Some New Fractional Calculus Results with I-Function

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Abstract)* — *In the present work we introduce a composition formula of the caputo – type MSM fractional derivatives with - I function. The obtained results are in terms of I – function. Certain special cases of the main results given in this paper generalize several known results obtained recently.

Keywords — *I-function, Caputo – type MSM fractional derivatives, Fractional Derivative.*

I. INTRODUCTION

Significant and importance of the fractional integral operators involving various special functions, have found applications in various sub-fields of applicable mathematical analysis. Since last four decades, a number of workers like Love [16], Marichev [9], Kalla [17], Kalla and Saxena [20,21], Saigo and Maeda [17]Srivastava[26,27] ,Kataria [20] and Vellaisamy[28] , Mishra[4,5,7,31] etc. have studied in depth, the properties, applications and different extensions of various hypergeometric operators of fractional integration

The I-function is defined by Rathie[3] which is represented by the following manner:

$$I^{m,n}_{p,q}\left[Z\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L z^{-s}\chi(s)\,ds\ (z\neq 0) \tag{1.1}$$

$$\chi(s)=\frac{\prod_{j=1}^{m}\Gamma^{\beta_j}(b_j+B_js)\prod_{j=1}^{n}\Gamma^{\alpha_i}(1-a_j-A_js)}{\prod_{j=m+1}^{q}\Gamma^{\beta_j}(1-b_j-B_js)\prod_{j=n+1}^{p}\Gamma^{\alpha_i}(a_j+A_is)} \tag{1.2}$$

It may be noted that m,n, p,q are integers lies between $0 \le m \le q$ and $0 \le n \le p$ and $\chi(s)$ denotes the fractional power of some of the gamma function with α_i, A_j are positive real numbers for i = 1,2,..., p and β_j, B_j are also positive real numbers for j = 1,2,...,q. Also, a_i's and b_j's are complex numbers .

The sufficient conditions of convergence of contour L which is defined in (1.1) and other representations, details of I- function can be seen in [3]. A useful generalization of I-function of several complex variables studied (see [9]) and including its applications in wireless communication can be found in the research work [15].

Throughout our present investigation, we use the following standard notations:

The corresponding fractional differential operators [17] have their respective form as

$$\left(\mathcal{D}^{\alpha,\alpha',\beta,\beta',\gamma}_{0+}f\right)(x)=\left(\frac{d}{dx}\right)^{[Re(\gamma)]+1}\left(I^{-\alpha',-\alpha,-\beta'+[Re(\gamma)]+1,-\beta,-\gamma+[Re(\gamma)+1]}_{0+}f\right)(x) \quad \dots (5)$$

$$\left(\mathcal{D}_{-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\left(-\frac{d}{dx}\right)^{[Re(\gamma)]+1}\left(I_{-}^{-\alpha',-\alpha,-\beta'+[Re(\gamma)]+1,-\beta,-\gamma+[Re(\gamma)+1]}f\right)(x)$$

… (6)

where m = [Re(γ)]+ 1 and [Re(γ)] denotes the integer part of Re(γ).

Saigo [14] introduced the fractional integral and differential operators involving Gauss hyper geometric function $_2F_1$ as the kernel. For $\alpha,\beta,\gamma \in \mathbb{C}$ and $x > 0$ with $Re(\alpha) > 0$, the left- and right-hand sided Saigo fractional integral operators are defined by

$$\left(I_{0+}^{\alpha,\beta,\gamma}f\right)(x)=\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}{}_2F_1\left(\alpha+\beta,-\gamma;\alpha;1-\frac{t}{x}\right)f(t)dt$$

… (7)

and

$$\left(I_{-}^{\alpha,\beta,\gamma}f\right)(x)=\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}t^{-\alpha-\beta}{}_2F_1\left(\alpha+\beta,-\gamma;\alpha;1-\frac{t}{x}\right)f(t)dt,$$

… (8)

respectively.

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}t^{-\alpha'}F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt$$

… (9)

And

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\frac{x^{-\alpha'}}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}t^{-\alpha}F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt$$

… (10)

respectively.

The third Appell function $F3$ (also known as Horn function [15]), is defined by

$$F_3(\alpha,\alpha',\beta,\beta';\gamma;x;y)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_m(\alpha')_n(\beta)_m(\beta')_n}{(\gamma)_{m+n}}\frac{x^m y^n}{m!\,n!}$$

such that max {|x|,|y|} < 1.

