On sgωα-Separation Axioms, sgωα-regular and sgωα- normal spaces

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Abstract

The present paper introduces a new class of separation axioms called a sgωα- separation axioms using sgωα-open sets. Also two new classes of spaces called sgωα- regular and sgωα-normal spaces are introduced and studied their fundamental properties, relationships and characterizations by utilizing sgωα-open and sgωα-closed sets.

Key Words: *semi open set, sgωα-open set, sgωα-closed set, sgωα-closed/open /continuous mappings, semi irresolute function, sgωα-T_k (0,1,2) spaces, sgωα-regular spaces, sgωα-normal spaces.*

I. INTRODUCTION

The notion of semi open set plays a significant role in general topology. In 1963 Levine N [6] started the study of generalized open sets with the introduction of semi open sets. Then Njastad [1] introduced and defined α-open/ closed sets. Benchlli et.al.[8] introduced and studied gωα-closed sets. Recently Rajeshwari K et.al.[9] introduced and defined sgωα-open/closed sets. Also Rajeshwari K et.al.[5] defined and studied sgωα-open/closed functions, sgωα- homeomorphism in topological spaces.

In 1970 Levine [7] generalized the concept of closed sets to generalized closed sets. Since then many topologists have utilized these concepts to the various notions of subsets, weak separation axioms, weak regularity, weak normality, weaker and stronger forms of covering axioms in the literature. Maheshwari and Prasad [10,11] introduced and studied the concept of s-regular and s-normal spaces in topology. Munshi [4] introduced and studied the notions of g-regular and g- normal spaces in topology.

In this paper we introduce sgωα-separation axioms using the concepts of sgωα-open sets called sgωα-T_k spaces. Also we define and study the two new classes of spaces called sgωα-regular, sgωα-normal spaces in topology and we characterize some of their basic properties.

II. PRELIMINARIES

Throughout this paper (X, τ) , (Y, σ) and (Z, η) represent non empty topological spaces on which no separation axioms are assumed unless explicitly stated and they are simply written X,Y and Z respectively. For a subset A of (X, τ) the closure of A, the interior of A with respect to τ are denoted by cl(A), int(A) respectively. The compliment of A is denoted by A^c.

The following definitions and results are listed because of their use in the sequel.

Definition 2.1: A subset A of a topological space (X, τ) is called semi-open [6] (resp. pre-open[13] and α-open [1]) set if A $\subseteq$ cl(int(A)) (resp. A$\subseteq$ int(cl(A)) and A $\subseteq$ int(cl(int(A))). The compliment of semi-open (resp. pre-open and α-open) set is called semi-closed (resp. pre-closed and α -closed) set.

For a subset A of X, the intersection of all semi-closed (resp. semi-open) subsets of (X,τ) containing A is called semi-closure (resp. semi-kernel) of A and is denoted by scl(A) (resp. sker(A)) and semi-interior of A is the union of all semi-open sets contained in A in (X,τ) and is denoted by sint(A). A subset A of a space X is said to be semi-regular if A is both semi-open and semi-closed.

Definition 2.2: A subset A of a topological space (X, **τ**) is called

i) ω-closed [12] if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
ii) ωα-closed [15] if αcl(A) ⊆ U whenever A ⊆ U and U is ω-open in X.
iii) generalized ωα-closed (briefly gωα-closed) set[8] if αcl(A) ⊆ U whenever A ⊆ U and U is ωα-open in X.
iv) semi generalized ωα-closed[9] (briefly sgωα-closed) if scl(A) ⊆ U whenever A ⊆ U and U is ωα- open in X.

Definition 2.3 [9]: Let A ⊆ X

i) The intersection of all sgωα- closed sets containing A is called sgωα-closure (sgωα-cl) of A and is denoted by sgωα-cl(A).
ii) A subset A of X is called sgωα-neighbourhood of a point x in X if there exists a sgωα- open set U such that x∈ U ⊆ A.

Theorem 2.4 [9]: Let A be a subset of a space X and x ∈ X. The following properties hold for the sgωα-cl(A):

1) x ∈ sgωα-cl(A) if and only if $A \cap U \neq \emptyset$, for every U ∈ SO(X) containing x.
2) A is sgωα-closed if and only if A = sgωα-cl(A).
3) sgωα-cl(A) is sgωα-closed.
4) sgωα-cl(A) ⊆ sgωα-cl(B) if A ⊆ B.
5) sgωα-cl(sgωα-cl(A)) = sgωα-cl(A).

