Binary Pre Generalized Regular Beta Closed Sets in Binary Topological spaces

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ABSTRACT

In this paper we introduce a new class of binary closed sets called binary pre generalized regular beta closed sets (briefly ^{b}pgrβ-closed sets) in binary topological spaces and study some of their properties.

Keywords :

Binary topological spaces, pgrβ -closed sets, ^{b}pgrβ -closed sets.

I. INTRODUCTION

S. Nithyanantha Jothi and P.Thangavelu [10, 11, 5] introduced the concept of a binary Topology between two non-empty sets which satisfies certain axioms that are analogous to the axioms of Topology and studied about binary topological spaces in the year 2011 and introduced the concept of generalized binary regular closed sets in binary topological spaces in the year 2016.Also S.Nithyanantha Jothi[12] have defined semi open sets in Binary Topological spaces and studied some of their properties in 2016. A.Manonmani and S.Jayalakshmi [7] introduced binary rβ closed sets and binary rβ open sets in topological spaces. Also A.Manonmani and S.Jayalakshmi [8] studied about pgrβ closed sets in topological spaces. In this paper, ^{b}pgrβ closed sets in binary topological spaces are introduced and their basic properties are studied. Throughout this paper, ℘(X) and ℘(Y) denotes the power set of X and Y respectively. Also binary closure of (A,B) and binary interior of (A, B) are denoted by b-cl(A, B) and b-int(A, B) respectively. The compliment of A and the compliment B are denoted by A^C and B^C respectively. For basic definitions and results of a binary topological space, the reader may refer Engelking [3].

II. PRELIMINARIES

Let X and Y be any two nonempty sets. A binary topology[10] from X to Y is a binary structure $\mathcal{M}$ $\subseteq \wp(X)\times\wp(Y)$ that satisfies the axioms namely (i) $(\emptyset,\emptyset)$ and $(X,Y)\in \mathcal{M}$, (ii) $(A_1\cap A_2 , B_1\cap B_2)\in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and (iii) If $\{(A_\alpha, B_\alpha): \alpha\in\Delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\Delta} A_\alpha , \bigcup_{\alpha\in\Delta} B_\alpha) \in \mathcal{M}$. If $\mathcal{M}$ is a binary topology from X to Y then the triplet (X, Y, $\mathcal{M}$) is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space (X,Y, $\mathcal{M}$). The elements of X×Y are called the binary points of the binary topological space (X, Y, $\mathcal{M}$). If Y=X then $\mathcal{M}$ is called a binary topology on X in which case we write (X, $\mathcal{M}$) as a binary topological space.

Definition 2.1[10]:

Let X and Y be any two non- empty sets and let (A, B) and (C, D) ∈ ℘(X) × ℘(Y). We say that (A, B) ⊆ (C, D) if A ⊆ C and B ⊆ D.

Definition 2.2 [10]:

Let X and Y be any two nonempty sets and let (A, B) and (C, D) ∈ ℘(X) ×℘(Y). We say that: (A, B) ⊄ (C, D) if one of the following holds:

(i) A⊆C and B ⊄ D (ii) A ⊄ C and B ⊆D (iii) A ⊄ C and B⊄ D.

Definition 2.3 [10]:

Let (X, Y, $\mathcal{M}$) be a binary topological space and A ⊆ X, B ⊆Y. Then (A, B) is called binary closed in (X, Y, $\mathcal{M}$) if (X- A, Y- B) ∈ $\mathcal{M}$

Proposition 2.4 [10]:

Let (X, Y, $\mathcal{M}$) be a binary topological space and (A,B) ⊆ (X,Y). Let $(A, B)^{1*} = \bigcap\{ A_\alpha : (A_\alpha ,B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \bigcap\{ B_\alpha: (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha,B_\alpha)\}$.

Definition 2.5 [10] :

The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 2.4 is called the binary closure of (A, B), denoted by b-*cl*(A, B) in the binary space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$. Here it is to be noted that $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*},(A, B)^{2*})$.

Proposition 2.6 [10]:

Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A,B) \subseteq (X,Y)$. Let $(A, B)^{1°} = \cup\{A_\alpha : (A_\alpha,B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2°} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Proposition 2.7 [10]:

Let $(X, Y, \mathcal{M})$ be a binary topological space. Then (i) $\mathcal{M}_X = \{A\subseteq X : (A, B) \subseteq \mathcal{M}$ for some $B \subseteq Y\}$ is a topology on X.(ii) $\mathcal{M}_Y = \{ B\subseteq Y : (A, B) \subseteq \mathcal{M}$ for some $A\subseteq X\}$ is a topology on Y.

