A Study of Intuitionistic Fuzzy Transportation Problem Using Vogel's Approximation Method

K.Ganesan[#1], D.Dheebia[*2]

[#] Assistant Professor, Department of Mathematics, Sri Manakula Vinayagar Engineering College, Pondicherry, India

[*] Assistant Professor, Department of Mathematics, Sri Manakula Vinayagar Engineering College, Pondicherry, India.

Abstract — *The transportation problem is a special type of linear programming problem where the objective consists in minimizing transportation cost of a given commodity from a number of sources or origins to a number of destinations. We have various methods to optimize the transportation cost. In this paper we are going to take demands and supply as Intuitionistic Fuzzy Number (IFN) and find the initial basic feasible solution by applying Vogel's approximation method in terms of Intuitionistic Fuzzy Number.*

Keywords — *Fuzzy Set, Fuzzy Transportation problem, Intuitionistic Fuzzy Number, Vogel's Approximation Method, Optimal Solution.*

1. INTRODUCTION

The study under taken in this thesis is mainly intended to study the applications of Fuzzy theory.

"The theory of Fuzzy set" was founded by "Loffi zadeh" However, some of the key ideas of the theory were envisioned by "Max black". A philosopher almost 30 year prior to zadeh's seminar paper (1935).

Thousands of applications are available in this new area, current contributions to the theory are scattered in many journals and books of collected papers. But the most important source that we have referred is the specialized journal "Fuzzy sets and systems".

If x denotes universal set then the membership function of a $\mu_{\tilde{A}}$: x → [0, 1] where [0, 1] denotes set of all real number between 0 & 1 including 0&1.

The Transportation Problem plays a vital role in operation research. It contains many practical applications. In this problem we determine optimal solutions between origins or sources and destinations. Consider m origins are to supply n destinations with a certain product. Let a_i be the amount of the product available at origin i, and b_j be the amount of the product required at destination j. Further, we assume that the cost of shipping a unit amount of the product from origin i to destination j is c_{ij}, we then let x_{ij} represent the amount shipped from origin i to destination j. If shipping cost is assumed to be proportional to the amount shipped from each origin to each destination so as to minimize the total shipping cost turns out be a linear programming problem.

Transportation models have huge applications in logistics and supply chain for reducing the cost. When the cost coefficients and the supply and demand quantities are known exactly many algorithms have been developed for solving the Transportation Problem. But, in the real world, there are many cases that the cost co-efficient, and the supply and demand quantities are fuzzy quantities. A fuzzy Transportation Problem is a Transportation Problem in which the transportation cost, supply and demand are fuzzy quantities. Most of the existing techniques provide only crisp solution for the Fuzzy Transportation Problem.

In fuzzy transportations problem, all parameters are fuzzy numbers. Comparing between two or multi fuzzy numbers, and ranking such numbers is one of the important subjects, and how to set the rank of fuzzy numbers has been one of main problems. Various methods are introduced for ranking of fuzzy numbers. Here we want to use a method which is introduced for ranking of fuzzy numbers by Basizadeh at et al [3]. Now we want to apply this method for all fuzzy Transportation problems where all parameter can be triangular fuzzy number.

At the end, the optimal solution of a problem can be obtained in a fuzzy number or a crisp number form.

II. PRELIMINARIES

A. Definition

If V is a collection of objects denoted generally by V, then a fuzzy set A in V is a set of order pairs. $\tilde{A} = \{V, \mu_{\tilde{A}}(x))/x \in V\}$, Where $\mu_{\tilde{A}}(x)$ is called membership function or grade of membership.

Example: Let V is a ten natural numbers

(i.e.) $V = \{2,3,6,8,9,11,12,14,16,17\}$

$\tilde{A} = \{(2,0.1),(3,0.2),(6,0.3),(8,0.4),(9,0.5),(11,0.6),(12,0.7),(14,0.8),(16,0.9),(17,1)\}$

B. Definition

A fuzzy set $\tilde{A}$ is convex if, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, x_1, x_2 \in X,\ \lambda \in [0,1]$

Ultimately, A fuzzy set is convex if all $\alpha -$ level set are convex.