Where $(z)_n$ is the Pochhammer symbol, which is defined as

$$(z)_n=\begin{cases}1 & if\ n=0,\\ z(z+1)(z+2)\dots(z+(n-1)) & if\ n\in\mathbb{N}.\end{cases}$$

,for $z \in \mathbb{C}$

The corresponding fractional differential operators are

$$\left(\mathcal{D}_{0+}^{\alpha,\beta,\gamma}f\right)(x)=\left(\frac{d}{dx}\right)^{[Re(\alpha)]+1}\left(I_{0+}^{-\alpha+[Re(\alpha)]+1,-\beta-[Re(\alpha)-1,\alpha+\gamma-[Re(\alpha)]-1]}f\right)(x) \quad \dots (11)$$

$$\left(\mathcal{D}_{-}^{\alpha,\beta,\gamma}f\right)(x)=\left(-\frac{d}{dx}\right)^{[Re(\alpha)]+1}\left(I_{-}^{-\alpha+[Re(\alpha)]+1,-\beta-[Re(\alpha)]-1,\alpha+\gamma}f\right)(x) \quad \dots (12)$$

where now m = [Re(α)]+1. For $\beta = -\alpha$ and $\beta = 0$ in (7)-(10), we get the corresponding Riemann – Liouville and Erdelyi – Kober fractional operators respectively

The Gauss hyper geometric function is related to third Appell function as

$$F_3(\alpha,\gamma-\alpha,\beta,\gamma-\beta,\gamma;x;y) = {}_2F_1(\alpha,\beta,\gamma\,;\,x+y-xy).$$

The MSM fractional operators (3)-(6) are connected to Saigo operators (7)-(10) by

$$\left(I_{0+}^{\alpha,0,\beta,\beta',\gamma}f\right)(x)=\left(I_{0+}^{\gamma,\alpha-\gamma,-\beta}f\right)(x),\quad \left(I_{-}^{\alpha,0,\beta,\beta',\gamma}f\right)(x)=\left(I_{0+}^{\gamma,\alpha-\gamma,-\beta}f\right)(x) \quad \dots (13)$$

and

$$\left(\mathcal{D}_{0+}^{0,\alpha',\beta,\beta',\gamma}f\right)(x)=\left(\mathcal{D}_{0+}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x),\quad \left(\mathcal{D}_{-}^{0,\alpha',\beta,\beta',\gamma}f\right)(x) = \left(\mathcal{D}_{-}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x) \quad \dots (14)$$

II In Section 2, some preliminary results are stated, which will be used in the proofs of subsequent theorems.

The following are well known results for MSM integral operators of power functions (see [14]).

Lemma 2.1. Let α,α',β,β',γ, ρ ϵ ℂ such that Re(γ) > 0.

(a) If Re(ρ) > max{0,Re(α' –β'), Re(α +α'+β –γ)}, then

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\right)(x) = \frac{\Gamma(\rho)\Gamma(-\alpha'+\beta'+\rho)\Gamma(-\alpha-\alpha'-\beta+\gamma+\rho)}{\Gamma(\beta'+\rho)\Gamma(-\alpha-\alpha'+\gamma+\rho)\Gamma(-\alpha'-\beta+\gamma+\rho)}x^{-\alpha-\alpha'+\gamma+\rho-1}$$

(b) If Re(ρ) > max{Re(β),Re(−α –α'+γ),Re(−α –β'+γ)}, then

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{-\rho}\right)(x) = \frac{\Gamma(-\beta+\rho)\Gamma(\alpha+\alpha'-\gamma+\rho)\Gamma(\alpha+\beta'-\gamma+\rho)}{\Gamma(\rho)\Gamma(\alpha-\beta+\rho)\Gamma(\alpha+\alpha'+\beta'+\gamma+\rho)}x^{-\alpha-\alpha'+\gamma-\rho} \quad \dots (16)$$

In subsequent theorems, the conditions for the absolute convergence of the integral involved in (1) are assumed. Also, the contour C of integration is assumed to be the imaginary axis, *i.e.*, Re(s) = 0.

III. MARICHEV-SAIGO-MAEDA (MSM) FRACTIONAL INTEGRATION REPRESENTAIONS RELATED TO THE I -FUNCTION

Our first set of results are contained in Theorem 3.1 below which related to the left-hand sided MSM fractional integration of the I -function.

