, we will have to face the fact that the obvious public ignorance of such publications

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appears to provide a kind of an indirect proof that these and similar papers must simply be wrong because ignoring the same suffices for the scientific community to date. With regard to the conceptual differences between the geometrized gravitational field and the classical Maxwellian theory of the electromagnetic filed the obviously more than insurmountable old borders remain at least in scientist's consciousness. Nonetheless and needless to say that only a minority of inveterate optimists try once and again to dispute accepted wisdom in this context.

CLAIM.

In general, the completely geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is given by

$$b_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F^{\mu c} \right) - \left(F_{de} \times F^{de} \right) \right) \right) \times g_{\mu\nu}$$
(64)

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{65}$$

is true, then the conclusion

$$b_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F^{\mu c} \right) - \left(F_{de} \times F^{de} \right) \right) \right) \times g_{\mu\nu}$$
(66)

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1) \tag{67}$$

is true. Multiplying this premise by the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu},$ we obtain

$$(+1) \times b_{\mu\nu} \equiv (+1) \times b_{\mu\nu} \tag{68}$$

or

$$b_{\mu\nu} \equiv b_{\mu\nu} \tag{69}$$

Rearranging equation according to the definition above, it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right) \times \left(\left(\left(F_{\mu c} \times F_{\nu d} \times g^{cd}\right)\right) - \left(\left(\frac{1}{4}\right) \times \left(F_{de} \times F^{de}\right) \times g_{\mu\nu}\right)\right)$$
(70)

Rearranging equation before again it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right) \times \left(\left(\mathbf{1} \times \left(F_{\mu c} \times F_{\nu d} \times g^{cd}\right)\right) - \left(\frac{\left(F_{de} \times F^{de}\right)}{4}\right) \times g_{\mu\nu}\right)$$
(71)

and equally

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right) \times \left(\left(\frac{4\times\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right) - \left(\frac{\left(F_{d e}\times F^{d e}\right)}{4}\right) \times g_{\mu\nu}\right)$$
(72)

According to definition above, this equation changes to

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right) \times \left(\left(\frac{\left(g^{\mu\nu}\times g_{\mu\nu}\right)\times\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right) - \left(\frac{\left(F_{d e}\times F^{d e}\right)}{4}\right)\times g_{\mu\nu}\right)$$
(73)

and equally to

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right) \times \left(\left(\left(\frac{\left(g^{\mu\nu}\times F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right)\times g_{\mu\nu}\right) - \left(\left(\frac{\left(F_{de}\times F^{de}\right)}{4}\right)\times g_{\mu\nu}\right)\right)$$
(74)

A further simplification of the relationship before yields the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ determined by the metric tensor of general relativity $g_{\mu\nu}$ as

$$b_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times \pi} \right) \times \left(\left(\frac{1}{4} \right) \times \left(\left(F_{\mu c} \times g^{\mu\nu} \times g^{cd} \times F_{\nu d} \right) - \left(F_{de} \times F^{de} \right) \right) \right) \right) \times g_{\mu\nu}$$
(75)

However, the term

$$\left(\left(\frac{1}{4}\right) \times \left(\left(F_{\mu c} \times g^{\mu \nu} \times g^{cd} \times F_{\nu d}\right) - \left(F_{de} \times F^{de}\right) \right) \right)$$
(76)

of the equation before can be simplified further. For an order-2tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[27] raises each index. In other words, it is according to definition view lines before that

$$F^{\mu c} \equiv g^{\mu v} \times g^{cd} \times F_{vd}$$
(77)

which simplifies the term before as

$$\left(\left(\frac{1}{4}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{d e} \times F^{d e}\right)\right)\right)$$
(78)

or as

$$\left(\frac{1}{4}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{d e} \times F^{d e}\right)\right) \tag{79}$$

This relationship simplifies the geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ further as

$$_{\mu\nu} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi} \right) \times \left(\left(F_{\mu c} \times F^{\mu c} \right) - \left(F_{de} \times F^{de} \right) \right) \right) \times g_{\mu\nu}$$
(80)

while F is called the (traceless) Faraday/electromagnetic/field strength tensor. Even this relationship can be simplified. However, the stress-energy momentum tensor of the electromagnetic field is geometrized completely, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

b

Theorem 3.2 (THE GEOMETRICAL FORM OF THE STRESS-ENERGY TENSOR OF EENERGY/MATTER).

The starting point of Einstein's theory of general relativity was that gravity as such is related to space-time geometry or even a property of space-time geometry. Consequently, Einstein published[5] a geometric theory of gravitation while Einstein's initial hope to construct a purely geometric theory of gravitation in which even the sources of gravitation themselves would be of geometric origin has still not been fulfilled. Einstein's field equations have a source term, the stress-energy tensor of matter, radiation, vacuum et cetera, which is of order two but still devoid of any geometry and regrettably free of any geometrical significance.

