An Effectiveness of Single Server And Multiple Servers Queuing Models In Hospital

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ABSTRACT

Queuing Theory is mostly applied in Healthcare which provides service to random demands. Queuing theory concerned with the mathematical study of modeling and analysis of the systems. In this paper, Various types of queues and multiple number of servers concerned in the process are observed by using probability distribution. The arrival process measured by Poisson distribution and service process measured by Exponential distribution. Single server and multiple servers queuing models are used to analyze the system.

Keywords: *Queuing Theory, Single server queuing model, Multiple server queuing model.*

1. INTRODUCTION TO QUEUING THEORY

The history of Queuing Theory goes back nearly 100 years. It was born with the work of A.K.Erlang. Erlang laid the foundation for the place of Poisson distribution in queuing theory. It has been applied to a large of service industries including banks, airlines and telephone call centers as well as emergency system such as police patrol, fire and ambulance. It has also applied in various healthcare setting.

Queuing Models can be very useful in identifying appropriate levels of staff, equipment and beds as well as in making decisions about resource allocation and the design of new services. Queuing Theory is a cluster of mathematical models of different queuing systems. It is used extensively to examine production and service processes exhibiting random variability in market demand and service time. It is also gives the technique for maximizing range to meet the demand. So that waiting time is reduced drastically.

2. ELEMENTS OF QUEUING SYSTEM

The element of queuing system is concerned with the pattern in which the customers arrive for service. Input source can be described by following factor:

- Arrival pattern of customers.
- The service pattern.
- The service mechanism
- System capacity.

3. QUEUE DISCIPLINE

Queue Discipline represents the way the queue is organized (rules of inserting and removing customers to/from the queue). There are these ways:

- FIFO (First In First Out) also called FCFS (First Come First Serve) - If the customers are served in the arrangement of their arrival, then this is known as the first come first served (FCFS) service discipline.
- LIFO (Last In First Out) also called LCFS (Last Come First Serve) - stack.

- SIRO (Serve In Random Order).
- Priority Queue, which may be viewed as a number of queues for various priorities.
- Many other more complex queuing methods that typically change the Customer's position in the queue according to the time spent already in the queue, expected service duration, and/or priority. These methods are typical for computer multi-access systems.

4. QUEUING MODELS

- **(M/M/1):(∞/FCFS)**
  The model deals with a queuing system having Single service channel. Poisson input, exponential service and there is no limit on the system capacity while the customers are served on a "first come, first served" basis.
- **(M/M/1):(N/FCFS)**
  The model deals with a queuing system having Single service channel. Poisson input, exponential service and there is limited number system capacity while the customers are served on a "first come, first served" basis.
- **(M/M/C):(∞/FCFS)**
  The model deals with a queuing system having multiple service channel. Poisson input, exponential service and there is no limit on the system capacity while the customers are served on a "first come, first served" basis.
- **(M/M/C):(N/FCFS)**
  The model deals with a queuing system having multiple service channel. Poisson input, exponential service and there is no limit on the system capacity while the customers are served on a "first come, first served" basis.

5. BASIC NOTATIONS

N denotes Number of customers in the system both waiting and in service.

λ denotes Average number of customers arriving per unit of time.

μ denotes Average number of customers being served per unit of time.

ρ : (λ/μ) denotes Traffic intensity or Utilization factor.

C denotes Number of parallel service channel(servers).

L_s(or) E(n) denotes Average number of customers in the system both waiting

and in the service.

L_q (or) E(m) denotes Average number of customers waiting in the queue.

W_s(or) E(v) denotes Average waiting time of a customer in the system, both waiting and inService.

W_q (or) E(w) denotes Average waiting time of a customer in queue.

$P_n(t)$ denotes Probability that there are n customers in the system at any time t, both waiting andin service.

6.1 SINGLE SERVER QUEUING MODEL(M/M/1):(∞/FCFS)

In this model deals with a queuing system having a single server, Poisson arrival, exponential service and there is no limit on the system's capacity while the customers are served on a "first in, first out" basis.

