Relationship of Lucas Matrix and *p*-Tribonacci Matrix

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Abstract — *This article discusses the relationship of Lucas matrix L_n and p-Tribonacci matrix $T_p(n)$, and the relationship of Fibonacci matrix F_n and p-Tribonacci matrix $T_p(n)$. these relationship are to obtain a new matrix called E_n^P matrix and H_n^P matrix, Then the E_n^P matrix and H_n^P matrix can be expressed in the form $L_n = T_p(n)E_n^P$ and $F_n = T_p(n)H_n^P$.*

Keywords — *Lucas Number, Lucas Matrix, p-Tribonacci Number, p-Tribonacci Matrix, Fibonacci Number, Fibonacci Matrix.*

I. INTRODUCTION

Fibonacci number is proposed by Italian Mathematician named Leonardo Pisano in 1202, Fibonacci number is started with 0,1, then the next number is the sum of two previously consecutive terms. Fibonacci number can be formed to be square matrix symbolized with F_n [5]. Then there is number which has similar pattern to Fibonacci number, such as Lucas number. Lucas number is introduced by France Mathematician Francois Edouard Anatole in 1877. Lucas number can be formed to be Lucas matrix, until each entries of Lucas matrix is Lucas number and symbolized by L_n[10]. Beside Lucas number, *p*-Tribonacci number also can be formed to be matrix in each of its entry is *p*-Tribonacci number, *p*-Tribonacci number is the generalization of Tribonacci number [4].

Some research discuss about Fibonacci number, Lucas and *p*-Tribonacci number such as Predrag Stanimirovi [11] discussing some matrix identity which is Lucas matrix and Fibonacci matrix, and discusses invers of Lucas matrix. Z. Zhang [13] discusses about Lucas matrix and some of its identities and discusses Relationship ofLucas matrix and Pascal matrix until constructed two matrix in which both of the matrixes are bottom triangle matrix. Kuhapatanaku [4] discusses about *p*-Tribonacci number and constructs *p*-Tribonacci matrix until obtains some interesting identities and then apply the linkages between *p*-Tribonacci number and pascal triangle. Ulfa [12] discusses some of identities of Fibonacci-Like sequence base on the relation between Fibonacci-Like and Lucas Number, then Fibonacci-Like can just be defined by Lucas number.

Then Lee [5] discusses about relationship of Pascal matrix and Fibonacci matrix which from that relationship a matrix is constructed. Sabeth [2] gives definition of Tribonacci matrix that is defined as bottom triangle matrix and each of its entry is Tribonacci number then he also discusses about relationship of Pascal matrix and Tribonacci matrix which from that relationship can be constructed two matrixes and two factorization. Rasmi [8] has discussed relationship of Stirling matrix and Tribonacci matrix then from that relationship of two matrixes are constructed which with those matrixes can be obtain two factorization. Mirfaturiqa [10] define the tetranacci matrix and discuss the factorization of the Pascal matrix with the tetranacci matrix.

This article discusses relationship of Lucas matrix and *p*-Tribonacci matrix and relationship of Fibonacci matrix and *p*-Tribonacci matrix. To show relationship of Lucas matrix and *p*-Tribonacci matrix and Fibonacci matrix and *p*-Tribonacci matrix, used the concept of matrix equation, with first finding the invers of *p*-Tribonacci matrix $T_p^{-1}(n)$, because the invers of *p*-Tribonacci matrix has the pattern then determined the general form of *p*-Tribonacci matrix invers, then conducted multiplication operation of Lucas matrix and invers of *p*-Tribonacci, then from the result of matrix multiplication operation can be constructed the matrix notated with E_n^P until can be showed $E_n^P = T_p^{-1}(n)L_n$ then $T_p(n)\, E_n^P = L_n$. Then conducted the multiplication of Fibonacci matrix and invers of *p*-Tribonacci matrix until constructed a matrix such as H_n^P matrix and can be showed H_n^P=$T_p^{-1}(n)F_n$ then $T_p(n)\, H_n^P = F_n$.

II. Fibonacci Matrix, Lucas Matrix, and *p*-Tribonacci Matrix

In this part given the definition of Fibonacci matrix, Lucas matrix and matrix of p-Tribonacci. Lee [5] defines Fibonacci number in 2.1 Definition.