CLAIM.

In general, the completely geometrical form of the stress-energy momentum tensor of energy/matter Einstein's theory of general relativity is given by

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{81}$$

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{82}$$

is true, then the conclusion

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(83)

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1) \tag{84}$$

is true. Multiplying this premise by the stress-energy momentum tensor of matter, we obtain $\begin{pmatrix} (8 \times \pi \times \nu) \\ (8 \times \pi \times \nu) \end{pmatrix} = \begin{pmatrix} (8 \times \pi \times \nu) \\ (8 \times \pi \times \nu) \end{pmatrix}$

$$(+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right) \equiv (+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right)$$
(85)

or

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right) \equiv \left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right)$$
(86)
rearranged as

The equation before can be rearranged as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times \mathbf{4} \times T_{\mu\nu}\right)$$
(87)

According to the definition at the begin of this paper, this equation changes too

$$\frac{(8 \times \pi \times \gamma)}{c^4} \times T_{\mu\nu} = \left(\left(\frac{2 \times \pi \times \gamma}{c^4} \right) \times \left(\boldsymbol{g}_{\mu\nu} \times \boldsymbol{g}^{\mu\nu} \right) \times T_{\mu\nu} \right)$$
(88)

or to

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right) \equiv \left(\left(\frac{2\times\pi\times\gamma}{c^4}\right)\times\left(g^{\mu\nu}\times T_{\mu\nu}\right)\times\left(g_{\mu\nu}\right)\right)$$
(89)

Referring to the definition of Laue's scalar T, this equation can be rearranged as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times \mathbf{T} \times (g_{\mu\nu})\right)$$
(90)

The desired geometrical representation of the stress-energy momentum tensor energy/matter of Einstein's general theory of relativity follows as

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{91}$$

In other words, our conclusion is true. QUOD ERAT DEMONSTRANDUM.

However, from the geometrical point of view the stress–energy momentum tensor of energy/matter $E_{\mu\nu}$ is more or less identical with the metric tensor $g_{\mu\nu}$, enriched only by view "constants" and a scalar as

$$E_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{92}$$

Using geometrized units where $\gamma = c = 1$, the geometrical form of the stress–energy momentum tensor of energy/matter $E_{\mu\nu}$ can be rewritten as

$$E_{\mu\nu} \equiv (2 \times \pi \times T) \times g_{\mu\nu} \tag{93}$$

However, describing the fundamental stress-energy momentum tensor of energy/matter, the source term of the gravitational field in Einstein's general theory of relativity, as an inherent geometrical structure, as being determined or dependent on the metric field $g_{\mu\nu}$ is associated with several and far reaching consequences. Theoretically, the properties of energy, momentum, mass, stress et cetera need no longer to be seen as intrinsic properties of matter with the consequence that the properties which material systems possess could be determined in virtue of their relation to space-time structures too. The question thus far arises whether the energy tensor $E_{\mu\nu}$ at the end could be in different aspects less fundamental than the metric field $g_{\mu\nu}$ itself. Briefly, is and why is matter more fundamental[40] than space-time? In contrast to such a position, is the assumption justified that without the space-time structure encoded in the metric no energy tensor? To bring it to the point, can space-time (and its geometric structure) exist without matter and if yes, what kind of existence could this be? Einstein's starting point was to derive space-time structure from the properties of material systems. However, the theorem before does not exclude the possibility that the energy tensor depends on the metric field too or is completely determined by the metric field. Consequently, matter fields themselves could be derivable from the structure of space-time or the very definition of an energy tensor is determined by space-time structures too. Thus far, the question is not answered definitely, which came first, either space-time structure or energy tensor. So, it is reasonable to ask, is the energy-momentum tensor of matter only dependent on the structure of space-time or even determined by the structure of space-time or both or none? In other words, granddaddies either chicken or the egg dilemma is revitalized and asking for an innovative and a comprehensive solution and may end up in an Anti-Machian^[41] theory. However, this leads us at this point too far afield.

Theorem 3.3 (EINSTEIN'S FIELD EQUATION'S COMPLETELY GEOMETRIZED).

Now, we can derive a completely geometrical form of Einstein's field equations. **CLAIM.**

In general, the completely geometrical form of Einstein's field equations is given by

$$\left(S \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{94}$$

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{95}$$

is true, then the conclusion

$$\left(S \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{96}$$

is also true, the absence of any technical errors presupposed. The premise $(+1) \equiv (+1)$

is true. Multiplying this premise by the stress-energy momentum tensor of matter, we obtain

$$(+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right) \equiv (+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right)$$
(98)

or

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right) \equiv \left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right)$$
(99)

This equation simplifies according to the theorem before as

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right) \equiv \left(\frac{2\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\nu}$$
(100)

Einstein's field equations are given as

(97)

$$\left(R_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(101)

Since $R_{\mu\nu} = S \times g_{\mu\nu}$, we obtain Einstein's field equations completely geometrized as

$$\left(S \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(102)

In other words, our conclusion is true. **OUOD ERAT DEMONSTRANDUM.**

Theorem 3.4 (THE RICCI TENSOR AND WEYL'S TENSOR). CLAIM.