The following assumptions made on this model:

- Arrivals are described by Poisson probability distribution and come from an uncountable population.
- Single waiting line and every arrival waits to be served whateverof the length of the queue and no balking and reneging take place.
- Queue discipline is first come first server.
- Single server and service time follows an exponential distribution.
- Average service rate is more than average arrival rate.

probability of an arrival in a time interval of length h>0 is given by

$$e^{\lambda h}(\lambda h) = \lambda h(1 - \lambda h + \frac{(\lambda h)^2}{2!} - \cdots)$$

$$= \lambda h + o(h)$$

Then $\lambda_n = \lambda$ n=0, 1, 2, 3…

hypothesis, service time distribution is given by

$$W_s(t) = P[s \le t] = 1 - e^{-\mu t}, \quad t \ge 0.$$

Then, when a customer is receiving service, the probability of a service completion (death) is a short time interval, h, is given by

$$1 - e^{-\mu t} = 1 - (1 - \mu h + \frac{(\mu h)^2}{2!} - \cdots) = \mu h + o(h)$$

Thus $\mu_n = \mu$, n=1, 2, 3…

Since $\rho = \frac{\lambda}{\mu}$ and C_n is equal to ρ^n,

$$S = 1 + \rho + \rho^2 + \cdots + \rho^n + \cdots = 1/(1-\rho)$$

Hence, $P_n = P[N=n] = (1-\rho)\rho^n$, n=0, 1, 2…

Mean number of customers in the system both waiting and in service

$$L_s = \frac{\lambda}{\mu - \lambda}$$

Mean number of customers waiting in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Mean waiting time of a customer in the system both waiting and in service

$$W_s = \frac{1}{\mu - \lambda}$$

Mean waiting at a customer in queue

$$W_q = \frac{\rho}{\mu - \lambda}$$

we calculate

P[server busy]= 1-P[N=0]=1-(1-ρ)= ρ

The law of huge numbers this probability can be interpreted as the fraction of time that the server is busy, it is denoted by ρ or server utilization.

Probability of no customer in the system

$$P_0 = 1- \rho$$

PROBLEM-6.1.1:

Let λ = 10 patients per hour and μ = 16 patients per hour.

By using single server queuing model we have to find the following

Traffic intensity $\rho = \frac{\lambda}{\mu} = 0.625$

Mean number of patients in the system (both waiting and in service)

$$L_s = \frac{\lambda}{\mu-\lambda} = 1.67 \text{ patients per hour}$$

Mean number of patients waiting in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 1.04 \text{ patients per hour}$$

Mean waiting time of a patient in the system (both waiting and in service)

$$W_s = \frac{1}{\mu-\lambda} = 0.167 \text{ hours per patient}$$

Mean waiting at a patients in queue

$$W_q = \frac{\rho}{\mu-\lambda} = 0.1042 \text{ hours per patients}$$

Probability of no customer in the system

$$P_0 = 1- \rho = 0.375$$

PROBLEM-6.1.2:

Let λ = 8 patients per hour and μ = 10 patients per hour.

By using single server queuing model we have to find the following

Traffic intensity $\rho = \frac{\lambda}{\mu} = 0.8$

Mean number of patients in the system (both waiting and in service)

$$L_s = \frac{\lambda}{\mu-\lambda} = 4 \text{ patients per hour}$$

Mean number of patients waiting in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = 3.2 \text{ patients per hour}$$

Mean waiting time of a patient in the system (both waiting and in service)

$$W_s = \frac{1}{\mu-\lambda} = 0.5 \text{ hours per patient}$$

Mean waiting at a patients in queue

$$W_q = \frac{\rho}{\mu-\lambda} = 0.4 \text{ hours per patients}$$

Probability of no customer in the system

$$P_0 = 1-\rho = 0.2$$

6.2 MULTIPLE SERVER QUEUINGMODELS (M/M/C):(∞/FCFS)

In this queuing model having the following assumptions

- Arrivals are described by Poisson probability distribution and from infinite population.
- Multiple waiting lines and every arrival wait to be served regardless of the length of the queue and no balking and reneging take place.
- Queue discipline is first come first served.
- Service time follows exponential distribution.
- Average service rate is more than average arrival rate.