Definition 2.1 *(Fibonacci Number) Fibonacci Number is given by:*

$$\mathcal{F}_n = \begin{cases} 0, & if\ n = 0 \\ 1, & if\ n = 1 \\ \mathcal{F}_{n-1} + \mathcal{F}_{n-2}, & if\ \forall n \geq 2 \end{cases} \tag{1}$$

Fibonacci number can be formed into Fibonacci matrix $n \times n$.Lee [5] gives Definition 2.2 for Fibonacci matrix.

Definition 2.2 *(Fibonacci Matrix) for each natural number n, Fibonacci matrix* $n \times n$ with each of its entry $F_n = [l_{i,j}]$, for each $i, j = 1,2,3, \ldots, n$ defined as:

$$f_{i,j} = \begin{cases} \mathcal{F}_{i-j+1}, & if\ i - j \geq 0 \\ 0, & if\ i - j < 0 \end{cases} \tag{2}$$

Then from the definition of Fibonacci matrix at equation (2), as the example for $n = 6$ obtained Fibonacci matrix F_6 as follows:

$$F_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 1 & 0 & 0 \\ 5 & 3 & 2 & 1 & 1 & 0 \\ 8 & 5 & 3 & 2 & 1 & 1 \end{bmatrix} \tag{3}$$

Ramirez [9] defines recursively Lucas number in Definition 2.3.

Definition 2.3 *(Lucas Number) Lucas number is given by:*

$$\mathcal{L}_n = \begin{cases} 2, & if\ n = 0 \\ 1, & if\ n = 1 \\ \mathcal{L}_{n-1} + \mathcal{L}_{n-2}, & if\ \forall n \geq 2 \end{cases} \tag{4}$$

Lucas number can be formed into Lucas matrix $n \times n$. Z.Zhang [13] gives Definition 2.4 for Lucas matrix.

Definition 2.4 *(Lucas Matrix) for each natural number n, Lucas matrix* n x n with each of its entry $L_n = [l_{i,j}]$, for each $i, j = 1,2,3, \ldots, n$ defined as:

$$l_{i,j} = \begin{cases} \mathcal{L}_{i-j+1}, & if\ i - j \geq 0 \\ 0, & if\ i - j < 0 \end{cases} \tag{5}$$

Then from the definition of Lucas matrix at equation (5), as the example for $n = 6$ obtained the value of Lucas matrix L_6 as follows:

$$L_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 \\ 4 & 3 & 1 & 0 & 0 & 0 \\ 7 & 4 & 3 & 1 & 0 & 0 \\ 11 & 7 & 4 & 3 & 1 & 0 \\ 18 & 11 & 7 & 4 & 3 & 1 \end{bmatrix} \tag{6}$$

Kuhapatanakul [14] gave definition of p-Tribonacci number as mentioned in definition 2.5.

Definition 2.5 *(p-Tribonacci Number) For instance* $p \in N$. *p-Tribonacci number is given by:*

$$\mathcal{T}_p(n+2) = \mathcal{T}_p(n+1) + \mathcal{T}_p(n) + \mathcal{T}_p(n-p);\ \text{for } (n \in Z).$$

with $\mathcal{T}_p(0) = 0, \mathcal{T}_P(1) = 1$ and $\mathcal{T}_p(2) = 1$.

(7)

p-Tribonacci number can be formed into p-Tribonacci matrix $n \times n$. Kuhapatanakul [4] also defines p-Tribonacci matrix, matrix is defined by [4] is not bottom triangle matrix, then by reviewing the previous research such as [10],[11],[5], and [8] who involve bottom triangle matrix until it is different from Kuhapatanakul [4] then in this article p-Tribonacci matrix is defined as bottom triangle matrix that its entry is p-Tribonacci number and obtained the definition as follows:

Definition 2.6 *(p-Tribonacci Matrix) for each natural number n, p-Tribonacci matrix $n \times n$ with each entry of $T_p(n) = [t_{i,j}], \forall i; j = 1,2,3, \dots; n$* is defined as:

$$t_{p(i,j)}(n) = \begin{cases} \mathcal{T}_p(i+j-1) & if\ \ i \geq j \\ 0 & if\ \ i < j \end{cases}$$

(8)