In general, the Ricci tensor is determined by Weyl's tensor as

 $(R_{\mu\nu}) \equiv (\underline{W}_{\mu\nu}) + (W_{\mu\nu})$ (103)

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{104}$$

is true, then the conclusion

$$\begin{pmatrix} R \ \mu\nu \end{pmatrix} \equiv \begin{pmatrix} \underline{W} \ \mu\nu \end{pmatrix} + \begin{pmatrix} W \ \mu\nu \end{pmatrix}$$
(105)

is also true, the absence of any technical errors presupposed. The premise $(+1) \equiv (+1)$ (106)

is true. Multiplying this premise by the Riemann curvature tensor $R_{kl\mu\nu}$, we obtain

$$(+1) \times \left(R_{kl\mu\nu}\right) \equiv (+1) \times \left(R_{kl\mu\nu}\right) \tag{107}$$

or

$$(R_{kl\mu\nu}) \equiv (R_{kl\mu\nu}) \tag{108}$$

Adding zero to this tensor, it is

$$\binom{R_{kl\mu\nu}}{H} + 0 \equiv \binom{R_{kl\mu\nu}}{-W_{kl\mu\nu}} + W_{kl\mu\nu}$$
(109)

According to our definition, it is

$$\underline{W}_{kl\mu\nu} \equiv \left(R_{kl\mu\nu}\right) - W_{kl\mu\nu} \tag{110}$$

Substituting this relationship into the equation before, we obtain

$$\left(R_{kl\mu\nu}\right) \equiv \left(\underline{W}_{kl\mu\nu}\right) + \left(W_{kl\mu\nu}\right)$$
(111)

The *massless, traceless* Weyl's curvature tensor $W_{k|\mu\nu}$, as already mentioned, does not convey information on how the volume of a certain body changes, but rather how the shape of this body is distorted. In contrast to this, the trace-full massive *anti Weyl's curvature tensor* $\underline{W}_{k|\mu\nu}$ convey information on how the volume of a certain body changes, but does not convey information how the shape of this body is distorted. A further manipulation (decomposition) of this equation before yields the following relationship.

$$\left(g^{kl} \times (R_{kl\mu\nu})\right) \equiv \left(g^{kl} \times (\underline{W}_{kl\mu\nu})\right) + \left(g^{kl} \times (W_{kl\mu\nu})\right)$$
(112)

which simplifies the relationship as

$$(R_{\mu\nu}) \equiv (\underline{W}_{\mu\nu}) + (W_{\mu\nu})$$
 (113)

In other words, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

The Ricci tensor $R_{\mu\nu}$ is determined completely by Weyl's tensor $W_{\mu\nu}$ and anti Weyl's tensor $W_{\mu\nu}$. The Ricci tensor $R_{\mu\nu}$ is equally the trace of the Riemann tensor $R_{kl\mu\nu}$ which describes among other the curvature of spacetime. However, due to the Einstein equations and in the absence of sources, the trace part of the Riemann curvature tensor will vanish. In particular, even in the absence of any sources, there can be small variations of curvature (i.e. gravitational waves) or a non-zero curvature of space time. In other words, under these circumstances, the Weyl curvature tensor W_{kluv} as the trace-free part of the Riemann curvature tensor R_{kluv} can still be non-zero. In addition to this, electromagnetic waves propagating in vacuum can be described by the source-free Maxwell equations without specifying anything about charges or currents that might have produced them. In point of fact, changes in the gravitational field propagate as gravitational waves. In this context, the Weyl tensor $W_{\mu\nu}$, and the potential role it plays in describing gravitational waves is related to the 3-Cotton[42]-York[43] tensor too which is rarely mentioned in this connection. The 3-Cotton-York tensor like Weyl's co-variant second rank tensor $W_{\mu\nu}$ is not locally determined by a matter field and is to some extent suitable[44] for describing gravitational waves in the 1+3 formalism too. It is necessary to note that Weyl's tensor, denoted as $W_{\mu\nu}$, comes from the contraction of the Weyl's [19] curvature tensor $W_{k|\mu\nu}$. The so called anti Weyl's tensor, denoted as $\underline{W}_{\mu\nu}$, comes from the contraction of anti Weyl's curvature tensor $\underline{W}_{kl\mu\nu}$. However, Weyl's curvature tensor $W_{kl\mu\nu}$ is related to nonlocality but the question is justified whether Weyl's curvature tensor Wkluv describes non-locality completely.

