Two-Dimensional Discrete Random Variables with Conditional Distribution

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Abstract --- *Conditional probability is the probability of one thing being true given that another thing is true, and is the key concept in Bayes' theorem. It reflects in the random variables, but two-dimensional random variables Conditional Distribution have some limitations. To extend the content of conditional distribution this paper gives the extension of conditional distribution under discrete random variables, and some examples. So, it can get conditional distributions that are extended into n-dimensional random variables, thereby enriching the contents of the conditional distribution.*

Keywords - *Random Variables, Bayes' theorem , Discrete Random Variables, Two-dimensional Random Variables, Conditional Distribution*

I. INTRODUCTION

A random variable's possible represent the possible outcomes of an experiment, or the possible outcomes of a past experiment whose already-existing value is uncertain. It may also conceptually represent either the results of an "objectively" random process (rolling a die) or the "subjective" randomness that results from incomplete knowledge of a quantity. The probabilities assigned the values of a random variable is not part of probability theory itself but it is related to philosophical arguments over the interpretation of probability. The mathematics also works the same regardless of the particular interpretation in use.

In an experiment a person may be selected at random, and one random variable may be the person's height. Mathematically, the random variable is involved as a function which maps the person to the person's height. Associated with the random variable is a probability distribution that allows the computation of the probability that the height is in any subset of possible values, such as the probability that the height is between 180 and 190 cm, or the probability that the height is either less than 150 or more than 200 cm.

If the possible values of (X, Y) are finite or countably infinite, then (X, Y) is called a two-dimensional discrete random variable. When (X,Y) is a two-dimensional discrete random variable the possible values of (X,Y) may be represented as (xi, yj), i= 1, 2, 3, …n, j = 1, 2, 3, …m. If the possible values of (X,Y) are finite or countably infinite, then (X,Y) is called a two-dimensional discrete random variable. When (X, Y) is a two-dimensional discrete random variable the possible values of (X, Y) may be represented as (xi, yj), i varies from 1 to n and j varies from 1 to m .

Bayes' theorem of two events A and B is important of both probability and statistics. For two events A and B Bayes' theorem (also called Bayes' rule and Bayes' formula) says

$$P\left(\frac{B}{A}\right)=\frac{P\left(\frac{A}{B}\right)P(B)}{P(A)}.$$

Comments:

1. To 'invert' conditional probabilities Bayes rule is used, i.e. to find P(B|A) from P(A|B).

2. In practice, to find P(A) using the law of total probability.

Conditional distribution for random variables comes from conditional probability of random events, so the relationship between the two random variables and the approaches are the same, but conditional distribution is more complicated in this system.

In recent years, the main research direction in some countries is the research of conditional eigenvalues and application of conditional distribution. Conditional eigenvalues are applied to conditional expectation, which comes in the expectation of conditional distribution. Zhang Mei used some examples to analyze the application of conditional expectation in practical prediction problems; Xu hui and Wu Guogeng discuss the total probability formula of discrete and continuous random variables in conditional expectation and . For extension of conditional distribution, Cheng Feiyue had a research to generalize the condition distribution of the Poisson process. Under the assumption that the number of hypothesis belongs to the family consisting of Poisson, binomial and negative binomial. Hu Duanping derive the formula of condition distribution of elliptically contoured matrix distribution is elliptically contoured distribution yet. In the above study, they focused only the application of the conditional distribution and also showing the research significance of the conditional distribution, but there are few studies researching the nature of the conditional distribution.

In comparison, the studies of conditional distribution is very depth For example, Rodney C.L. Wolff, etc studied the methods of solving conditional distribution function; Jushan Bai investigated the dynamic model to finding parametric conditional distributions Peter Hall and Qiwei Yao discussed conditional distribution function by using dimension reduction Bruce E. Hansen studied non-parametric estimation of smooth conditional distributions. Therefore, we show that the researches of conditional distribution give multi-faceted and more complex ideas undergraduate teaching.

Therefore, this paper begins to discuss and analyze of the two-dimensional random variable conditional distribution and given the conditions to obtain the extensions of conditional distribution and when there are three-dimensional random variables. This paper gives the ideas to solve the conditional distribution of multidimensional random variables for some conditions and its results can be used for teaching, expending the knowledge of the conditional distribution and facilitating people's calculations.