Here μ is the service rate of one server, but here we have c number of servers therefore cμ will be the service rate. If there are n customers in the queuing system at any point in the time, then following two cases may arise:

i. If n<c, then there will be no queue. However (c-n) numbers of servers are not busy. Then combined service rate will be $\mu_n = n\mu$; n<c.
ii. If n=c, then all servers will be busy and the atmost number of customers in the queue will be (n-c). the combined service rate will be $\mu_n = c\mu$; n=c. Thus we have $\lambda_n = \lambda$ for all n=0, 1, 2…

and
$$\mu_n = \begin{cases} n\mu & n = 1, 2, 3, \ldots..c \\ \mu_n = c\mu & n = c, c+1, c+2, \ldots \ldots. \end{cases}$$

with u=λ/μ and ρ=u/c

$$S=\frac{1}{P_0} = 1+u+\frac{u^2}{2!}+\cdots+\frac{u^{c-1}}{(c-1)!}+\frac{u^c}{c!}\left(1+\frac{u}{c}+\left(\frac{u}{c}\right)^2+\cdots\right)$$

$$=\sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!}\sum_{n=0}^{\infty}\rho^n = \sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!(1-\rho)}$$

$$P_0 = \left[\sum_{n=0}^{c-1}\frac{u^n}{n!}+\frac{u^c}{c!(1-\rho)}\right]^{-1}$$

And

$$P_n = \begin{cases} \frac{u^n}{n!}P_0 & if\ n = 0,\ 1,\ 2,\ \ldots.c \\ \frac{u^n}{c!c^{n-c}}P_0 & if\ n \geq c \end{cases}$$

Mean number of customers waiting in the queue

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2}P_0$$

Mean number of customers in the system both waiting and in service

$$L_s = L_q + \frac{\lambda}{\mu}$$

Mean waiting at a customer in queue

$$W_q = \frac{L_q}{\lambda}$$

Mean waiting time of a customer in the system both waiting and in service

$$W_s = W_q + \frac{1}{\mu}$$

PROBLEM-6.2.1

Let λ = 10 patients per hour and μ = 16 patients per hour and c= 2 doctors

By using multiple server queuing model we have to find the following

Traffic intensity $\rho = \frac{\lambda}{c\mu} = \frac{10}{2*16} = 0.312$

Probability of no patient in the system

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!(1-\rho)}\right]^{-1} = 0.524$$

Mean number of patients waiting in the queue

$$L_q = L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} P_0 = 0.0676 \text{ patients per hour}$$

Mean number of patients in the system both waiting and in service

$$L_s = L_q + \frac{\lambda}{\mu} = 0.6926 \text{ patients per hour}$$

Mean waiting at a patient in queue

$$W_q = \frac{L_q}{\lambda} = 0.00676 \text{ hour per patient}$$

Mean waiting time of a patient in the system both waiting and in service

$$W_s = W_q + \frac{1}{\mu} = 0.06926 \text{ hour per patient}$$

PROBLEM-6.2.2

Let λ = 8 patients per hour and μ = 10 patients per hour and c= 3 doctors

By using multiple server queuing model we have to find the following

Probability of no patient in the system

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!(1-\rho)}\right]^{-1} = 0.447$$

Mean number of patients waiting in the queue

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} P_0 = 0.41612 \text{ patients per hour}$$