Definition 2.8 [10]:

The ordered pair $((A, B)^{1°}, (A, B)^{2°})$ defined in proposition 2.6 is called the binary interior of (A, B), denoted by b-*int* (A, B). Here $((A, B)^{1°}, (A, B)^{2°})$ is binary open and $((A, B)^{1°}, (A, B)^{2°}) \subseteq (A, B)$.

Definition 2.9 [10]:

Let $(X, Y, \mathcal{M})$ be a binary topological space. Let $(A, B) \subseteq (X,Y)$. Define $\mathcal{M}_{(A, B)} = \{(A \cap U, B \cap V) : (U,V)\in \mathcal{M}\}$. Then $\mathcal{M}_{(A, B)}$ is a binary topology from A to B. The binary topological space $(A, B, \mathcal{M}_{(A, B)})$ is called a binary subspace of $(X, Y, \mathcal{M})$.

Definition 2.9 [12]:

A subset (A,B) of a Binary topological space $(X, Y, \mathcal{M})$ is called

(i) a binary regular open set (shortly ^{b}regular open set) if (A, B) = b-int(b-*cl*(A, B)) and binary regular closed(shortly ^{b}regular closed set) if (A, B)= b-*cl*(b-int(A, B)),

(ii) a binary semi open set (shortly ^{b}semi open set) in $(X, Y, \mathcal{M})$ if b-int (b-cl (A, B)) $\subseteq$ (A, B) and its compliment is called a binary semi closed set (shortly ^{b}semi closed set) in $(X, Y, \mathcal{M})$.

Definition 2.10:

A subset A of a Topological space (X, τ) is said to be

(i) a pre - open set [9] in (X, τ) if A □ int (cl (A)) and pre - closed if cl (int (A)) □ A,

(ii) a β-open set [1] (or semi preopen set[2]) in (X, τ) if A □ cl(int (cl (A))) and β- closed set [1] (or semi pre closed set [2]) if int (cl (int (A))) □ A,

(iii) a pgrβ- closed set [8] in (X, τ) if there exists a rβ - open set U in (X, τ) such that p*cl*(A) $\subseteq$ U whenever A $\subseteq$ U.

Definition 2.11 [4]:

A subset (A, B) of a Binary topological space $(X, Y, \mathcal{M})$ is called

(i) a binary pre closed set (shortly ^{b}pre closed set) in $(X, Y, \mathcal{M})$ if b-cl (b-int(A, B)) $\subseteq$(A, B) ,

(ii) a binary semi pre closed set (shortly ^{b}semi pre closed set) or a binary β closed set (shortly ^{b}β - closed set) in $(X, Y, \mathcal{M})$ if b-cl (b-int (b-cl (A, B))) $\subseteq$ (A, B),

(iii) a binary generalized closed set(shortly ^{b}g - closed set) in $(X, Y, \mathcal{M})$ if there exist a binary open set (U, V) in $(X, Y, \mathcal{M})$ such that (b-cl (A, B) $\subseteq$ (U, V),

(iv) a binary rβ closed set (shortly ^{b}rβ - closed set) in $(X, Y, \mathcal{M})$ if there exists a ^{b}β - open set (U, V) in $(X, Y, \mathcal{M})$ such that b-r*cl*(A, B) $\subseteq$ (U, V) whenever (A, B) $\subseteq$ (U, V).

The compliment of the above binary closed sets are their respective binary open sets in $(X, Y, \mathcal{M})$.

III. BINARY pgrβ CLOSED (^{b}pgrβ -CLOSED) SETS IN BINARY TOPOLOGICAL SPACES

In this section, we give the definition of binary pre generalized regular beta closed sets (shortly ^{b}pgrβ - closed sets) in a binary topological space and study some of their properties.

Definition 3.1:

Let $(X, Y, \mathcal{M})$ be a binary topological space. Let $(A,B) \subseteq (X,Y)$. Then (A, B) is called binary pre generalized regular βeta closed set (shortly ^{b}pgrβ - closed set) in $(X, Y, \mathcal{M})$ if there exists a binary regular beta open set (shortly ^{b}rβ - open set) [4] (U, V) in $(X, Y, \mathcal{M})$ such that b-p*cl*(A, B) $\subseteq$ (U, V) whenever (A, B) $\subseteq$ (U, V).