Theorem 3.1. The following MARICHEV-SAIGO-MAEDA (MSM) FRACTIONAL INTEGRATION representation formulas hold true for the $I-$function :

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{9-1}I_{p,q}^{m,n}\left[at^{u}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$

$$= x^{-\alpha-\alpha'+\gamma+\rho-1}I_{p+3,q+3}^{m,n+3}\left[ax^{\mu}\left|\begin{matrix}(1-\rho,\mu,1) & (1+\alpha'-\beta'-\rho,\mu,1)\\(b_j,B_j,\beta_j)_{1,q} & (1-\beta'-\rho,\mu,1)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1+\alpha+\alpha'+\beta-\gamma-\rho,\mu 1) & (a_i,A_i,\alpha_i)_{1,p}\\(1+\alpha+\alpha'-\gamma-\rho,\mu,1) & (1+\alpha'+\beta-\gamma-\rho,\mu,1)\end{matrix}\right] \quad \ldots (17)$$

Provided that x > 0 and each member of (17) exist i.e. α,α′,β,β′,γ,ρ,a ∈ ℂ be suchthatRe(γ),μ>0and Re(ρ) > max{0, Re(α0 −β 0),Re(α +α0+β −γ)}.

Proof. The left-hand side (lhs) of (15) is given by

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\frac{1}{2\pi i}\int_C \mathcal{X}(s)(at^{\mu})^{-s}ds,\right)\right)(x), \quad \ldots (18)$$

where $\chi(s)$ is given by (2). Interchanging the order of integration and using (15), (18) is equal to

$$\frac{1}{2\pi i}\int_C \mathcal{X}(s)a^{-s}\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-\mu s-1}\right)(x)ds$$

$$= x^{-\alpha-\alpha'+\gamma+\rho-1}\frac{1}{2\pi i}\int_C \mathcal{X}(s)\mathcal{X}_1(s)(ax^{\mu})^{-s}ds,$$

Where

$$\mathcal{X}_1(s) = \frac{\Gamma(\rho-\mu s)\Gamma(-\alpha'+\beta'+\rho-\mu s)\Gamma(-\alpha-\alpha'-\beta+\gamma+\rho-\mu s)}{\Gamma(\beta'+\rho-\mu s)\Gamma(-\alpha-\alpha'+\gamma+\rho-\mu s)\Gamma(-\alpha'-\beta+\gamma+\rho-\mu s)}$$

The result now follows from (1).

In view of (13), we have the following result for Saigo operators.

Corollary 3.2. For the I-function defined by (1), the following special cases of Theorem 1 were derived in [18]: Let α, β, γ, ρ, a ∈ ℂ be such that Re(α),μ > 0 and Re(ρ) > max{0, Re(β −γ)}. Then the left-hand sided generalized fractional integration $I_{0+}^{\alpha,\beta,\gamma}$ of the I-function is given for $x > 0$ by

$$\left(I_{0+}^{\alpha,\beta,\gamma}\left(t^{\rho-1}I_{p,q}^{m,n}\left[at^{u}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$
$$= x^{-\beta+\rho-1}I_{p+2,q+2}^{m,n+2}\left[ax^{\mu}\left|\begin{matrix}(1-\rho,\mu,1) & (1+\beta-\gamma-\rho,\mu,1) & (a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q} & (1-\beta-\rho,\mu,1) & (1+\alpha-\gamma-\rho,\mu,1)\end{matrix}\right.\right]$$

The above corollary leads to Erdelyi-Kober fractional integral as follows.

Corollary 3.3.

$$\left(I_{\gamma,\alpha}^{+}\left(t^{\rho-1}I_{p,q}^{m,n}\left[at^{u}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$
$$= x^{\rho-1}I_{p+1,q+1}^{m,n+1}\left[ax^{\mu}\left|\begin{matrix}(1-\gamma-\rho,\mu,1) & (a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q} & (1-\alpha-\gamma-\rho,\mu,1)\end{matrix}\right.\right]$$

Provided that $\alpha, \gamma, \rho, a \in \mathbb{C}$ be such that $Re(\alpha),\mu > 0$ and $Re(\rho) > \max\{0, Re(-\gamma)\}$. Then the left-hand sided Erdelyi-Kober fractional integration $I_{\gamma,\alpha}^{+}\left(= I_{0+}^{\alpha,0,\gamma}\right)$ of the I-function is given for $x > 0$.

The following result corresponds to the right-hand sided MSM fractional integration of the I-function.