C. Definition

Let X be a finite fuzzy set $\tilde{A}$, the cardinality $|\tilde{A}|$ is defined as $|\tilde{A}| = \sum_{x\in X} \mu_{\tilde{A}}(x)$, $\| \tilde{A} \| = \frac{|\tilde{A}|}{|X|}$ is called relative cardinality of $\tilde{A}$

Example:

Let $X = \{3,6,9,12,15,18\}$ be the set and $\tilde{A} = \{(3,0.2),(6,0.5),(9,1),(12,0.3),(15,0.6)\}$

The cardinality is, $|\tilde{A}| = 0.2 + 0.5 + 1 + 0.3 + 0.6 = 2.6$

Its relative cardinality is $\| \tilde{A} \| \frac{2.6}{6} = 0.43$

The relative cardinality can be interpreted as the fraction of elements of X being in $\tilde{A}$ weighted by their degrees of membership in $\tilde{A}$. For infinite X, the cardinality is defined by, $|\tilde{A}| = \int_x \mu_{\tilde{A}}(x)dx$ $|\tilde{A}|$ does not always exist.

III. TYPES OF FUZZY NUMBERS

There are two types of Fuzzy Numbers that are commonly used in Fuzzy theory, namely

1. **Triangular Fuzzy Number**
2. **Trapezoidal Fuzzy Number**

1. *Triangular Fuzzy number*:

There are various shapes of fuzzy number. In that triangular fuzzy number(TFN) is the most popular one.

Definition:

It is a fuzzy number represented with three points as follows: $A = (a_1, a_2, a_3)$

This representation is interpreted as membership function

$$\mu_A(x) = \begin{cases} 0, & x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & x \geq a_3 \end{cases}$$

Interval A_α shall be obtained as follows $\forall\, \alpha \in [0,1]$

From $\frac{a_1^{(\alpha)} - a_1}{a_2 - a_1} = \alpha$, $\frac{a_3 - a_3^{(\alpha)}}{a_3 - a_2} = \alpha$

Triangular fuzzy number $A = (a_1, a_2, a_3)$ we get $a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1$

$$a_3^{(\alpha)} = -(a_3 - a_2)\alpha + a_3$$

Thus $A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$

2. *Trapezoidal Fuzzy number:*

The next shape of fuzzy number is trapezoidal fuzzy number. This shape is taken from the fact that there are several points whose membership degree is maximum ($\alpha = 1$)

Definition:

A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4) where $a_1, a_2, a_3,\ and\ a_4$ are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below:

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & x > a_4 \end{cases}$$

IV. INTUITIONISTIC FUZZY NUMBER

A. *Definition*

Let X be denote a universe of discourse, then an intuitionistic fuzzy set in X is given by a set of ordered triples, $\tilde{A}^I = \{< x, \mu_A(x), \vartheta_A(x) >; x \in X\}$ where μ_A , $\vartheta_A: X \to [0,1]$, are functions such that $0 \le \mu_A(x) + \vartheta_{A(x)} \le 1, \forall\, x \in X$.
For each x the membership $\mu_A(x)$ and $\vartheta_A(x)$ represent the degree of membership and the degree of non – membership of the element $x \in X$ to $A \subset X$ respectively.

B. *Definition*

An Intuitionistic fuzzy subset $\tilde{A}^I = \{< x, \mu_A(x), \vartheta_A(x) >; x \in X\}$ of the real line R is called an intuitionistic fuzzy number (IFN) if the following holds:

i. There exist $m \in R$, $\mu_A(x) = 1$, and $\vartheta_A(m) = 0$, (m is called the mean value of A).
ii. μ_A is a continuous mapping from R to the closed interval [0,1] and $\forall\ x \in R$, the relation $0 \le \mu_A(x) + \vartheta_{A(x)} \le 1$ holds.

The membership and non – membership function of above fuzzy set is of the following form:

$$\mu_A(x) = \begin{cases} 0 & for -\infty < x \le m - \alpha \\ f_1(x) & for\ x \in [m - \alpha, m] \\ 1 & for\ x = m \\ h_1(x) & for\ x \in [m, m + \beta] \\ 0 & for\ m + \beta \le x < \infty \end{cases}$$

Where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $[m - \alpha, m]$ and $[m, m + \beta]$ respectively.

$$\vartheta_A(x) = \begin{cases} 1 & for -\infty < x \le m - \alpha' \\ f_2(x) & for\ \ x \in [m - \alpha', m]; 0 \le f_1(x) + f_2(x) \le 1 \\ 0 & for\ x = m \\ h_2(x) & for\ x \in [m, m + \beta'];\ 0 \le h_1(x) + h_2(x) \le 1 \\ 1 & for\ m + \beta' \le x < \infty \end{cases}$$

Here m is the mean value of A. $\alpha\ and\ \beta$ are called left and right spreads of membership function $\mu_A(x)$, respectively. $\alpha'\ and\ \beta'$ represents left and right spreads of non membership function $\vartheta_A(x)$, respectively. Symbolically, the intuitionistic fuzzy number $\tilde{A}^I$ is represented as $A_{IFN} = (m;\ \alpha, \beta; \alpha', \beta')$.