***Example 3.2*:**

Consider X ={a, b, c} and Y ={1, 2}.

Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Also {(∅,∅), ({a},{1}) ({c},{2}), ({a, c},{2}), ({a, c},Y), ({b, c},{2}),({X,{2}), (X, Y)} are binary closed sets in (X, Y, $\mathcal{M}$).

Then {(∅,∅), (∅,{1}), (∅,{2}), (∅, Y), ({a}, ∅), ({a},{1}), ({a},{2}), ({a},Y), ({b},{2}), ({b},Y), ({c}, ∅), ({c},{1}), ({c},{2}), ({c},Y),({a, b},{2}),({a, b},Y), ({a, c}, ∅), ({a, c},{1}), ({a, c},{2}), ({a, c},Y), ({b, c},∅), ({b, c},{1}),({b, c},{2}), ({b, c},Y), (X, ∅), (X,{1}), ({X,{2}), (X, Y)} are ^{b}pgrβ - closed sets in (X, Y, $\mathcal{M}$).

Definition 3.3:

Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) ⊆ (X, Y).Then (A, B) is called binary pre generalized regular βeta open set (shortly ^{b}pgrβ - open set) in (X, Y, $\mathcal{M}$) if the compliment of (A, B) is ^{b}pgrβ - closed set in (X, Y, $\mathcal{M}$).

Proposition 3.4:

The union of two ^{b}pgrβ -closed sets in (X, Y, $\mathcal{M}$) need not be ^{b}pgrβ -closed set in (X, Y, $\mathcal{M}$) which is shown in the following example.

Example 3.5:

Let X = {a, b, c} and Y = {1, 2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}),({a, b},{1}), ({ b, c},{2}),(X, Y)} is a binary topology from X to Y. Here (∅, {1}) and (, {∅,{2}) are ^{b}pgrβ - closed sets in (X, Y, $\mathcal{M}$), but their union (∅, Y) is not a ^{b}pgrβ - closed set in (X, Y, $\mathcal{M}$).

Proposition 3.6:

The intersection of two ^{b}pgrβ -closed sets in (X, Y, $\mathcal{M}$) need not be ^{b}pgrβ -closed set in (X, Y, $\mathcal{M}$) which is shown in the following example.

Example 3.7:

Let X = {a, b, c} and Y = {1, 2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b}, {1}), ({ b, c},{2}),(X, Y)} is a binary topology from X to Y. Here ({a, b},{2}) and ({b, c}, ∅) are ^{b}pgrβ – closed sets in (X, Y, $\mathcal{M}$), but their intersection ({b}, ∅) is not a ^{b}pgrβ - closed set in (X, Y, $\mathcal{M}$).

Proposition 3.8:

If (A, B) is ^{b}closed in (X, Y, $\mathcal{M}$), then (A,B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Proof:

Let (A, B) be a ^{b}closed in a binary topological space (X,Y, $\mathcal{M}$). Suppose that (A, B) ⊆ (U, V) where (U,V) is ^{b}rβ- open set in (X, Y, $\mathcal{M}$). Since (A, B) is ^{b}closed, b-cl(A,B) =(A,B) and b-pcl(A,B) ⊆b-cl(A,B) ⊆ (U,V). Hence b-pcl(A,B) ⊆ (U,V) and therefore, (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Remark 3.9:

The converse of Proposition 3.8 is not true. This is shown in the following example.

Example 3.10:

If (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$), then (A, B) need not be ^{b}closed in (X, Y, $\mathcal{M}$). For, let X = {a, b, c}, Y = {1,2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Here ({ c},{1}) is ^{b}pgrβ -closed in (X, Y, $\mathcal{M}$) but not ^{b}closed in (X, Y, $\mathcal{M}$) .

Proposition 3.11:

If (A, B) is ^{b}regular closed in (X, Y, $\mathcal{M}$), then (A,B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Proof:

Let (A, B) be a ^{b}regular closed in a binary topological space (X,Y, $\mathcal{M}$). Suppose that (A, B) ⊆ (U, V) where (U,V) is ^{b}rβ- open set in (X, Y, $\mathcal{M}$). Since (A, B) is ^{b}regular closed, b-rcl(A,B) =(A,B) and also b-pcl(A,B) ⊆b-rcl(A,B) ⊆ (U,V). Hence b-pcl(A,B) ⊆ (U,V) and so (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Remark 3.12:

The converse of Proposition 3.11 is not true. This is shown in the following example.