Theorem 3.2. Each of the following MARICHEV-SAIGO-MAEDA (MSM) FRACTIONAL INTEGRATION representation formulas hold true for the I –function**:**

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\rho}I_{p,q}^{m,n}\left[at^{-\mu}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$
$$= x^{-\alpha-\alpha'+\gamma-\rho}I_{p+3,q+3}^{m,n+3}\left[ax^{-\mu}\left|\begin{matrix}(1+\beta-\rho,\mu,1) & (1-\alpha-\alpha'+\gamma-\rho,\mu,1) & (1-\alpha-\beta'+\gamma-\rho,\mu,1) & (a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q} & (1-\rho,\mu,1) & (1-\alpha+\beta-\rho,\mu,1) & (1-\alpha-\alpha'-\beta'+\gamma-\rho,\mu,1)\end{matrix}\right.\right], \quad \dots (19)$$

for $x > 0$

provided that $\alpha, \alpha', \beta, \beta', \gamma, \rho, a \in \mathbb{C}$ be such that $Re(\gamma), \mu > 0$ and $Re(\rho) > \max\{Re(\beta), Re(-\alpha - \alpha' + \gamma), Re(-\alpha - \beta' + \gamma)\}$

Proof. Using (1), the lhs of (19) is equal to

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\rho}\frac{1}{2\pi i}\int_C \mathcal{X}(s)(at^{-\mu})^{-s}ds,\right)\right)(x), \quad \dots (20)$$

where $\chi(s)$ is given by (2). Interchanging the order of integration and using (16), (20) reduces to

$$\frac{1}{2\pi i}\int_{C} \mathcal{X}(s)a^{-s}\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{-(\rho-\mu s)}\right)(x)ds$$
$$= x^{-\alpha-\alpha'+\gamma-\rho}\frac{1}{2\pi i}\int_{C} \mathcal{X}(s)\mathcal{X}_2(s)(ax^{-\mu})^{-s}ds,$$

where

$$\mathcal{X}_2(s) = \frac{\Gamma(-\beta+\rho-\mu s)\Gamma(\alpha+\alpha'-\gamma+\rho-\mu s)\Gamma(\alpha+\beta'-\gamma+\rho-\mu s)}{\Gamma(\rho-\mu s)\Gamma(\alpha-\beta+\rho-\mu s)\Gamma(\alpha+\alpha'-\beta'-\gamma+\rho-\mu s)}$$

The result now follows from (1).

The Saigo and Erdelyi-Kober fractional integration of the $\overline{H}$-function follow as corollaries.

Corollary 3.5. Let α, β, γ, ρ, a ∈ ℂ be such that Re(α),μ > 0 and Re(ρ) > max{Re(−β), Re(−γ)}. Then the right-hand sided generalized fractional integration $I_{-}^{\alpha,\beta,\gamma}$ of the $\overline{H}$-function, for $x > 0$, is given by

$$\left(I_{-}^{\alpha,\beta,\gamma}\left(t^{-\rho}I_{p,q}^{m,n}\left[at^{-\mu}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$
$$= x^{-\beta-\rho}I_{p+2,q+2}^{m,n+2}\left[ax^{-\mu}\left|\begin{matrix}(1-\gamma-\rho,\mu,1) & (1-\beta-\rho,\mu,1) & (a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q} & (1-\rho,\mu,1) & (1-\alpha-\beta-\gamma-\rho,\mu,1)\end{matrix}\right.\right]$$

Corollary 3.6. Let α, γ, ρ, a ∈ ℂ be such that Re(α),μ > 0 and Re(ρ) > max{0, Re(−γ)}. Then the right-hand sided Erdelyi-Kober fractional integration K−γ,α (= I− α,0,γ) of the I-function is given for x > 0 by

$$\left(K_{\gamma,\alpha}^{-}\left(t^{-\rho}I_{p,q}^{m,n}\left[at^{-\mu}\left|\begin{matrix}(a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,p}\end{matrix}\right.\right]\right)\right)(x)$$
$$= x^{-\rho}I_{p+1,q+1}^{m,n+1}\left[ax^{\mu}\left|\begin{matrix}(1-\gamma-\rho,\mu,1) & (a_i,A_i,\alpha_i)_{1,p}\\(b_j,B_j,\beta_j)_{1,q} & (1-\alpha-\gamma-\rho,\mu,1)\end{matrix}\right.\right]$$

IV. Conclusions

The I-function is one of the most generalized function available in literature, which generalizes the H-function, H-function, Meijer, G-function, generalized, Wright function, hypergeometric function, generalized Mittag-Leffler function, and many other functions. On the other hand, the MSM fractional operators generalize, among others, Saigo, Riemann-Liouville, Weyl and Erdelyi-Kober fractional operators. In view of this fact, several recent results obtained by Srivastava et al.[27], Saigo et al. [14], Kilbas and Sebastian in a series of papers ,[8],[7],[6] become particular cases of our results.

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