Quantum theory is more or less kinematically nonlocal, while Einstein's theory of relativity (including relativistic quantum field theory) requires something like a dynamical locality ("Einstein locality"). One can hardly help thinking that quantum non-locality cannot be reconciled with Einstein locality. Nonetheless, might there be some circumstances which could have led to a change of mind? Under which conditions does general relativity leads to non-local[45] effects in the matter sector and can such non-local effects be described somehow mathematically or even measured? Another possible point of criticism is the unified field theory program. In this respect and due to the considerations being given before let us consider the following. Can and how can the for basic fields of nature be described from the point of view of locality and non-locality? *Table 2* may provide a preliminary overview of the issue mentioned and the definitions of the four basic fields of nature under conditions of Weyl's tensor

		Locality		
		Yes	No	
Momentum -	Yes	Α μν	b _{µv}	E×g _{µν}
	No	p _{µv}	q _{µv}	$t \times \underline{g}_{\mu\nu}$
		$\underline{W}_{\mu\nu}$	$W_{\mu u}$	$S \times g_{\mu\nu}$

Table 2. Four basic fields of nature and non-locality

while for preliminary purposes, the Weyl's tensor $W_{\mu\nu}$ is used to describe non-locality to some extent. Furthermore, in this context and only for the sake of the argument it is assumed that $(p_{\mu\nu})+(q_{\mu\nu}) = ((R/2)\times g_{\mu\nu})-(\Lambda\times g_{\mu\nu})$ and that $(p_{\mu\nu}) = (W_{\mu\nu})-(g_{\mu\nu})$. Under these conditions, the tensor $q_{\mu\nu}$ would to be determined as

$$q_{\mu\nu} \equiv W_{\mu\nu} - ((\frac{1}{4 \times 4 \times \pi}) (114)) \\ ((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de})) \\) \times g_{\mu\nu}$$

What kind of a particle can be ascribed to the field $q_{\mu\nu}$. Pert like ever it is difficult suppress the remark: none. Something, even if mathematically correct, need not to be epistemologically valuable too. Another consequence of such an approach to unified field theories would be the need to accept the following form of the tensor $p_{\mu\nu}$.

$$p_{\mu\nu} \equiv \underbrace{W_{\mu\nu}}_{-(} (\underbrace{2 \times \pi \times \gamma \times T}_{-(} \times g_{\mu\nu}) \times g_{\mu\nu})_{-(} (\underbrace{(\frac{1}{4 \times 4 \times \pi})}_{(} \times F^{\mu c}) - (F_{de} \times F^{de}))_{-(} \times g_{\mu\nu})_{-(} \times g_$$

which is again mathematically correct but still not very convincing. We may not have the final word on **the relationship between general relativity and non-locality**, but in any case, it is possible to approach even to the solution of this problem.

Theorem 3.5 (EINSTEIN'S FIELD EQUATION'S AND WEYL'S CURVATURE).

A reformulation of Einstein's field equations in much greater detail in terms of Weyl's curvature tensor highlights the path of general relativity to non-locality more than the original one field equations.

CLAIM.

In general, Einstein's field equations in terms of Weyl's curvature are given as

$$\left(\underline{W}_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) + \left(W_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{116}$$

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{117}$$

is true, then the conclusion

$$\left(\underline{W}_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) + \left(W_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(118)

is also true, the absence of any technical errors presupposed. The premise

$$1) \equiv (+1) \tag{119}$$

(+is true. Multiplying this premise by the stress-energy momentum tensor of matter, we obtain

$$(+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right) \equiv (+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \right)$$
(120)

or

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right) \equiv \left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right)$$
(121)

This equation simplifies according to the theorem before as

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(122)

Einstein's field equations are given as

$$\left(R_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(123)

As found before, it is $R_{\mu\nu} = W_{\mu\nu} + \underline{W}_{\mu\nu}$. Einstein's field equations in terms of Weyl's tensor follow as

$$\left(\left(\underline{W}_{\mu\nu}\right) + \left(W_{\mu\nu}\right)\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(124)

or as

$$\left(\underline{W}_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) + (\Lambda \times g_{\mu\nu}) + (W_{\mu\nu}) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(125)

In other words, our conclusion is (mathematically) true.

QUOD ERAT DEMONSTRANDUM.

However, the properties of the Weyl tensor $W_{\mu\nu}$ demand that its contraction vanishes[22] or it is

$$\left(g^{kl} \times \left(W_{kl\mu\nu}\right)\right) \equiv \left(W_{\mu\nu}\right) \equiv 0 \tag{126}$$

Thus far, even if this tensor is formally present inside the famous Einstein equation, the question is justified, how much information does Weyl tensor $W_{\mu\nu}$ contain (namely the information as regards the gravitational field in vacuum) under these circumstances about non-locality at all.

Theorem 3.6 (THE RICCI TENSOR AND LOCALITY AND NON-LOCALITY).