Example 3.13:

If (A,B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$), then (A, B) need not be ^{b}r- closed in (X, Y, $\mathcal{M}$). For, let X = {a, b, c},Y = {1,2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Here ({b, c},{2}) is ^{b}pgrβ -closed in (X, Y, $\mathcal{M}$) but not ^{b}r - closed in (X, Y, $\mathcal{M}$) .

Proposition 3.14:
If (A, B) is ^{b}pre closed in (X, Y, $\mathcal{M}$), then (A,B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Proof:
Let (A, B) be a ^{b}pre closed in a binary topological space (X,Y, $\mathcal{M}$). Suppose that (A, B) ⊆ (U, V) where (U,V) is ^{b}rβ- open set in (X, Y, $\mathcal{M}$). Since (A, B) is ^{b}pre closed in (X,Y, $\mathcal{M}$),b-pcl(A, B) =(A, B) and therefore b-pcl(A, B) ⊆ (U,V). Hence (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Remark 3.15:
The converse of the Proposition 3.14 is not true. This is shown in the following example.

Example 3.16 :
If (A, B) is ^{b}pgrβ- closed in (X, Y, $\mathcal{M}$), then (A, B) need not be ^{b}pre closed in (X,Y, $\mathcal{M}$). For, let X = {a, b, c},Y ={1,2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Here (X,{ 1} is ^{b}pgrβ -closed in (X, Y, $\mathcal{M}$) but not ^{b}pre closed in (X, Y, $\mathcal{M}$) .

Proposition 3.17:
If (A, B) is ^{b}g-closed in (X, Y, $\mathcal{M}$), then (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Proof:
Let (A, B) be a ^{b}g-closed in a binary topological space (X,Y, $\mathcal{M}$). Suppose that (A, B) ⊆ (U, V) where (U,V) is ^{b}rβ- open set in (X, Y, $\mathcal{M}$). Since (A, B) is ^{b}g-closed, b-gcl(A,B) =(A,B) and b-pcl(A, B) ⊆b-gcl(A, B) ⊆ (U,V). Hence b-pcl(A, B) ⊆ (U,V) and therefore, (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Remark 3.18:
The converse of Proposition 3.17 is not true. This is shown in the following example.

Example 3.19:
If (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$), then (A, B) need not be ^{b}g-closed in (X, Y, $\mathcal{M}$). For, let X = {a, b, c},Y = {1,2}. Clearly $\mathcal{M}$={(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Here (∅,{1}) is ^{b}pgrβ -closed in (X, Y, $\mathcal{M}$) but not ^{b}g-closed in (X, Y, $\mathcal{M}$) .

Proposition 3.20:
If (A, B) is ^{b}rβ-closed in (X, Y, $\mathcal{M}$), then (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Proof:
Let (A, B) be a ^{b}rβ-closed in a binary topological space (X,Y, $\mathcal{M}$). Suppose that (A, B) ⊆ (U, V) where (U,V) is ^{b}rβ- open set in (X, Y, $\mathcal{M}$). Since (A, B) is ^{b}rβ-closed, b-rcl(A, B) ⊆ (A, B) and b-pcl(A, B) ⊆ b-rcl(A, B) ⊆ (A, B) ⊆ (U, V). Hence b-pcl(A, B) ⊆ (U, V) and therefore, (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$).

Remark 3.21:
The converse of Proposition 3.19 is not true. This is shown in the following example.

Example 3.22:
If (A, B) is ^{b}pgrβ - closed in (X, Y, $\mathcal{M}$), then (A, B) need not be ^{b}rβ-closed in (X, Y, $\mathcal{M}$). For, let X = {a, b, c},Y ={1,2}. Clearly $\mathcal{M}$= {(∅,∅), (∅,{1}) ({a},{1}), ({b},∅), ({b},{1}), ({a, b},{1}),({ b, c},{2}), (X, Y)} is a binary topology from X to Y. Here ({ c},Y) is ^{b}pgrβ -closed in (X, Y, $\mathcal{M}$) but not ^{b}rβ-closed in (X, Y, $\mathcal{M}$) .