C. *Definition*

A Triangular Intuitionistic Fuzzy Number ($\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non membership function $\vartheta_A(x)$:)

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ \ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & for\ \ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1}, & for\ a_1' \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2}, & for\ a_2 \le x \le a_3' \\ 1, & otherwise \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_A(x)$, $\vartheta_A(x) \le 0.5$ for $\mu_A(x) = \vartheta_A(x)\ \forall\, x \in R$. This TrIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3)(a_1', a_2, a_3')$

D. Arithmetic Operations

- ***Addition*** : $\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a_1' + b_1', a_2 + b_2, a_3' + b_3')$
- ***Subtraction*** : $\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 + b_1)(a_1' - b_3', a_2 - b_2, a_3' - b_1')$
- ***Multiplication*** : $\tilde{A}^I \otimes \tilde{B}^I = (l_1, l_2, l_3)(l_1', l_2, l_3')$

  Where $l_1 = \min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$
  $l_2 = a_2b_2$
  $l_3 = \max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$
  $l_1' = \min\{a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'\}$
  $l_2 = a_2b_2$
  $l_3' = \max\{a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'\}$
- ***Scalar Multiplication*** : i) $k\tilde{A}^I = (ka_1, ka_2, ka_3)(ka_1', ka_2, ka_3')$, for K > 0
  ii) $k\tilde{A}^I = (ka_3, ka_2, ka_1)(ka_3', ka_2, ka_1')$, for K < 0.

E. Ranking of Triangular Intuitionistic fuzzy number

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ be a TIFN, then we defined a score function for membership and non-membership values respectively as

$S(\tilde{A}^{I\alpha}) = \frac{a_1 + 2a_2 + a_3}{4}$ & $S(\tilde{A}^{I\beta}) = \frac{a_1' + 2a_2 + a_3'}{4}$

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ be a TIFN, then we define

$$\tilde{A}^I = \frac{a_1 + 2a_2 + a_3 + a_1' + 2a_2 + a_3'}{8}$$

An accuracy function of $\tilde{A}^I$, to, defuzzify the given number.

V. TRANSPORTATION PROBLEM

A. Mathematical formulation of Transportation Problem:

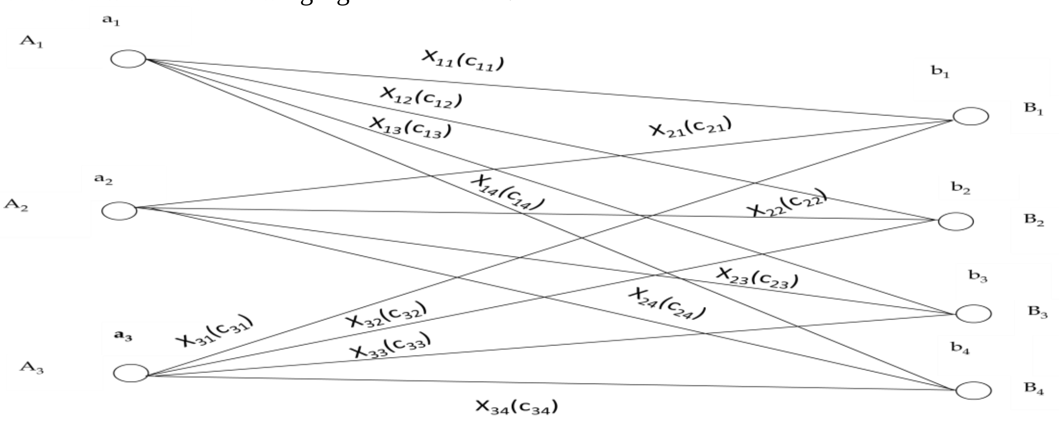
Let there be three units, producing car, say A_1, A_2 and A_3 from where the cars are to be supplied to four depots says B_1, B_2, B_3 and B_4.

Let the number of cars produced at A_1, A_2 and A_3 be a_1, a_2 and a_3 respectively and the demands at the depots be b_1, b_2, b_2 and b_4 respectively.