CLAIM.

In the following let $L_{\mu\nu}$ denote covariant second rank tensor of locality, let $\underline{L}_{\mu\nu}$ denote the covariant second rank tensor of non-locality. In general, the Ricci tensor is determined by a local and non-local part as

$$(R_{\mu\nu}) \equiv (\underline{L}_{\mu\nu}) + (L_{\mu\nu})$$
(127)

If the premise

$$(+1) \equiv (+1) \tag{128}$$

is true, then the conclusion

PROOF BY MODUS PONENS.

$$R_{\mu\nu}) \equiv (\underline{L}_{\mu\nu}) + (L_{\mu\nu}) \tag{129}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1)$$
 (130)

is true. Multiplying this premise by the Riemann curvature tensor $R_{kl\mu\nu}\!,$ we obtain

$$(+1) \times \left(R_{kl\mu\nu} \right) \equiv (+1) \times \left(R_{kl\mu\nu} \right)$$
(131)

or

$$\left(R_{kl\mu\nu}\right) \equiv \left(R_{kl\mu\nu}\right) \tag{132}$$

$$(R_{kl\mu\nu}) + 0 \equiv (R_{kl\mu\nu}) - L_{kl\mu\nu} + L_{kl\mu\nu}$$
(133)

while $L_{kl\mu\nu}$ may denote *the completely local part of Riemann curvature tensor* $R_{kl\mu\nu}$. In the following, we define

$$\underline{L}_{kl\mu\nu} \equiv \left(R_{kl\mu\nu}\right) - L_{kl\mu\nu} \tag{134}$$

while $\underline{L}_{kl\mu\nu}$ may denote *the completely non-local part of Riemann curvature tensor* $R_{kl\mu\nu}$. Substituting this relationship into the equation before, it is

$$(R_{kl\mu\nu}) \equiv (\underline{L}_{kl\mu\nu}) + (L_{kl\mu\nu})$$
(135)

It is necessary to note that *the non-local part* ($\underline{L}_{kl\mu\nu}$) *of Riemann curvature tensor* ($R_{kl\mu\nu}$) need not to be completely identical with Weyl's curvature tensor ($W_{kl\mu\nu}$). A further manipulation (decomposition) of this equation before yields the following relationship.

$$\left(g^{kl} \times (R_{kl\mu\nu})\right) \equiv \left(g^{kl} \times (\underline{L}_{kl\mu\nu})\right) + \left(g^{kl} \times (L_{kl\mu\nu})\right)$$
(136)
relationship as

which simplifies the relationship as

$$(R_{\mu\nu}) \equiv (\underline{L}_{\mu\nu}) + (L_{\mu\nu})$$
 (137)

In other words, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

The thought is unavoidable and it is difficult to suppress the possible consequences. In order to enable a logically consistent discussion about **locality and non-locality from the point of view of general relativity** it is possible and necessary to decompose Riemannian curvature tensor in an appropriate way. Finally, the need becomes clear, what could be the concrete geometrical structure of such tensors?

Theorem 3.7 (THE TENSORS OF LOCALITY AND NON-LOCALITY).

How local is an electromagnetic wave or an electromagnetic field? How local is a gravitational wave or a gravitational field?

CLAIM.

In the following let $L_{\mu\nu}$ denote covariant second rank tensor of locality, let $\underline{L}_{\mu\nu}$ denote the covariant second rank tensor of non-locality. In general, there are conditions where the tensor of non-locality is determined as

$$\underline{L}_{\mu\nu} \equiv \left(\left(R_{\mu\nu} \right) - \left(a_{\mu\nu} \right) \right)$$
(138)

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{139}$$

is true, then the conclusion

$$\underline{L}_{\mu\nu} \equiv \left(\left(R_{\mu\nu} \right) - \left(a_{\mu\nu} \right) \right)$$
(140)

is also true, the absence of any technical errors presupposed. The premise $(+1) \equiv (+1)$ (141)

is true. Multiplying this premise by the Ricci tensor
$$R_{\mu\nu}$$
, we obtain
 $(+1) \times (R_{\mu\nu}) \equiv (+1) \times (R_{\mu\nu})$
(142)

$$(R_{\mu\nu}) \equiv (R_{\mu\nu})$$
 (143)

$$(a_{\mu\nu}) + (b_{\mu\nu}) + (c_{\mu\nu}) + (d_{\mu\nu}) \equiv (R_{\mu\nu})$$
(144)

We assume that gravitons (field $c_{\mu\nu}$) and photons (field $b_{\mu\nu}$) are massless. Therefore, in our understanding, we define the tensor of non-locality $\underline{L}_{\mu\nu}$ as

$$(b_{\mu\nu}) + (c_{\mu\nu}) + (d_{\mu\nu}) \equiv (\underline{L}_{\mu\nu})$$
(145)

Under these assumptions, it is given that

$$\underline{L}_{\mu\nu} \equiv (b_{\mu\nu}) + (c_{\mu\nu}) + (d_{\mu\nu}) \equiv ((R_{\mu\nu}) - (a_{\mu\nu}))$$
(146)

In other words, our conclusion is true. QUOD ERAT DEMONSTRANDUM.