Remark: 3.23:
The concept of ^{b}semi closed sets in (X, Y, $\mathcal{M}$) and the concept of ^{b}β –closed sets in (X, Y, $\mathcal{M}$) are independent from the concept of ^{b}pgrβ-closed sets in (X, Y, $\mathcal{M}$) which is illustrated from the following example.

Example 3.24:
Let X = {a,b} , Y ={1,2}) and $\mathcal{M}$={(∅,∅),(∅,{2}), ({b},∅), ({b},{1}), ({b}, {2}), ({b},Y), (X,{2}), (X, Y)} is a binary topology from X to Y. Here =(∅,{2}) is ^{b} semi closed set in (X, Y, $\mathcal{M}$) and ({b},{1}) is a ^{b}β- closed set in(X, Y, $\mathcal{M}$) but not a ^{b}pgrβ- closed set in (X,Y, $\mathcal{M}$).
Also (X, {2}) is a ^{b}pgrβ- closed in (X, Y, $\mathcal{M}$) but neither ^{b} semi closed set nor a ^{b}β- closed set in (X, Y, $\mathcal{M}$).

Remark 3.25:

The relation between ^{b}pgrβ- closed set and other binary closed sets from the above discussions are implemented in the following figure.

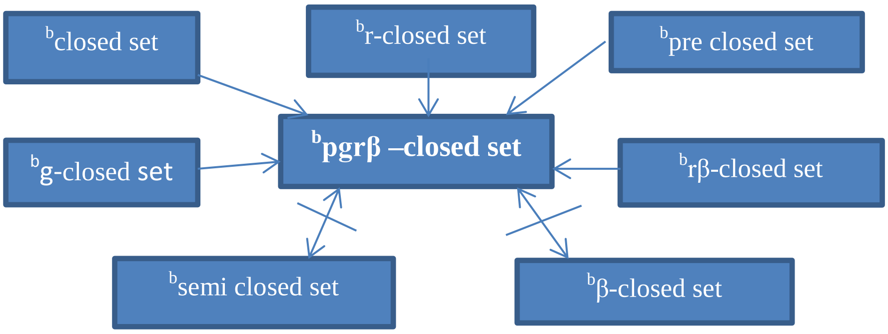

Figure 1: Relation between ^{b}pgrβ- closed set and other binary closed sets

A $\longrightarrow$ B means A implies B and A $\longleftrightarrow$ B means A and B are independent

Theorem 3.26:

Let (X, Y, $\mathcal{M}$) be a binary topological space. Let A ⊆ X, B ⊆Y. If (A, B) is binary open in (X, Y, $\mathcal{M}$), then A^C is pgrβ- closed in X and B^C is pgrβ closed in Y.

Proof:

By Proposition 2.14 of [10], we have A ∈ $\mathcal{M}_X$ and B ∈ $\mathcal{M}_Y$. That is A is open in (X, $\mathcal{M}_X$) and B is open in (B, $\mathcal{M}_Y$).Therefore A^C is closed in (X, $\mathcal{M}_X$) and B^C is closed in (B, $\mathcal{M}_Y$). But we know that every closed set in X is pgrβ-closed set [8] in X .So A^C is pgrβ closed in X and B^C is pgrβ closed in Y.

Remark 3.27:

The converse of the above theorem need not be true from the fact that every pgrβ closed sets need not be a closed set which is shown in the following example.

Example 3.28:

Let X = {a, b, c}, Y = {1,2}, $\mathcal{M}_X$ = {∅, {a}, {b}, {a, b}, X} and $\mathcal{M}_Y$ = {∅, {1}, {2}, Y }. Here {a} is a pgrβ- closed set in (X, $\mathcal{M}_X$), and ∅ is a pgrβ-closed set in (Y, $\mathcal{M}_Y$) . That is , their corresponding compliment {b, c} is a pgrβ- open set in (X, $\mathcal{M}_X$), and Y is a pgrβ-open set in (Y, $\mathcal{M}_Y$) . But ({b, c}, Y) is not a binary open set in (X, Y, $\mathcal{M}$).

Theorem 3.29:

Let (A, B) be a ^{b}pgrβ- closed set in a binary topological space (X, Y, $\mathcal{M}$) and suppose (A, B) ⊆ (C, D) ⊆ b - pcl(A,B). Then (C, D) is a ^{b}pgrβ- closed set in (X, Y, $\mathcal{M}$).