Let assume the condition $a_1 + a_2 + a_3 = b_1 + b_2 + b_3 + b_4$

i.e., all cars produced are supplied to the different depots. Let the transportation cost of one car from A_1 to B_1 be c_{11}. Similarly the cost of transportation in other cases are also shown in the figure and Tabel:1

Let out of a_1 cars available at A_1, x_{11}, be taken at B_1 depot, x_{12} be taken at B_2 depot and two other depots as well as shown in the following figure and table 1.



Total number of car, to be transported from A_1 to all destination, i.e., B_1, B_2, B_3 and B_4 must be equal to a_1.

$$\therefore x_{11} + x_{12} + x_{13} + x_{14} = a_1 \text{ ------------ } (1)$$

Similarly, from A_2 and A_3 the cars transported be equal to a_2 and a_3 respectively

$$\therefore x_{21} + x_{22} + x_{23} + x_{24} = a_2 \text{ ------------ } (2)$$

and $$x_{31} + x_{32} + x_{33} + x_{34} = a_3 \text{-------------} (3)$$

On the other hand it should be kept in mind that the total number as cars delivered to B_1 from all units must be equal to b_1, i.e., $x_{11} + x_{21} + x_{31} = b_1$----------- (4)

Similarly, $x_{12} + x_{22} + x_{32} = b_2$------------ (5)

$$x_{13} + x_{23} + x_{33} = b_3 \text{------------ } (6)$$

$$x_{14} + x_{24} + x_{34} = b_4 \text{------------- } (7)$$

With the help of the above information we construct the following table:

Depot / **Unit**	To B1	To B2	To B3	To B4	Stock
From A1	$x_{11}(c_{11})$	$x_{12}(c_{12})$	$x_{13}(c_{13})$	$x_{14}(c_{14})$	a_1
From A2	$x_{21}(c_{21})$	$x_{22}(c_{22})$	$x_{23}(c_{23})$	$x_{24}(c_{24})$	a_2
From A3	$x_{31}(c_{31})$	$x_{32}(c_{32})$	$x_{33}(c_{33})$	$x_{34}(c_{34})$	a_3
Requirement	b_1	b_2	b_3	b_4	

The cost of transportation from A_i (i= 1, 2, 3) to B_j(j=1,2,3,4) will be equal to $S = \sum_{i,j} c_{ij} x_{ij}$ ------------- (8) where the symbol put before $c_{ij}x_{ij}$ signifies that the quantities $c_{ij}x_{ij}$ *must* be summed over all i = 1, 2, 3 and all j= 1, 2, 3, 4. Thus we come across a linear programming problem given by equation (1) to (7) and a linear function (8)

We have to find the non-negative solutions of the system such that it minimize the function (8)

Note: We can think about a transportation problem in a general way where there are m sources (say A_1, A_2,…,A_m) and n-destination (say B_1, B_2,…. ,B_n). We can use a_i to denote the quantity goods concentrated at points A_i (i=1,2,…m) and b_j denotes the quantity of goods expected at points B_j (j=1,2,…,n). We assume the condition

$$a_1 + a_2 + \cdots + a_m = b_1 + b_2 + \cdots b_n$$

Which implies that the total stock of goods is equal to the summed demand for it.

The following terms are to be defined with reference to the transportation problem.

(a)Feasible solution (F.S)

A set of non-negative allocation $x_{ij} \geq 0$ which satisfies the row and column restriction is known as feasible solution.

(b)Basic Feasible solution (B.F.S)

The Feasible solution to a 'm' origin and 'n' destination problem is said to be basic feasible solution if the number of positive allocations are m+n-1

If number of allocation in a basic feasible solution are less than (m+n-1) it is called degenerate basic feasible solution (DBFS) (otherwise non-degenerate)

(c) Optimal solution

The feasible solution is said to be optimal if it minimizes the total transportation cost.

B. Initial Basic Feasible solution:

Three different methods to obtain the initial basic feasible solution are:

i) North-west corner rule
ii) Lowest cost entry method
iii) Vogel's approximation method.

In the cost of transportation by the above three methods, one thing is clear that Vogel's approximation method gives an initial basic feasible solution which is much closer to the optimal solution than the other two methods. It is always worthwhile to spend some time in finding a "good" initial solution because it can considerably reduce the total number of iterations required to reach an optimal solution.

C. Steps for Vogel's approximation method:

The Vogel's approximation method is an iterative procedure for computing a basic feasible solution of transportation problem.