Definition 2.5: A map f : X → Y is called

i) semi continuous [20] if $f^{-1}(A)$ is semi open in X for every open set A in Y.
ii) sgωα-continuous [16] if $f^{-1}(A)$ is sgωα-open in X for every open set A in Y.
iii) sgωα-irresolute [16] if $f^{-1}(A)$ is sgωα-closed in X for every sgωα-closed set A in Y.
iv) sgωα-closed [5] if f(A) is sgωα-closed in Y for every closed set A in X.
v) semi-irresolute [19] if $f^{-1}(A)$ is semi closed in X for every semi closed set A in Y.

III. Sgωα-T_k SPACES (k=0,1,2)

The following are the definitions concerned with sgωα-T_k spaces where (k=0,1,2).

Definition 3.1: A space (X, **τ**) is called

i) sgωα-T_0 space if for each x ,y ∈ X with x≠y, there exists sgωα-open set G in X such that x ∈ G ,y∉ G.

ii) sgωα-T_1 space if for each x, y ∈ X with x≠ y there exists sgωα-open set U such that x∈ U, y∉U and sgωα-open set V such that y ∈ V, x∉V.

iii) sgωα-T_2 space(sgωα-Hausdorff) if for each x ,y ∈ X with x≠y there exist disjoint sgωα-open sets U and V such that x ∈ U , y ∈ V .

Theorem 3.2: A space (X,τ) is sgωα-T_0 space if and only if for each pair of points x, y ∈ X with x ≠ y, sgωα-cl({x}) ≠ sgωα-cl({y}).

Proof: Let (X,τ) be sgωα-T_0 space .Let x, y ∈ X such that x ≠ y,then there exists sgωα-open set V containing one of the point but not the other, say x ∈V and y∉V. Then V^c is sgωα-closed set containing y but not x. But sgωα-cl({y}) is the smallest closed set containing y. Therefore sgωα-cl({y}) ⊂ V^c and hence x ∉ sgωα-cl({y}) .Thus, sgωα-cl({x}) ≠ sgωα-cl({y}).

Conversely, suppose x ,y ∈ X, x ≠ y and sgωα-cl({x}) ≠ sgωα-cl({y}). Let z ∈ X such that z ∈sgωα-cl({x}),but z ∉ sgωα-cl({y}). If x ∈ sgωα-cl({y}) then sgωα-cl({x}) ⊂ sgωα-cl({y}) and hence z ∈ sgωα-cl({y}) which is a contradiction. Therefore x ∉ sgωα-cl({y}). Thus x ∈ $(\text{sgωα-cl}(\{y\}))^c$. So, $(\text{sgωα-cl}(\{y\}))^c$ is sgωα-open set containing x but not y. Hence (X,τ) is sgωα-T_0 space.

Theorem3.3:A topological space X is sgωα-T_1 space if and only if for every x ∈ X, singleton{x}is sgωα-closed in X.

Proof: Let X be sgωα-T_1 space and let x ∈ X .We shall prove that X-{x} is sgωα-open..

Let y ∈ X-{x}, implies x ≠ y. Since X is sgωα-T_1 space there exist sgωα- open sets U,V such that x ∈ U, y ∉ U and y ∈ V ⊆ X-{x}. This implies X-{x} is sgωα-open set. Hence {x} is sgωα- closed set.

Conversely, let x ≠ y ∈ X. Then{x}, {y} are sgωα-closed sets. So X-{x} is sgωα-open set. Clearly x ∉ X-{x}and y ∈ X-{x}.Similarly X-{y} is sgωα-open set, y ∉ X-{y} and x ∈ X-{y}. Hence X is sgωα-T_1 space.

Theorem3.4: For a topological space (X,τ), the following statements are equivalent:

i) (X,τ) is sgωα-T_2 space.

ii) If x ∈ X, then for each y ≠ x, there is a sgωα-open set U containing x such that y ∉ sgωα-cl(U).

Proof: i) → ii) Let $x \in X$. If $y \in X$ is such that $y \neq x$ then there exists disjoint sgωα-open sets U and V such that $x \in U$, $y \in V$.Then $x \in U \subseteq X\text{-}V$ which implies X-V is sgωα-open and $y \notin X\text{-}V$. Therefore, $y \notin$ sgωα-cl(U).

ii) → i) Let $x, y \in X$ such that $x \neq y$. By ii) there exists sgωα-open set U containing x such that $y \notin$ sgωα-cl(U). Therefore $y \in$ X- (sgωα-cl(U)) , X- (sgωα-cl(U)) is sgωα-open and $x \notin$ X- (sgωα-cl(U)). Also U ∩ [X- (sgωα- cl(U))] = ∅ . Hence, (X,τ) is sgωα-T_2 space.