Proof:

Since (A,B) is a ^{b}pgrβ- closed set, there exists a ^{b}rβ open set (U, V) such that b-pcl(A, B) ⊆ (U, V). Since (C, D) ⊆ b-pcl(A, B), b-pcl(C, D) ⊆ b-pcl(b-pcl(A, B)) i.e., b-pcl(C, D) ⊆ b-pcl(A, B) ⊆ (U, V).Therefore (C, D) is also a ^{b}pgrβ- closed set.

Theorem 3.30:

Let (X, Y, $\mathcal{M}$) be a binary topological space and (A, B, $\mathcal{M}_{A,B}$) is a binary subspace of (X,Y, $\mathcal{M}$). Let (C, D) be a ^{b}pgrβ- closed set in (X,Y, $\mathcal{M}$) and (C, D) ⊆ (A, B). Then (C ,D) is a ^{b}pgrβ- closed set in (A, B, $\mathcal{M}_{A,B}$).

Proof:

Since (C, D) is a ^{b}pgrβ- closed set in (X, Y, $\mathcal{M}$), we have b-pcl(C, D) ⊆ (U, V) where (U, V) is ^{b}rβ-open in (X, Y, $\mathcal{M}$) and hence it should be in $\mathcal{M}$. By definition of a binary subspace, (U∩A, V∩B) ⊆ $\mathcal{M}_{A,B}$. Let (U,V) is a rβ- open set in (A, B, $\mathcal{M}_{A,B}$). Since (C, D) ⊆ (A, B) , (C, D) ⊆ (U,V). So *b-p*cl(C ,D) = *b-p*cl(C∩A,D∩B) ⊆b-pcl(U∩C,V∩D) ⊆ (U,V). Thus (C, D) is a ^{b}pgrβ- closed set in (A, B, $\mathcal{M}_{A,B}$).

Remark 3.31:

The converse of the above theorem need not be true as shown in the following example.

Example 3.32:

Let $X=\{a, b, c\}$, $Y =\{1,2\}$. Clearly $\mathcal{M}=\{(\emptyset,\emptyset), (\emptyset,\{1\}) (\{a\},\{1\}), (\{b\},\emptyset), (\{b\},\{1\}), (\{a, b\},\{1\}), (\{ b, c\},\{2\}), (X, Y)\}$ is a binary topology from X to Y.. Also $\{(\emptyset,\emptyset), (\{a\},\{1\}) (\{c\},\{2\}), (\{a, c\},\{2\}), (\{a, c\},Y), (\{b, c\},\{2\}),(\{X,\{2\}), (X, Y)\}$ are binary closed sets in $(X, Y, \mathcal{M})$. Let $(A,B) = (\{a,b\},\{1,2\})$ which is a subset of (X,Y). Clearly $\mathcal{M}_{A,B} =\{(\emptyset,\emptyset),(\emptyset,\{2\}), (\{b\},\emptyset), (\{b\},\{1\}), (\{b\}, \{2\}), (\{b\},B), (A,\{2\}), (A,B)\}$ is a binary topology from A to B. The binary closed sets of $(A, B, \mathcal{M}_{A,B})$ are $\{(\emptyset,\emptyset),(\emptyset,\{1\}),(\{a\},\emptyset), (\{a\}, \{1\}), (\{a\},\{2\}), (\{a\}, B), (A,\{1\}), (A,B)\}$. Consider $(C, D) = (\{a\},\emptyset) \subseteq (A,B)$. Clearly $(\{a\},\emptyset) \subseteq (\{a,b\},\{1,2\})$ is a ^{b}pgrβ- closed set in $(A, B, \mathcal{M}_{A,B})$ but (C, D) is not a ^{b}pgrβ- closed set in $(X,Y, \mathcal{M})$.

Theorem 3.33:

If A is pgrβ - closed in (X, τ) and B is pgrβ - closed in (Y, σ), then (A, B) is ^{b}pgrβ - closed in $(X,Y, \tau \times \sigma)$.

Proof.

By [10] , $\tau \times \sigma$ is a binary topology from X to Y.

The above Proposition is illustrated in the following example.