Mean number of patients in the system both waiting and in service

$$L_s = L_q + \frac{\lambda}{\mu} = 1.2161 \text{ patients per hour}$$

Mean waiting at a patient in queue

$$W_q = \frac{L_q}{\lambda} = 0.052 \text{ hour per patient}$$

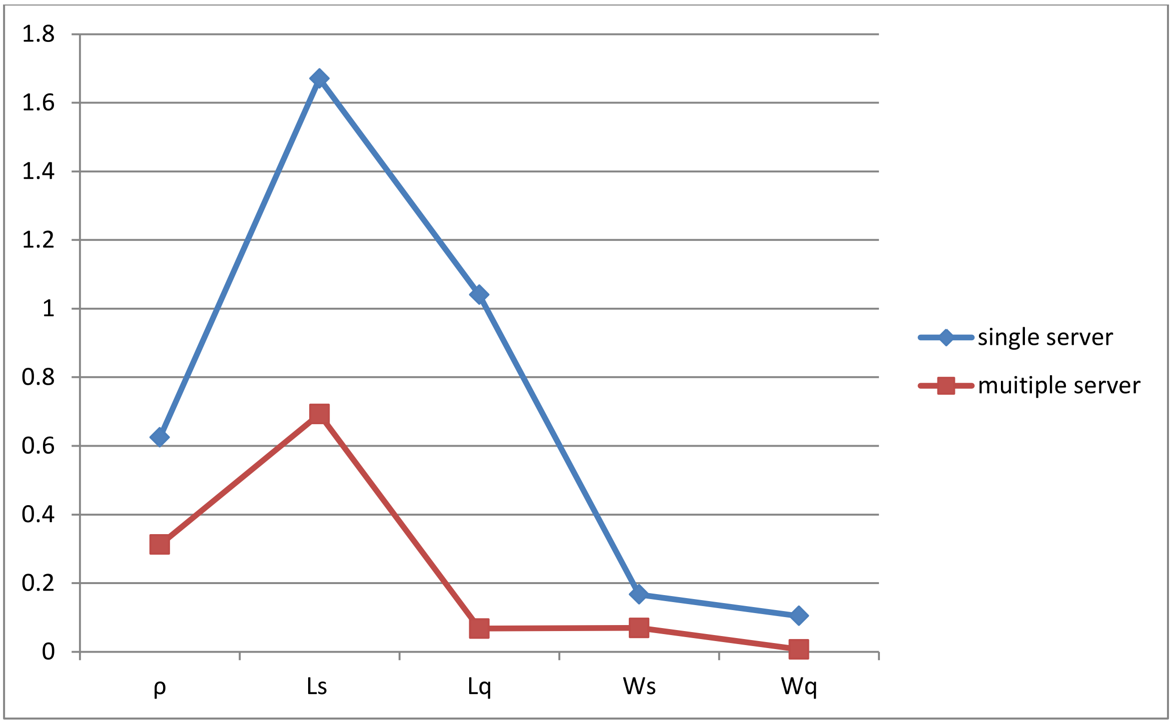
Mean waiting time of a patient in the system both waiting and in service

$$W_s = W_q + \frac{1}{\mu} = 0.152 \text{ hour per patient}$$

7. COMPARISION OF SINGLE SERVER AND MULTIPLE SERVERS QUEUING MODEL

7.1.1 COMPARISION OF PROBLEM-6.1.1 AND PROBLEM-6.2.1

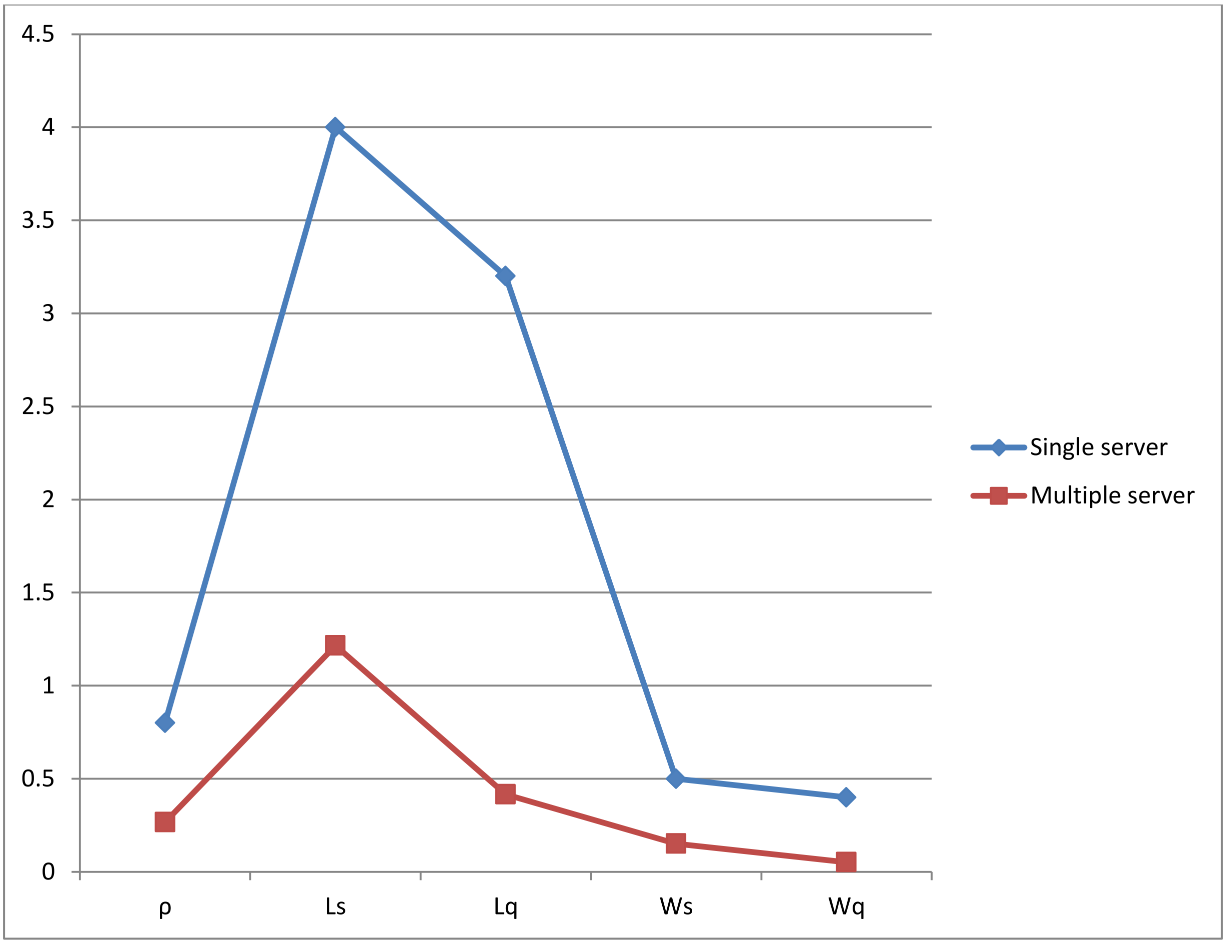
Parameters	Single server(M/M/1)	Multiple servers(M/M/2)
λ	10	10
μ	16	16
ρ	0.625	0.312
P(0)	0.375	0.524
L_s	1.67	0.6926
L_q	1.04	0.0676
W_s	0.167	0.06926
W_q	0.1042	0.00676