Theorem 3.5: If $f : X \to Y$ is injective and sgωα- irresolute function and Y is sgωα-T_0 space then X is sgωα-T_0.

Proof: Let $x,y \in X$ with $x \neq y$.Since f is injective and Y is sgωα-T_0, there exists is sgωα-open set U in Y such that $f(x) \in U$ and $f(y) \notin U$ or there exists a sgωα-open set V in Y such that $f(y) \in V$ and $f(x) \notin V$ with $f(x) \neq f(y)$. Since f is sgωα-irresolute $f^{-1}(U)$, $f^{-1}(V)$ are sgωα- open sets in X such that $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$ or $y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$. This shows that X is sgωα-T_0..

Theorem 3.6: If $f : X \to Y$ is injective and sgωα-irresolute function and Y is sgωα-T_1 space then X is sgωα-T_1.

Proof: The proof is similar to the proof of above Theorem.

Theorem 3.7: Let X be a topological space and Y be a sgωα-T_2 space. If $f : X \to Y$ is injective and sgωα- irresolute then X is sgωα-T_2 space.

Proof: Let $x, y \in X$ such that $x \neq y$. Since f is injective $f(x) \neq f(y)$. As Y is sgωα-T_2 space there exist disjoint sgωα-open sets U and V in Y such that $f(x) \in U$, $f(y) \in V$. Since f is sgωα- irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are sgωα-open sets in X such that $x \in f^{-1}(U)$, $y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence, X is sgωα-T_2 space.

Theorem 3.8: Let X be a topological space and Y a T_2 space. If $f : X \to Y$ is injective and sgωα- continuous then X is sgωα-T_2 space.

Proof: Suppose $x, y \in X$ such that $x \neq y$. Since f is injective $f(x) \neq f(y)$. As Y is T_2 space, there exist disjoint open sets U and V in Y such that $f(x) \in U$, $f(y) \in V$. Since f is sgωα-continuous, $f^{-1}(U)$, $f^{-1}(V)$ are sgωα-open sets in X such that $x \in f^{-1}(U)$, $y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence, X is sgωα- T_2 space.

Theorem 3.9: Every open subspace of sgωα-T_2 space is sgωα-T_2.

Proof: Let Y be an open subspace of sgωα-T_2 space (X,τ). Let x, y be distinct points in Y. Since $Y \subseteq X$, $x, y \in X$. Then there exist disjoint sgωα - open sets U and V in X such that $x \in U$, $y \in V$. So $Y \cap U$ and $Y \cap V$ are sgωα-open sets in Y such that $x \in Y \cap U$, $y \in Y \cap V$. Also $(Y \cap U) \cap (Y \cap V) = \emptyset$.Thus, subspace (Y, τ_Y) is sgωα-T_2.

IV. Sgωα-REGULAR SPACES

In this section we introduce sgωα-regular spaces in topological space. We obtain several characterizations of sgωα-regular spaces.

Definition 4.1: A space (X,τ) is said to be sgωα-regular if for every sgωα-closed set F and a point $x \notin F$, there exist disjoint semi-open sets U and V such that $F \subseteq U$ and $x \in V$.

Theorem 4.2: For a topological space (X,τ) the following are equivalent:

i) (X,τ) is sgωα-regular.

ii) Every sgωα-open set U is a union of semi-regular sets.

iii) Every sgωα-closed set A is an intersection of semi-regular sets.

Proof: i) → ii) Let U be a sgωα-open set and let $x \in U$. If A= X- U then A is sgωα-closed. As (X,τ) is sgωα-regular there exist disjoint semi open subsets G_1 and G_2 of X such that $x \in G_1$ and $A \subseteq G_2$. If V= scl(G_1), then V is semi-closed and $V \cap A \subseteq V \cap G_2 = \emptyset$. It follows that $x \in V \subseteq U$. Thus, U is a union of semi-regular sets.

ii) → iii) : This is obvious.

iii) → i): Let A be sgωα-closed and let $x \notin A$. By assumption , there exists a semi regular set V such that $A \subseteq V$ and $x \notin V$. If U = X-V, then U is semi open set containing x and $U \cap V = \emptyset$. Thus (X,τ) is sgωα-regular.