In more detail. Under some conditions, the tensor of non locality $\underline{L}_{\mu\nu}$ is determined as

$$\underline{L}_{\mu\nu} \equiv ($$

$$\frac{S}{-}$$

$$(\frac{2 \times \pi \times \gamma \times T}{c^{4}})$$

$$-$$

$$(\frac{1}{4 \times 4 \times \pi})$$

$$\times$$

$$((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de}))$$

$$)$$

$$) \times g_{\mu\nu}$$

$$(147)$$

Logically, the tensor of locality $L_{\mu\nu}$ is determined under the same conditions as

$$(L_{\mu\nu}) \equiv (R_{\mu\nu}) - (b_{\mu\nu}) - (c_{\mu\nu}) - (d_{\mu\nu}) \equiv (R_{\mu\nu}) - (\underline{L}_{\mu\nu})$$
 (148)

It is of course a matter of further research, whether such an approach to the issue of locality and non-locality can lead to the desired solution of the problem of locality and non-locality from the point of view of general theory of relativity. Considering this new insight, Einstein's field equation can be reformulated in terms of locality $(L_{\mu\nu})$ and non-locality $(\underline{L}_{\mu\nu})$ as

$$\left(L_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) + \left(\Lambda \times g_{\mu\nu}\right) + \left(\underline{L}_{\mu\nu}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(149)

In this context, Einstein's field equations enable the existence of elementary particles with opposite though otherwise equal electric charge but unequal mass. Deriving quantum theory as a consequence of unified field theory appears to be theoretically possible. It may remain a highly considerable privilege of future studies to provide clear evidence whether $a_{\mu\nu}$ is or is not the tensor of locality. A proof that $a_{\mu\nu}$ is the tensor of locality would equally provide evidence against the so called **"Big bang"**[46] **theory**, a term coined by Sir Fred Hoyle (1915 – 2001) on BBC radio in 1949. In the same context, Georges Lemaître (1894 – 1966) hypothesis[47] that this universe can be traced back in time to an originating single "primeval atom" would collapse. In any way, the **cosmic background radiation**[48] as discovered by Arno Penzias and Robert Wilson[49] would have to be understood in another[50] way. Assumed that there has been something like the beginning of our world, the same cannot have begun with something concrete like ordinary matter/mass as described by the tensor $a_{\mu\nu}$. The matter or mass had to develop or had be created in what follows after the beginning of this world. In other words, one condition of the beginning of this world must have been that $a_{\mu\nu} = 0$ or that $L_{\mu\nu} = 0$. A straightforward consequence of such a logic it that this universe must have begun from the state of pure non-locality, the state of infinite and pure flatness. The following table 3 may provide an overview.

		Locality		
		Yes	No	
Momentum	Yes	0	b _{μν}	Εμν
	No	0	$\mathbf{c}_{\mu\nu} + \mathbf{d}_{\mu\nu}$	t _{µv}
		$L_{\mu u}$	$\underline{L}_{\mu\nu}$	$R_{\mu u}$

Table 3. Locality and non-locality

Under these assumptions, Einstein's field equation must be able to describe the beginning of our world by the equation

$$\left(\underline{L}_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$
(150)

Under the further assumption of **symmetry**, this equation changes to

$$\left(\underline{L}_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu\nu}\right)\right) - \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv 0$$
(151)
with $(h_{\mu\nu} + q_{\mu\nu} + d_{\mu\nu}) + (h_{\mu\nu} - q_{\mu\nu}) = (h_{\mu\nu} + d_{\mu\nu}) - (F_{\mu\nu}) = 0 \text{ as } h_{\mu\nu} = 0$ We obtain

hich equal with
$$(\mathbf{b}_{\mu\nu} + \mathbf{c}_{\mu\nu} + \mathbf{d}_{\mu\nu}) + (\mathbf{b}_{\mu\nu} - \mathbf{c}_{\mu\nu}) - (\mathbf{b}_{\mu\nu} + \mathbf{d}_{\mu\nu}) - (\mathbf{E}_{\mu\nu}) = 0$$
 or $\mathbf{b}_{\mu\nu} - \mathbf{E}_{\mu\nu} = \mathbf{0}$. We obtain
 $\begin{pmatrix} \mathbf{b}_{\mu\nu} \end{pmatrix} \equiv \begin{pmatrix} \left(\frac{\mathbf{2} \times \boldsymbol{\pi} \times \boldsymbol{\gamma} \times \boldsymbol{T}}{c^4}\right) \times \boldsymbol{g}_{\mu\nu} \end{pmatrix}$
(152)

Therefore, before the beginning of our world, the stress energy tensor of the electromagnetic field has been determined by the equation $\mathbf{b}_{\mu\nu} = \mathbf{R}_{\mu\nu} - ((\mathbf{R}/2) \times \mathbf{g}_{\mu\nu}) - (\mathbf{\Lambda} \times \mathbf{g}_{\mu\nu}))$. It is crystal clear, and almost implicit in these

w

equations, that, before the beginning of our world, all energy, stress and momentum et cetera must have been contained within the stress energy tensor of the electromagnetic field. However, why should nature and how can nature escape from zero, the black whole of mathematics?