Step 1: Find the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.

Step 2: Find the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column.

Step 3: Identify the maximum penalty. If it is along the side of the table, make maximum allotment at the box having minimum, cost of transportation in that row. If it is along bottom of the table make maximum allotment to the box having minimum cost of transportation in that column.

Step 4: If the penalties corresponding to two or more rows or columns are equal, then select the top most row and the extreme left column.

Numerical Example: 5.1

Let us consider the numerical problem stated in the introduction and the mathematical formulation of the same. (All terms are in hundreds)

Unit \ Depot	B1	B2	B3	B4	Stock
A1	$c_{11} = 2$	$c_{12} = 3$	$c_{13} = 5$	$c_{14} = 1$	$a_1 = 8$
A2	$c_{21} = 7$	$c_{22} = 3$	$c_{23} = 4$	$c_{24} = 6$	$a_2 = 10$
A3	$c_{31} = 4$	$c_{32} = 1$	$c_{33} = 7$	$c_{34} = 2$	$a_3 = 20$
Requirement	$b_1 = 6$	$b_2 = 8$	$b_3 = 9$	$b_4 = 15$	

Solution:

Now using the step 1 to step 4 of the vogel's approximation method, we have the following reduced transportation Table I

Table I

Unit \ Depot	B1	B2	B3	B4	Stock	Penalties
A1	(2)	(3)	(5)	(1)	8	(1)
A2	(7)	(3)	(4)	(6)	10	(1)
A3	(4)	(1)	(7)	(2)	20	(1)
Requirement	6	8	9	15	38	
Penalties	(2) ↑	(2)	(1)	(1)		

Table II

Unit \ Depot	B1	B2	B3	B4	Stock	Penalties
A1		(3) ↓	(5)	(1)	2	← (2)
A2		(3)	(4)	(6)	10	(1)
A3		(1)	(7)	(2)	20	(1)
Requirement		8	9	15	32	
Penalties		(2)	(1)	(1)		

Table III

Depot / Unit	B_1	B_2	B_3	B_4	Stock	Penalties
A_1						
A_2		(3)	(4)	(6)	10	(1)
A_3		(1)	(7)	(2) →	20	(1)
Requirement		8	9	13	32	
Penalties		(2)	(3)	(4) ↑		

Table IV

Depot / Unit	B_1	B_2	B_3	B_4	Stock	Penalties
A_1						
A_2		(3)	(4)		10	(1)
A_3		(1) ↓	(7)		7	(6) ←
Requirement		8	9		32	
Penalties		(2)	(3)			

Table V

Depot / Unit	B_1	B_2	B_3	B_4	Stock	Penalties
A_1						
A_2		(3)	(4) →		10	(1)
A_3						
Requirement		1	9			
Penalties		(3)	(4) ↑			

Table VI

Depot / Unit	B_1	B_2	B_3	B_4	Stock	Penalties
A_1						
A_2		(3)			6	
A_3						
Requirement		1				
Penalties		(3)				

Finally we get the allotment table

Depot / Unit	B_1	B_2	B_3	B_4	Stock	Penalties
A_1	**6(2)**			**2(1)**	8	
A_2		**1(3)**	**9(4)**		10	
A_3		**7(1)**		**13(2)**	20	
Requirement	6	8	9	15	38	
Penalties						

From the above fact calculate the cost of transportation as

=6X2+2X1+1X3+9X4+7X1+13X2 = 86

(i.e.) Rs. 8600

We take another numerical example by taking the supply and demands are as Intuitionistic Fuzzy number and apply Vogel's approximation method

Numerical example: 5.2

Consider the following 4X4 Intuitionistic Fuzzy Transportation Problem (IFTP)

	IFD1	IFD2	IFD3	IFD4	IF Supply
IFO1	2	3	11	7	< (4,6,9) (2,6,10) >
IFO2	1	0	6	1	< (0.5,1,3) (0,1,5) >
IFO3	5	8	15	9	< (8.5,10,12)(8,10,14)>
IF	<(6,7,9)(5,7,11)>	<(4,5,7)(3,5,8)>	<(2,3,5)(1.5,3,6)>	<(1,2,3)(0.5,2,4)>	

Solution:

Now using the step 1 to step 4 of the Vogel's approximation method, we have the following reduced Intuitionistic Fuzzy Transportation Table I