Theorem4.3: Suppose $B \subseteq A \subseteq X$, B is sgωα-closed relative to A and that A is open and sgωα-closed in (X,τ) .Then B is sgωα-closed in (X,τ).

Theorem 4.4: If (X,τ) is sgωα-regular space and Y is an open and sgωα-closed subset of (X,τ) , then the subspace Y is sgωα- regular.

Proof: Let F be any sgωα-closed subset of Y and let $y \notin F$. By Theorem 4.3 F is sgωα closed in (X,τ) . Since (X,τ) is sgωα-regular , there exist disjoint semi-open sets U and V of (X,τ) such that $y \in U$ and $F \subseteq V$. As Y is open and hence semi open, we get $U \cap Y$ and $V \cap Y$ as disjoint semi open subsets of the space Y such that $y \in Y \cap U$ and $F \subseteq Y \cap V$. Hence, the subspace Y is sgωα-regular.

Theorem 4.5: Let (X,τ) be a topological space. Then, the following statements are equivalent:

i) (X,τ) is sgωα-regular.

ii) For each point $x \in X$ and for each sgωα- neighbourhood N of x, there exists a semi open set V of x such that scl(V) $\subseteq$ N.

iii) For each point $x \in X$ and for each sgωα-closed set F not containing x ,there exists a semi open set V of x such that $scl(V) \cap F = \emptyset$.

Proof: i) → ii): Let N be any sgωα-neighbourhood of x . Then there exists a sgωα-open set G such that $x \in G \subseteq N$. Since G^c is sgωα-closed and $x \notin G^c$, by hypothesis, there exist semi open sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \emptyset$ and so $V \subseteq U^c$. Now $scl(V) \subseteq scl(U^c) = U^c$ and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq N$. Therefore, $scl(V) \subseteq N$.

ii)→ iii) Let $x \in X$ and F be a sgωα-closed set such that $x \notin F$. Then F^c is a sgωα-neighbourhood of x and by hypothesis, there exists a semi open set V of x such that $scl(V) \subseteq F^c$ and hence $scl(V) \cap F = \emptyset$.

iii)→ i) : Let $x \in X$ and F be a sgωα-closed set such that $x \notin F$. By hypothesis there exists semi open set V of x such that $scl(V) \cap F = \emptyset$. This implies that $F \subseteq (scl(V))^c$ which is semi open. Also $V \cap (scl(V))^c = \emptyset$. Thus, (X, τ) is sgωα-regular.

Theorem4.6 : A topological space (X,τ) is sgωα-regular if and only if for each sgωα-closed set F of (X,τ) and each $x \notin F$, there exist semi open sets U and V of (X,τ) such that $x \in U$, $F \subseteq V$ and $scl(U) \cap scl(V) = \emptyset$.

Proof: Let F be a sgωα-closed set of (X,τ) and $x \notin F$. Then there exist semi-open sets U_x and V such that $x \in U_x$, F $\subseteq$V and $U_x \cap V = \emptyset$ which implies that $U_x \cap scl(V) = \emptyset$. Since (X,τ) is sgωα- regular there exists semi open sets G and H of (X,τ) such that $x \in G$, $scl(V) \subseteq H$ and $G \cap H = \emptyset$. This implies $scl(G) \cap H = \emptyset$. Now let us take $U=U_x \cap G$, then U and V are semi open sets of (X,τ) such that $x\in U$ and $F \subseteq V$ and $scl(U) \cap scl(V) = \emptyset$.

Converse is obvious.

Theorem 4.7: Let X and Y be topological spaces and let Y be sgωα- regular space. If $f: X \to Y$ is sgωα-closed, semi-irresolute and one to one function then X is sgωα-regular space.

Proof: Let F be closed set in X, hence sgωα-closed in X. Let $x \notin F$. Since f is sgωα-closed mapping, it follows that f(F) is sgωα-closed set in Y, $f(x) = y \notin f(F)$. As Y is sgωα-regular space, there exist semi open sets U,V in Y such that $y\in U$, $f(F) \subseteq V$ and $U \cap V = \emptyset$. Since f is semi-irresolute mapping and one to one it follows that $f^{-1}(U)$, $f^{-1}(V)$ are two semi-open sets in X such that $x \in f^{-1}(U)$, $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ .Thus, X is sgωα-regular space.