Example 3.34:

Consider $X= \{a, b, c\}$ and $Y = \{1, 2\}$. Clearly $\tau = \{\emptyset,\{a\},\{b\},\{a, b\},X\}$ is a topology on X and $\sigma =\{\emptyset,\{1\},\{2\},Y\}$ is a topology on Y. Here $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, X are pgrβ -closed sets in X and $\emptyset$, $\{1\}$, $\{2\}$, Y are pgrβ - closed sets in Y. Now , $\tau \times \sigma = \{((\emptyset,\emptyset), (\emptyset,Y), (\emptyset,\{1\}), (\emptyset,\{2\}), (\{a\},\emptyset), (\{a\},Y), (\{a\},\{1\}), (\{a\},\{2\}), (\{b\},\emptyset), (\{b\},Y), (\{b\},\{1\}), (\{b\},\{2\}), (\{a,b\},\emptyset), (\{a,b\},Y), (\{a, b\}, \{1\}),(\{a, b\}, \{2\}), (X, \emptyset), (X,\{1\}), (X,\{2\}),(X,Y)\}$ is a binary topology from (X, τ) to (Y, σ). Here $\{(\emptyset,\emptyset), (\emptyset,Y), (\emptyset,\{1\}),(\emptyset,\{2\}), (\{a\},\emptyset), (\{a\},Y), (\{a\},\{1\}), (\{a\},\{2\}), (\{b\},\emptyset), (\{b\},Y), (\{b\},\{1\}), (\{b\},\{2\}), (\{c\},\emptyset), (\{c\},Y), (\{c\},\{1\}), (\{c\},\{2\}), (\{a,b\},\emptyset), (\{a,b\},Y), (\{a, b\}, \{1\}), (\{a, b\}, \{2\}), (\{a,c\},\emptyset), (\{a,c\},Y), (\{a, c\}, \{1\}), (\{a, c\}, \{2\}), (\{b, c\},\emptyset), (\{b, c\},Y), (\{b, c\}, \{1\}), (\{b,c\}, \{2\}) , (X, \emptyset), (X,\{1\}), (X,\{2\}),(X,Y)$ are ^{b}pgrβ - closed sets in $(X, Y, \tau \times \sigma)$.

IV. CONCLUSION

In this paper, Binary Pre generalized regular beta closed sets in Binary topological spaces have been introduced and some of their properties are studied. The relationship between ^{b}pgrβ - closed sets in $(X, Y, \mathcal{M})$ and some other binary closed sets in $(X, Y, \mathcal{M})$ are also we have studied.

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REFERENCES

[1] M.E.Abd El-Monsef ,S.N.El-Deeb and R.A.Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ., 12, 1983, 77-90

[2] D.Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1), 1986, 24-32 .

[3] Engelking, *General Topology*, Polish Scientific Publishers, Warszawa, 1977.

[4] S.Jayalakshmi and A.Manonmani, *Binary regular beta closed sets and Binary regular beta open sets in Binary topological spaces*, The International Journal of Analytical and Experimental Modal Analysis, Vol 12(4), 2020 ,494 – 497.

[5] Jothi, S.N, & Thangavelu, P**.**, *Generalized Binary Regular Closed Sets*. International Journal of Applied Sciences (ISSN 2455-4499), 4(2), 2016, 259-263.

[6] N.Levine, *Generalized closed sets in Topology*, Rend. Circ. Mat. Paleramo, 2 , 1970, 89-96.

[7] A,Manonmani and S,Jayalakshmi, *On regular beta closed sets in a Topological spaces*, Int. J. Sci and Research, Vol 8 ,2019, 1756-1760.

[8] A.Manonmani and S.Jayalakshmi, *On Pre generalized regular beta closed sets in Topological spaces*, Int. J. Innovative Research in Sci, Eng and Technology, ISSN 2347-6710, Vol . 9(2), 2020, 13873-13881.

[9] A.S. Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, *On pre-continuous and weak pre- continuous mappings*, Proc. Math and Phys. Soc. Egypt ,53,1982, 47-53.

[10] S. Nithyanantha Jothi and P.Thangavelu, *Topology between two sets***,** Journal of Mathematical Sciences & Computer Applications, 1(3), 2011, 95-107.

[11] S. Nithyanantha Jothi and P.Thangavelu, *On binary topological spaces***,** Pacific–Asian Journal of Mathematics, 5(2), 2011,133-138.

[12] S. Nithyanantha Jothi, *Binary Semi open sets in Binary topological Spaces*, International journal of Mathematical Archieve 7(9), 2016,73-76.