Table I

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1	2	3	11	7	< (4,6,9) (2,6,10) >	(1)
IFO2	1	0	6	**1**	< (0.5,1,3) (0,1,5) >	(1)
IFO3	5	8	15	9	<(8.5,10,12) (8,10,14)>	(3)
IF demand	<(6,7,9) (5,7,11)>	<(4,5,7) (3,5,8)>	<(2,3,5) (1.5,3,6)>	<(1,2,3) (0.5,2,4)>		
Penalties	(1)	(3)	(5)	**(6)**		

Table II

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1	2	3	11		< (4,6,9) (2,6,10)>	(1)
IFO2	1	0	**6**		<(-2.5,-1,2) (-4,-1,4.5)>	(1)
IFO3	5	8	15		< (8.5,10,12) (8,10,14) >	(3)
IF demand	<(6,7,9) (5,7,11) >	<(4,5,7) (3,5,8)>	<(2,3,5) (1.5,3,6) >			
Penalties	(1)	(3)	**(5)**			

Table III

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1	2	3			< (4,6,9) (2,6,10)>	(1)
IFO2	1	0			<(-7.5,-4,0) (-10,-4,3)>	(1)
IFO3	**5**	8			< (8.5,10,12) (8,10,14) >	(3)
IF demand	< (6,7,9) (5,7,11) >	< (4,5,7) (3,5,8) >				
Penalties	(1)	(3)				

Table IV

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1	2	3			< (4,6,9)(2,6,10)>	(1)
IFO2	1	**0**			<(-7.5,-4,0)(-10,-4,3)>	(1)
IFO3						
IF demand	<-6,-3,0.5)(-9,-3,3)>	< (4,5,7)(3,5,8) >				
Penalties	(1)	(3)				

Table V

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1	2				< (4,6,9)(2,6,10)>	(2)
IFO2	1				<-14.5,-9,-4)(-18,-9,0)>	(1)
IFO3						
IF demand	<-6,-3,0.5)(-9,-3,3)>					
Penalties	(1)					

Table VI

	IFD1	IFD2	IFD3	IFD4	IF Supply	Penalties
IFO1						
IFO2	1				<(-14.5,-9,-4)(-18,-9,0)>	
IFO3						
IF demand	<-15,-9,-3.5)(-19,-9,1)>					
Penalties						

Hence, the total intuitionistic fuzzy transportation minimum cost (by Vogel's approximation method) is given by

Min $\tilde{Z}^I$ = (1)[<(1,2,3)(0.5,2,4)]+(6)[<(2,3,5)(1.5,3,6)>]+(5)[<(8.5,10,12)(8,10,14)>]+(0)[<(4,5,7)(3,5,8)>]+ (2)[(4,6,9)(2,6,10)]+(1)[<(-14.5,-9,-4)(-18,-9,0)]

Min $\tilde{Z}^I$ = <(49,73,107)(35.5,73,130)>

VI. CONCLUSION

This Study is focused on obtaining the optimal solution for transportation problem where the supply and demand are Intuitionistic Fuzzy Numbers. Here we use the Vogel's Approximation Method to obtain the solution for Intuitionistic fuzzy transportation problem. And we conclude that Vogel's approximation is efficient method to solve the Intuitionistic Fuzzy Transportation Problem to get the optimal solution.

REFERENCE

[1] Atanassova K.T(9186), *Intuitionistic Fuzzy Set*, Fuzzy sets and system, Vol.20, pp.87-96.

[2] Ismail Mohideen.S, Senthil kumar.P(2010), *A Comparative study o Transformation problem in Fuzzy environment*, International Journal of Mathematics Research; Vol.2 No.1, pp.151-451.

[3] Kaufmann. A, Gupta.M(1991), *Introduction to fuzzy arithmetic Theory and applications*, Van Nostrand Reinhold; Newyork.

[4] Nagoor Gani.A(2012), *A New Operation on Triangular Fuzzy Number for solving Fuzzy Linear Programming Problem*, Vol.6, No.11, 525-532.

[5] Nagoor Gani.A, Abbas., *Intuitionistic Fuzzy Transportation Problem*, proceedings of the heber international conference pp.445-451.

[6] H.J. Zimmermann, *Fuzzy set theory and its applications*, Kluwer – Nijhoff, Boston, 1996.

[7] P.Pandian and G.Natarajan., *A new Algorithm for finding a fuzzy Optimal solution for Fuzzy Transportation problems*, Applied mathematics sciences, Vol.4, 2010, no.2, 79 -90