II. Discrete random variables conditional distribution-Extension

A. ***Extension 2.1*** Let X and Y are the Discrete random variables and X ,Y are independent , We know that the distribution series of X and Y , under the given condition of X+Y=n then the conditional distribution is given by

$$P\left(X=k/X+Y=n\right)=\frac{P\left(X=k\right)P\left(Y=n-k\right)}{P\left(X+Y=n\right)} \qquad \text{--------------(1)}$$

B. ***Example 2.1.1***

I) By using basal examples In the case of two-dimensional random variables, X and Y are independent and $X \sim P(\lambda_1)$, $Y \sim P(\lambda_2)$ .Given the condition of $X+Y=n$ to solve the conditional distribution of X .

To solve: For the sum of the independent Poisson variables is still Poisson variable, viz $X+Y \sim P(\lambda_1+\lambda_2)$, so

$$P(X=k/X+Y=n)=\frac{P(X=k,X+Y=n)}{P(X+Y=n)}=\frac{P(X=k)P(Y=n-k)}{P(X+Y=n)}$$

$$=\frac{\frac{\lambda_1^k}{k!}e^{-\lambda_1}\frac{\lambda_2^{n-k}}{(n-k)!}}{\frac{(\lambda_1+\lambda_2)^n}{n!}e^{-(\lambda_1+\lambda_2)}}$$

$$=\frac{n!}{k!(n-k)!}\frac{\lambda_1^k\lambda_2^{n-k}}{(\lambda_1+\lambda_2)^n}$$

$$=nCk\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k},k=0,1,2,....n.$$

That is under the condition of $X+Y=n,$ X subjects to binomial distribution $b(n,p)$, there into $p=\frac{\lambda_1}{\lambda_1+\lambda_2}$.

II) Two-dimensional discrete variables (X,Y) subject to trinomial distribution $M(n,p_1,p_2,p_3)$. Given the condition of $X=i$ to solve the conditional distribution of Y.

To solve: (X,Y) subject to trinomial distribution $M(n,p_1,p_2,p_3)$, then theirs joint distribution is

$$P(X=i,Y=j)=\frac{n!}{i!j!(n-i-j)!}p_1^ip_2^j(1-p_1-p_2)^{n-i-j},i,j=1,2,...,n,i+j\le n$$

For the marginal distribution of multinomial distribution is still distribution and the marginal distribution of trinomial distribution, so $X \sim b(n,p_1),Y \sim b(n,p_2).$

$$P(y=j/x=i)=\frac{P(x=i,y=j)}{P(x=i)}=\frac{\frac{n!}{i!j!(n-i-j)!}p_1^ip_2^j(1-p_1-p_2)^{n-i-j}}{\frac{n!}{i!(n-i)!}p_1^i(1-p_1)^{n-i}}$$

$$=\frac{(n-i)!}{j!(n-i-j)!}\frac{p_2^j(1-p_1-p_2)^{n-i-j}}{(1-p_1)^{n-i}}$$

$$=(n-i)Cj\left(\frac{p_2}{1-p_1}\right)\left(\frac{1-p_1-p_2}{1-p_1}\right)^{n-i-j}$$

That is under the condition of $X=i,Y$ subjects to binomial distribution $b(n-i,p)$, there into $p=\frac{p_2}{1-p_1}$.

C. ***Extension2.2*** Set X,Y,Z for the discrete random variables and X,Y,Z are mutual independent. Known the distribution series X,Y,Z of under the given condition of $X+Y+Z=n$ the conditional distribution of X is

$$P(X=k/X+Y+Z=n)=\frac{P(X=k,X+Y+Z=n)}{P(X+Y+Z=n)}=\frac{P(X=k)P(Y+Z=n-k)}{P(X+Y+Z=n)}.$$

D. ***Extension2.2.1***

I) X,Y,Z are mutual independent, and $X \sim P(\lambda_1),Y \sim P(\lambda_2),Z \sim P(\lambda_3)$.

1) Given the condition of $Y+Z=n$ to solve the conditional distribution of X.

To Solve: For the sum of the independent Poisson variables is still Poisson variable, viz $Y+Z \sim P(\lambda_2+\lambda_3)$, so $P\left(X=k/Y+Z=n\right)=\dfrac{P\left(X=k,Y+Z=n\right)}{P(Y+Z=n)}=P(X=k) \sim P\left(\lambda_1\right)$. that is X still subjects to $P\left(\lambda_1\right)$.

2) Given the condition of $X+Y+Z=n$ to solve the conditional distribution of X.