Based on definition 2.6, it can be concluded that p-Tribonacci matrix $T_p(n)$ its main diagonal is 1 and determinant value (det) from p-Tribonacci matrix $T_p(n)$ is the multiplication result of its diagonal entries, until obtained $\det T_p(n) = 1$. Because $\det T_p(n) \neq 0$, then p-Tribonacci matrix $T_p(n)$ has invers. By using maple conducted invers calculation of p-Tribonacci matrix $T_p^{-1}(n)$, as example for p = 2 and n = 6 obtained invers of p-Tribonacci matrix as follows:

$$T_2^{-1}(6) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 \end{bmatrix}$$

(9)

From example (9) prevails entry pattern in column $t'_{p(i,j)}(n)$ with value 1,-1,-1 and 0. By using maple obtained that this pattern prevails for matrix n x n in which entry pattern $t'_{p(i,j)}(n)$ does not change. Therefore, given definition for invers of p-Tribonacci matrix as follows:

Definition 2.7 *for instance $T_p^{-1}(n)$ is invers of p-Tribonacci matrix for each natural number n with each entry from invers of p-Tribonacci matrix $T_p^{-1}(n) = [t'_{i,j}(p)]$* for each $i, j = 1, 2, 3, \dots, n$, is defined as:

$$t'_{p(i,j)}(n) = \begin{cases} 1, & jika\ i = j, \\ -1, & jika\ i-2 \leq j \leq i-j, \\ -1, & jika\ (i-j)-2 = p, \\ 0, & \quad otherwise \end{cases}$$

(10)

III. Relationship of Lucas Matrix and *p*-Tribonacci Matrix and Relationship of Fibonacci Matrix and *p*-Tribonacci Matrix

This part discusses relationship of Lucas matrix and p-Tribonacci matrix and relationship of Fibonacci matrix and p-Tribonacci matrix. Relationship of Lucas and p-Tribonacci matrix which involves p-Tribonacci matrix generates a matrix E_n^P for each n is the natural number then Relationship of Fibonacci matrix and p-Tribonacci matrix produces a matrix H_n^P for each n is the natural number.

A. Relationship of Lucas Matrix and p-Tribonacci Matrix

Based on definition 2.6 for $n = 6\ dan\ p = 2$ obtained invers value of p-Tribonacci $T_2^{-1}(6)$ as in example (9). Then from definition of Lucas matrix in equation (5) for $n = 6$ obtains the value of Lucas matrix in example (6). Then conducted invers multiplication of p-Tribonacci matrix $T_p^{-1}(n)$ with Lucas matrix, until from those matrix multiplication obtains matrix called as E_n^P matrix. For $n = 6$ and $p = 2$ and $n = 7$ and $p = 2$ obtain:

$$E_6^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ -1 & 0 & 0 & 2 & 1 & 0 \\ -3 & -1 & 0 & 0 & 2 & 1 \end{bmatrix} \quad E_7^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & 1 & 0 & 0 \\ -3 & -1 & 0 & 0 & 2 & 1 & 0 \\ -4 & -3 & -1 & 0 & 0 & 2 & 1 \end{bmatrix} \tag{11}$$

And with the same stages obtained for $n = 6$ and $p = 3$ and $n = 7$ and $p = 3$ obtained:

$$E_6^3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ -1 & 0 & 0 & 0 & 2 & 1 \end{bmatrix} \quad E_7^3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 2 & 1 & 0 \\ -3 & -1 & 0 & 0 & 0 & 2 & 1 \end{bmatrix} \tag{12}$$

By considering each entry on E_n^P matrix from equation (11) and (12) obtained that for different p value, the result of matrix multiplication obtained is also different, however it has the same pattern such as for $i - j = 1$ then $eij = 2$, for $i = j$ then $eij = 1$, and other 0, and for $i - j \geq p + 2$ all values of its entry listed in Table 3.1 and Table 3.2.