Theorem 3.8 (WEYL'S TENSOR AND NON-LOCALITY).

CLAIM.

Weyl's curvature tensor (Wkluv) is a very important and determining part of the non-locality curvature tensor ($\underline{L}_{kl\mu\nu}$). In general, it is

$$\left(\underline{L}_{kl\mu\nu}\right) \equiv \left(W_{kl\mu\nu}\right) + \left(Z_{kl\mu\nu}\right)$$
PROOF BY MODUS PONENS.
(153)

If the premise

$$(+1) \equiv (+1) \tag{154}$$

is true, then the conclusion

$$\left(\underline{L}_{kl\mu\nu}\right) \equiv \left(W_{kl\mu\nu}\right) + \left(Z_{kl\mu\nu}\right) \tag{155}$$

is also true, the absence of any technical errors presupposed. The premise $(+1) \equiv (+1)$ (156)

is true. Multiplying this premise by the Riemann curvature tensor
$$R_{kl\mu\nu}$$
, we obtain
 $(+1) \times (R_{kl\mu\nu}) \equiv (+1) \times (R_{kl\mu\nu})$
(157)

$$(157) \times (R_{kl\mu\nu}) \equiv (+1) \times (R_{kl\mu\nu})$$

or

$$\left(R_{kl\mu\nu} \right) \equiv \left(R_{kl\mu\nu} \right)$$
 (158)

Adding zero to this tensor, it is

$$(R_{kl\mu\nu}) + 0 \equiv (R_{kl\mu\nu}) - L_{kl\mu\nu} + L_{kl\mu\nu}$$
 (159)

while
$$L_{kl\mu\nu}$$
 may denote the completely local part of Riemann curvature tensor $R_{kl\mu\nu}$. As defined before, it is

$$\frac{L}{kl\mu\nu} \equiv (R_{kl\mu\nu}) - L_{kl\mu\nu} \qquad (160)$$
while $\underline{L}_{kl\mu\nu}$ denote the completely non-local part of Riemann curvature tensor $R_{kl\mu\nu}$. Substituting this

relationship into the equation before, we obtain

$$\left(R_{kl\mu\nu}\right) \equiv \left(\underline{L}_{kl\mu\nu}\right) + \left(L_{kl\mu\nu}\right)$$
 (161)

However, the *Riemann curvature tensor* $R_{kl\mu\nu}$ is determined by the Weyl's curvature tensor $(W_{kl\mu\nu})$ too. In other words, it is equally valid that

$$\begin{pmatrix} L_{kl\mu\nu} \end{pmatrix} + \begin{pmatrix} \underline{L}_{kl\mu\nu} \end{pmatrix} \equiv \begin{pmatrix} W_{kl\mu\nu} \end{pmatrix} + \begin{pmatrix} \underline{W}_{kl\mu\nu} \end{pmatrix}$$
(162)

Rearranging equation, we obtain

$$\left(\underline{L}_{kl\mu\nu}\right) \equiv \left(W_{kl\mu\nu}\right) + \left(\underline{W}_{kl\mu\nu}\right) - \left(L_{kl\mu\nu}\right)$$
According to our definition before, this equation simplifies as
$$(163)$$

$$\left(\underline{L}_{kl\mu\nu}\right)^{T} \equiv \left(W_{kl\mu\nu}\right) + \left(Z_{kl\mu\nu}\right)$$
(164)

In other words, our conclusion is true. QUOD ERAT DEMONSTRANDUM.

Theorem 3.9 (THE IDENTITY AND DIFFERENCE BETWEEN WEYL'S TENSOR AND THE **TENSOR OF NON-LOCALITY).**

CLAIM.

Under some special circumstances, Weyl's curvature tensor (Wkluv) is identical with the non-locality curvature tensor ($\underline{L}_{kl\mu\nu}$) or **describes non-locality** completely. However, the identity of Weyl's curvature tensor ($W_{kl\mu\nu}$) and the non-locality curvature tensor ($\underline{L}_{kl\mu\nu}$) is **not** given under any circumstances or in general but only under conditions where

$$\left(Z_{kl\mu\nu}\right) \equiv 0 \tag{165}$$

PROOF BY MODUS PONENS.