V. Sgωα-NORMAL SPACE

In this section we introduce and study the weak form of normality called sgωα-normality in topological spaces.

Definition 5.1: A topological space (X,τ) is said to be sgωα -normal if for any pair of disjoint sgωα -closed sets A and B in X there exist disjoint semi open sets U ,V such that $A\subseteq U$, $B\subseteq V$.

Theorem 5.2: If (X,τ) is sgωα -normal space and Y is an open and sgωα-closed subset of (X,τ) then the subspace Y is sgωα-normal.

Proof: Let A and B be any two disjoint sgωα -closed sets in Y. By Theorem 4.3 A and B are sgωα-closed in (X,τ) . Since $(,\tau)$ is sgωα-normal, there exist disjoint semi open sets U and V of (X,τ) such that A $\subseteq$ U and B $\subseteq$ V. Since Y is open and hence semi open, Y∩ U and Y ∩ V are disjoint semi open sets of the subspace Y such that A $\subseteq$ Y∩ U and B $\subseteq$ Y ∩ V . Hence, the subspace Y is sgωα -normal.

Now we characterize the sgωα -normal spaces.

Theorem 5.3 : Let (X,τ) be a topological space. Then the following statements are equivalent:

i) (X,τ) sgωα-normal

ii) Foe each sgωα-closed set F and for each sgωα -open set U containing F, there exists a semi open set V containing F such that scl(V) $\subseteq$ U.

iii) For each pair of disjoint sgωα-closed sets A and B in (X,τ) , there exists a semi open set U containing A such that scl(U) ∩ B = ∅ .

iv) For each pair of disjoint sgωα-closed sets A and B in (X,τ) , there exists semi open sets U and V such that A $\subseteq$ U, B $\subseteq$ V and scl(A) ∩ scl (B) = ∅.

Proof: i) → ii) Let F be a sgωα-closed set and U be a sgωα-open set such that F $\subseteq$ U. Then F ∩ U^c = ∅ . By assumption, there exist semi open sets V and W such that F $\subseteq$ V, U^c $\subseteq$ W and V∩ W = ∅, which implies scl(V) ∩W = ∅ .Now scl (V) ∩ U^c $\subseteq$ scl(V) ∩ W = ∅ and so scl(V) $\subseteq$ U.

ii) →iii): Let A and B be any two sgωα-closed sets of (X,τ) such that A ∩ B =∅ .Then A $\subseteq$ B^c and B^c is sgωα-open . By assumption there exists semi open set U containing A such that scl (U) $\subseteq$ B^c and so scl(U) ∩ B = ∅ .

iii) → iv) : Let A and B be any two disjoint sgωα-closed sets of (X,τ) . Then by assumption , there exists a semi open set U containing A such that scl(U) ∩ B = ∅ . Since scl(A) is semi closed, it is sgωα-closed and so B and scl(A) are disjoint sgωα-closed sets in (X,τ) .Therefore, by assumption there exists semi open set V containing B such that scl(A) ∩ scl(B) = ∅.

iv) → i) Let A and B be any disjoint sgωα-closed sets of (X,τ) . By assumption there exists semi open sets U and V such that A $\subseteq$ U and B $\subseteq$ V and scl(U) ∩ scl(V) = ∅ , so U ∩ V = ∅ and thus (X,τ) is sgωα- normal.

Theorem 5.4: If f : $(X, \tau) \rightarrow (Y ,\sigma)$ is sgωα -closed and semi- irresolute function and Y is sgωα-normal, then X is sgωα-normal.

Proof: Let A and B be any two disjoint closed sets of (X, τ) . Since every closed set is sgωα- closed it follows that A and B are disjoint sgωα-closed sets of (X, τ). As f is sgωα-closed map, f(A)and f(B) are sgωα-closed sets in Y. Since

Y is sgωα-normal there exist semi open sets U,V in Y such that f(A)⊆ U, f(B) ⊆ V and U ∩ V = ∅ .Since f is semi-irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are semi open sets in (X, τ) such that A⊆ $f^{-1}(U)$, B⊆ $f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Thus, X is sgωα- normal.

VI. CONCLUSION

Separation Axioms, sgωα- regular/normal spaces in terms of sgωα-open sets have been formulated and their structural properties have also been discussed and emphasized which opens the further scope of respective compact spaces, connected spaces and bi topological spaces.

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