Table 3.1: Value of Matrix Element E_n^P $for\ p = 2$

Entry of E_n^2 Matrix	**Entry Value of E_n^2 Matrix**
e_{51}	$-1 \times 1 = -1$
e_{62}	$-1 \times L_1 = -1 \times 1 = -1$
e_{71}	$-1 \times L_3 = -1 \times 4 = -4$
e_{72}	$-1 \times L_{7-2-3} = -1 \times L_2 = -1 \times 3 = -3$
e_{73}	$-1 \times L_{7-3-3} = -1 \times L_1 = -1 \times 1 = -1$
$\vdots$	$\vdots$
e_{ij}	$= (-1)(L_{i-j-3})$

Table 3.2: The Element Value of Matrix E_n $for\ p = 3$

Entry of Matrix E_n^3	**Entry Value of Matrix E_n^3**
e_{61}	$-1 \times 1 = -1$
e_{71}	$-1 \times 3 = -3$
e_{72}	$-1 \times L_1 = -1 \times 1 = -1$
$\vdots$	$\vdots$
e_{ij}	$= (-1)(L_{i-j-3})$

Then by considering each step from Table 3.1 and 3.2 obtained definition of matrix E_n^P.

Definition 3.1 *for each n* $\in$ N, E_n^P is matrix *n x n* with each entry $E_n^P = [eij]$, $\forall i - j \geq p + 2$, defined as:

$$e_{ij} = (-1)(L_{i-j-3}) \tag{13}$$

By considering each entry at matrix E_n^p from equation (11) , (12) and definition 3.1 can be detracted Notion 3.2.

Notion 3.2 *For instance matrix E_n^p in equation (11) and (12), Lucas matrix in equation (5) and invers of p-Tribonacci matrix $T_p(n)$ in equation (10) in such way until $E_n^p = -1(L_n), untuk\ \ n \geq p+3$.*

(14)

Based on the definition of matrix E_n^p in equation (13), Lucas matrix in equation (5) and p-Tribonacci matrix in equation (8) can be detracted Theorem 3.3.

Theorem 3.3 *For example matrix E_n^p in equation (13), Lucas matrix in equation (5) and p-Tribonacci matrix $T_p(n)$ in equation (8) in such way until $L_n = T_p(n)E_n^p$..*

Proof. Because p-Tribonacci matrix has invers, will be proven that:

$$E_n^p = T_p^{-1}(n)L_n \tag{15}$$

For instance $T_p^{-1}(n)$ is the invers from p-Tribonacci matrix that is defined at equation (9) obtains the main diagonal from the invers of matrix $T_p^{-1}(n)$ =1. Next based on Lucas matrix in equation (5) obtains main diagonal of matrix $L_n = 1$. By considering the left side of equation (11) and (12), if $i = j$ then $e_{i,j} = 1$, if $i - j = 1$ then $e_{i,j} = 2$. Until for $i - j \geq p + 2$ obtained:

$$\begin{aligned} e_{i,j} &= t'_{p(i,k)}L_{k,j} \\ &= t'_{p(i,i)}L_{i,j} + t'_{p(i,i-1)}L_{i-1,j} + t'_{p(i,i-2)}L_{i-2,j} + t'_{p(i,i-3)}L_{i-3,j} + \cdots + t'_{p(i,n)}L_{n,j} \\ &= \textstyle\sum_{k=1}^{n} t'_{p(i,k)}L_{k,j} \end{aligned}$$

Thus obtained $T_p^{-1}(n)L_n = E_n^p$. Next from the definition of matrix E_n^p in equation (13), Lucas matrix L_n in equation (5) and p-Tribonacci matrix in equation (8), obtained $L_n = T_n(p)E_n^p$.

Then by using maple conducted the multiplication of Lucas matrix and invers of p-Tribonacci matrix and obtained the same matrix with matrix E_n^p until $T_n^{-1}(p)L_n = L_nT_n^{-1}(p) = E_n^p$ can be applied and the multiplication of Lucas matrix and invers matrix of commutative p-Tribonacci.

B. Relationship of Fibonacci Matrix and p-Tribonacci Matrix

By conducting the multiplication of Fibonacci matrix F_6 in equation (2) with invers of p-Tribonacci matrix in equation (8). Until obtained matrix that is called H_n^P matrix. For $n = 6$ and $p = 2$ and $n = 7$ and $p = 2$ obtained:

$$H_6^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad H_7^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 \\ -2 & -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

And with the same step obtained for $n = 6$ and $p = 3$ and $n = 7$ and $p = 3$ obtained:

$$H_6^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad H_7^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

Based on the calculation from equation (16) and (17) obtained that for each entry of matrix H_n^P is influenced by p value, however it remains owning the same pattern such as for $i = j$ then $hij = 1$, and 0 other, then for $i - j \geq p + 2$. all values of its entry listed in Table 3.3 and Table 3.4.