If the premise

$$(+1) \equiv (+1) \tag{166}$$

is true, then the conclusion

$$\left(Z_{kl\mu\nu}\right) \equiv 0 \tag{167}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1)$$
 (168)

is true. Multiplying this premise by the non-locality curvature tensor ($\underline{L}_{kl\mu\nu}$), we obtain (+1) ×

$$(169) + 1) \times (\underline{L}_{kl\mu\nu}) \equiv (+1) \times (\underline{L}_{kl\mu\nu})$$

or

$$\left(\underline{L}_{kl\mu\nu}\right) \equiv \left(\underline{L}_{kl\mu\nu}\right) \tag{170}$$

According to the theorem before, it is equally

$$(\underline{L}_{kl\mu\nu}) \stackrel{f}{\equiv} (W_{kl\mu\nu}) + (Z_{kl\mu\nu})$$
(171)

Thus far, only under conditions where the tensor $Z_{kl\mu\nu}$ vanish, i.e. where $Z_{kl\mu\nu} = 0$, we obtain $\left(\underline{L}_{kl\mu\nu}\right) \equiv \left(W_{kl\mu\nu}\right) + 0$ (172)

$$(\underline{L} k l \mu v) = (W k l \mu v) + 0$$
 (172)
but not in general. In other words, only under circumstances where

$$\left(Z_{kl\mu\nu}\right) \equiv 0 \tag{173}$$

Weyl's curvature tensor $(W_{kl\mu\nu})$ describes non-locality completely or it is

$$\frac{L}{k l \mu v} \equiv (W_{k l \mu v})$$
(174)

but not in general. Finally, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

Weyl's curvature tensor is not completely identical with the non-locality curvature tensor but contributes to the same essentially. The following table 4 may provide an overview.

		Weyl's curvature		
		Yes	No	
Locality	Yes	U klµv	V klµv	$L_{kl\mu\nu}$
Locality	No	X klμν	y klµv	<u>L</u> klμν
		W _{klµv}	$\underline{W}_{kl\mu\nu}$	Rklµv

Table 4. Locality and non-locality

5. Discussion

General relativity as a theory for continuous fields introduced a more complicated geometry of space and time than the Euclidean geometry but failed to geometrize especially mass/matter and the electromagnetic field. Therefore, the various proposals for a unified field theory, were not able to bridge the gap between classical field theory and quantum theory. In view of this, geometrization remained the guiding principle throughout most of the attempts at unification and equally a weak spot of these undertakings. Tonnelat for example defines a unified field theory as "a theory joining the gravitational and the electromagnetic field into one single hyperfield ... " [51]. However, such a field cannot be devoid of any geometrical significance. Even Einstein's long lasting unified field theory program [52] was at the end completely in vain but still justified in every conceivable way. However, any successful enterprise to geometrize all fundamental interactions of nature should provide in the same respect a completely geometrized[8] theory of relativity too and will simplify the work of at least view scientists. In spite of the fact that Einstein's general theory of relativity (GTR) is an excellent theory of gravity which is tested to a very high degree of precision and which works extremely well at large distances, in the infrared (IR) regime et cetera, it is more than quite surprising to be confronted with claims that GTR breaks down at short distances, in smalltime scales or in the ultraviolet (UV) regime et cetera. In this context, it is worth noting that the Ricci tensor $(R_{\mu\nu})$ determines the contribution to the space-time curvature more or less from *the local sources* with energymomentum tensor of matter/energy $T_{\mu\nu}$ while and in addition to this, the Weyl curvature tensor gives primarily the contribution due to the non local sources. However, there is some evidence that the Ricci tensor and the Ricci scalar are not[53] vanishing completely and for sure in the region of non-locality, meaning that the Ricci tensor has the potential to describe even locality and non-locality, as proofed by the theorems before. It must be taken into account that the Weyl curvature tensor ($W_{kl\mu\nu}$) as the traceless part of the Riemann curvature tensor alone and as proofed before is not enough to describe non-locality completely. In addition to this, there is some evidence that the Weyl tensor ($W_{\mu\nu}$) vanishes [53] especially at short distances. In general, neither does the Ricci tensor describe only locality nor does Weyl's tensor describe only non-locality. In point of fact, Weyl's tensor $W_{\mu\nu}$ is only one part of the tensor on non-locality $\underline{L}_{\mu\nu}$ as proofed by the theorems before. In other words, in the region of *non-locality*, the spacetime metric is not completely described by Weyl's tensor $W_{\mu\nu}$.

5. Conclusion

Weyl's curvature tensor $(W_{k|\mu\nu})$ is not completely identical with a non-locality curvature tensor $(L_{k|\mu\nu})$. The results of this publication justify the view that there is no contradiction at all between Einstein's field equations and the concept of locality and non-locality.

Nil.

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