From the above chart the efficiency of parameter under two different queuing models of server the traffic intensity has deviated from 62.5% to 31.2%. This shows that more severs (doctors) are introduced, then the hospital becomes less busy.

7.2 COMPARISION OF PROBLEM-6.1.2 AND PROBLEM-6.2.2

Parameters	Single server	Multiple servers
λ	8	8
μ	10	10
ρ	0.8	0.2667
P(0)	0.2	0.447
L_s	4	1.2161
L_q	3.2	0.41612
W_s	0.5	0.152
W_q	0.4	0.052



From the above chart the efficiency of parameter under two different queuing models of servers the traffic intensity has deviated from 80% to 26.67%. This shows that more severs(doctors) are introduced, then the hospital becomes less busy.

CONCLUSION

In this paper, Queuing theory in single server model and multiple server models with infinite queue length are used to analyze the system. Comparison of these two models by using flow chart, multiple server models gives the more effectiveness of the system.

REFERENCES

[1] Anindita Sharma, ParimalBakulBarua, "*Application of queuing theory in a small enterprise*", International journal of engineering trends and technology, vol-27, No.2, Sep 2015.

[2] Adamu Muhammad, "*Analysis of single queue-single server and single queue-multiserver system using simulation*", International journal of scientific and engineering research, vol-6.

[3] Adam.I.J.B.F, Boxma.O,J and Resing.J.A.C, "*Queuing models with multiple waiting lines.*

[4] AugustinDushime, Dr.Joseph K. Mungatu and Dr.MarcelNdengo, "*Queuing model for healthcare services in public health facilities*", International journal of mathematics and physical sciences research, vol-3, issue-1, april 2015

[5] Bakari.H.R, Chamalwa.H.A and Baba.A.M, "*Queuing process and its applications to customer service delivery*", International journal of mathematical and statistics invention, vol-2, issue-1.

[6] Boenzi.F "*A simple queue model for an appointment system and application in a hospital CT scan facility*".

[7] Ezelorachukwuemeka Daniel, OgunohArinze Victor, UmeshMaryroseNgozi, MbeledeoguNjide N, "*Analysis of Queuing System using Single line Multiple servers System*", International journal of scientific and technology research, vol-3, issue3, march 2014.

[8] Foad MahdaviPajouh and ManjunathKamath, "*Applications of queuing models in hospital*", MWAIS 2010.

[9] Filipowicz.B and Kwiecien.J, "*Queuing system and networks. Models and applications*", Bulletin of the polish academy of sciences technical sciences, vol-56, no.4,2008.