To Solve: For the sum of the independent Poisson variables is still Poisson variable, viz, $X+Y+Z \sim P\left(\lambda_1+\lambda_2+\lambda_3\right)$, so

$$P\left(X=k/X+Y+Z=n\right)=\frac{P\left(X=k,X+Y+Z=n\right)}{P\left(X+Y+Z=n\right)}=\frac{P\left(X=k\right)P\left(Y+Z=n-k\right)}{P\left(X+Y+Z=n\right)}$$

$$=\frac{\dfrac{\lambda_1^k}{k!}e^{-\lambda_1}\dfrac{\left(\lambda_2+\lambda_3\right)^{n-k}}{(n-k)!}e^{-(\lambda_2+\lambda_3)}}{\dfrac{\left(\lambda_1+\lambda_2+\lambda_3\right)^n}{n!}e^{-(\lambda_1+\lambda_2+\lambda_3)}}=\frac{n!}{k!(n-k)!}\frac{\lambda_1^k\left(\lambda_2+\lambda_3\right)^{n-k}}{\left(\lambda_1+\lambda_2+\lambda_3\right)^n}$$

$$=nCk\left(\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\right)^k\left(\frac{\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)^{n-k},k=0,1,\ldots,n.$$

That is under the condition of $X+Y+Z=n,\ X$ subjects to binomial distribution $b\left(n,p_1\right)$, thereinto $p_1=\dfrac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}$.

In a similar way, under the condition of $X+Y+Z=n,\ Y$ subjects to binomial distribution $b\left(n,p_2\right)$, thereinto $p_2=\dfrac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}$; Z subjects to binomial distribution $b\left(n,p_3\right)$, there into $p_3=\dfrac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}$.

II) Let $X_1,X_2,\ldots,X_n$ are mutual independent and $X_i \sim P\left(\lambda_1\right),i=1,2,\ldots,n$. Given the condition of $X_1+\ldots+X_n=m$ to solve the conditional distribution of X_i.

To Solve: $P\left(X_i=k/\sum_{j=1}^{n}X_j=m\right)=\dfrac{P\left(X_i=k,\sum_{j=1}^{n}X_j=m\right)}{P\left(\sum_{j=i}^{n}X_j=m\right)}$

$$=\frac{P\left(X_i=k\right)P\left(\sum_{\substack{j=1\\ j\neq i}}^{n}X_j=m-k\right)}{P\left(\sum_{j=1}^{n}X_j=m\right)}$$

$$=\frac{\frac{\lambda_1^k}{k!}e^{-\lambda_1}\frac{\left(\sum_{\substack{j=1\\j\neq i}}^{n}\lambda_j\right)^{m-k}}{(m-k)!}e^{-\sum_{\substack{j=1\\j\neq i}}^{n}\lambda_j}}{\frac{\left(\sum_{j=1}^{n}\lambda_j\right)^{m}}{m!}e^{-\sum_{j=1}^{n}\lambda_j}}$$

$$=\frac{m!}{k!(m-k)!}\frac{\lambda_1^k\left(\sum_{\substack{j=1\\j\neq i}}^{n}\lambda_j\right)^{m-k}}{\left(\sum_{j=1}^{n}\lambda_j\right)^{m}}$$

$$=mCk\left(\frac{\lambda_1}{\sum_{j=1}^{n}\lambda_j}\right)^{k}\left(\frac{\sum_{\substack{j=1\\j\neq i}}^{n}\lambda_j}{\sum_{j=1}^{n}\lambda_j}\right)^{m-k},k=0,1,\ldots n.$$

That is under the condition of $X_1+\ldots+X_n=m,\ X_i$ subjects to binomial distribution $b(n,p_i)$, thereinto $p_i=\frac{\lambda_1}{\sum_{j=1}^{n}\lambda_j}$.

when $\lambda_1=\lambda_2=\ldots=\lambda_n\ \ p_1=p_2=p_3\ldots=\frac{1}{n}\ \ X_i \sim b\left(n,\frac{1}{n}\right),i=1,2,3\ldots n$

The solution of the above two extensions is relatively simple so it is omitted.

III.CONCLUSION

This paper mainly deals the conditional distributions of multidimensional random variables and consider some relevant examples given in the case of discrete situation. It changes the original condition of one fixed variable into more different conditions, for example the above condition is says that the sum of two variables is fixed in two-dimensional situation.

In addition to that this paper extends two-dimensional into three and n-dimensional random variables and gives the conditional distributions. For example, in the case of three-dimension, we can get one variables conditional distribution givens the sum of the other two or the three variables are fixed. This article gives the above results strictly according to two-dimensional random variables conditional distribution.

We can apply the Conditional distribution in the real life and work to resolve practical problems. The application of conditional distribution theory to carry out scientific analysis and calculations of real data is an important reflection of the usefulness of conditional distribution. In future , for there are more and more discussions about multi -dimensional random variables in reality this paper extends the conditional distribution to provide new ideas of thought for the research in some extent and it aims at enriching the content of conditional distribution, deeply the understanding of it and applying it well in practical .

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