Table 3.3: The element value of matrix H_n^p $for\ p = 2$

The Entry of Matrix H_n^2	Entry value of Matrix H_n^2
h_{62}	$-1 \times f_1 = -1 \times 1 = -1$
h_{71}	$-1 \times f_3 = -1 \times 2 = -2$
h_{72}	$-1 \times f_{7-2-3} = -1 \times f_2 = -1 \times 1 = -1$
h_{73}	$-1 \times f_{7-3-3} = -1 \times f_1 = -1 \times 1 = -1$
$\vdots$	$\vdots$
h_{ij}	= (-1)(f_{i-j-3})

Table 3.4: The element value of matrix H_n^p $for\ p = 3$

The Entry of Matrix H_n^3	Entry Value of Matrix H_n^3
h_{61}	$-1 \times 1 = -1$
h_{71}	$-1 \times 1 = -1$
h_{72}	$-1 \times f_1 = -1 \times 1 = -1$
$\vdots$	$\vdots$
h_{ij}	= (-1)(f_{i-j-3})

Then by considering each step from Table 3.3 and Table 3.4 obtained the definition of matrix H_n^p.

Definition 3.4 for each $n \in$ N, H_n^p is the matrix n×n with each entry H_n^p = [hi,j], $\forall\ i - j \geq p + 2$, defined as:

$$h_{ij} = (-1)(f_{i-j-3}) \tag{18}$$

By considering each entry in matrix H_n^p from equation (16) and (17) and the definition 3.4 can be detracted Notion 3.5.

Notion 3.5 *For instance matrix H_n^p in equation (16) and (17), Fibonacci matrix in equation (2) and invers of p-Tribonacci matrix $T_p(n)$ in equation (10) in such way until $H_n^p = -1(F_n), untuk\ \ n \geq p + 3$.*

(19)

Based on the definition of matrix H_n^p in equation (18), Fibonacci matrix in equation (2) and p-Tribonacci in equation (8) can be detracted Theorem 3.6.

Theorem 3.6 *For instance matrix H_n^p in equation (18), Fibonacci matrix which in equation (2) and p-Tribonacci matrix $T_p(n)$ in equation (6) in such way until $F_n = T_p(n)H_n^p$.*

Proof. Because p-Tribonacci matrix has invers, will be proven that:

$$H_n^p = T_p^{-1}(n)F_n \tag{20}$$

For instance $T_p^{-1}(n)$ is the invers of p-Tribonacci matrix which is defined at equation (9) obtained the main diagonal of matrix $T_p^{-1}(n)$ = 1 invers. Next based on equation (2) obtained the main diagonal of matrix $F_n = 1$. By considering the left side at equation (16) and (17), if $i = j$ then $h_{i,j} = 1$, then 0 other. Until for $i - j \geq p + 2$ based equation (9) and equation (2) obtained:

$$\begin{aligned} h_{i,j} &= t'_{p(i,k)} f_{k,j} \\ &= t'_{p(i,i)} f_{i,j} + t'_{p(i,i-1)} f_{i-1,j} + t'_{p(i,i-2)} f_{i-2,j} + t'_{p(i,i-3)} f_{i-3,j} + \cdots + t'_{p(i,n)} f_{n,j} \\ &= \textstyle\sum_{k=1}^{n} t'_{p(i,k)} f_{k,j} \end{aligned}$$

Thus obtained $T_p^{-1}(n)F_n = H_n^p$. Next from the definition of matrix H_n^p in equation (18), Fibonacci matrix at equation (2) and p-Tribonacci matrix in equation (6), obtained $F_n = T_p(n)H_n^p$ ■

Then by using maple also conducted the multiplication of Fibonacci matrix in equation (2) and invers of p-Tribonacci matris in equation (9) and obtained the same matrix with matrix H_n^p until $T_n^{-1}(p)F_n = F_nT_n^{-1}(p) = H_n^p$ can be applied in the multiplication of Fibonacci matrix and invers matrix comutative p-Tribonacci.

IV. CONCLUSIONS

This paper discusses Relationship of Lucas matrix and p-Tribonacci matrix and Relationship of Fibonacci matrix and p-Tribonacci matrix. Then, from these Relationships constructed the matrix that is called as matrix E_n^p and matrix H_n^p.

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