Some Properties of Induced and Second Order Induced Intuitionistic Fuzzy Sets

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Abstract: *In this paper we have discussed some basic properties like union , intersection etc. of four different types of induced intuitionistic fuzzy sets, namely,* $A^{\circ}(B), A^{*}(B), A_{\circ}(B)$ *and* $A_{*}(B)$. *We also discussed some properties of second order induced intuitionistic fuzzy sets.*

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1. INTRODUCTION

The notion of Intuitionistic fuzzy set (IFS) was introduced in [1]. Later various properties on it were discussed in [2- 4,6]. The concept of induced intuitionistic fuzzy sets introduced by B.C. Mondal in [5].

In section 2, we have described a few basic definitions, notations, properties and also considered an example which deals with the concept of intuitionistic fuzzy set and induced intuitionistic fuzzy set.

In section 3, we have discussed some properties of induced intuitionistic fuzzy sets.

Finally, in section 4, we have discussed some properties of second order induced intuitionistic fuzzy set and established a few relationship among them .

2. *Preliminaries*

This section contains some basic definitions, notations, properties, properties and an example which are used through-out the paper.

Definition 2.1 [1,2]: Let E be any non-empty set. An intuitionistic fuzzy set A of E is an object of the form $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \}$, where the functions $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denotes the degree of membership and the non - membership of the element x ε E respectively and for every $x \in E, 0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.2 [1,2]: If A and B are two intuitionistic fuzzy sets of a non - empty set E, then $A \subseteq B$ if and only if for all $x \in E$, $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$;

$A = B$ .if and only if for all $x \in E$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$;

$A \cap B = \{ (x, \min(\mu_A(x) = \mu_B(x)), \max(\nu_A(x), \nu_B(x)) : x \,\varepsilon\, E \}$;

$A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) : x \varepsilon E \}$.

Considering the degree of memberships $\mu_A(x), \mu_B(x)$ and the non - memberships $\nu_A(x), \nu_B(x)$of each element $x \in E$ of the intuitionistic fuzzy sets A and B respectively of a non-empty set E, the four types of induced ituitionistic fuzzy sets are defined as follows:

Definition 2.3 [5]: If A and B are two intuitionistic fuzzy sets of a non - empty set E then

$$A^{\circ}(B) = A^{\circ} = \{ \left(x, \mu_A(x), \min\left(\nu_A(x), \nu_B(x)\right) : x \in E \right\};$$

$$A_{\circ}(B) = A_{\circ} = \left\{\left(x, \mu_A(x), \max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E \right\};$$

$$A^{*}(B) = A^{*} = \left\{ (x, \max\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)) : x \in E \right\};$$

$$A_{*}(B) = A_{*} = \left\{(x, \min\left(\mu_A(x), \mu_B(x)\right), \nu_A(x) : x \in E)\right\}.$$

Example2.4 [5]: Let $E = \{a, b, c\}$ and A, B be two intuitionistic fuzzy sets on it with their degree of membership functions μ_A, μ_B and the degree of non-membership functions ν_A, ν_B respectively defined as follows:

$\mu_A(a) = 0.6, \nu_A(a) = 0.2; \mu_A(b) = 0.3, \nu_A(b) = 0.4; \mu_A(c) = 0.2, \nu_A(c) = 0.3;$

$\mu_B(a) = 0.7, \nu_B(a) = 0.1; \mu_B(b) = 0.4, \nu_B(b) = 0.5; \mu_B(c) = 0.1, \nu_B(c) = 0.4$.

Therefore,

$$A^{\circ} = \{(a, 0.6, 0.1), (b, 0.3, 0.4), (c, 0.2, 0.3)\}$$

$$A^{*} = \{(a, 0.7, 0.2), (b, 0.4, 0.4), (c, 0.2, 0.3)\}$$

$$A_{\circ} = \{(a, 0.6, 0.2), (b, 0.3, 0.5), (c, 0.2, 0.4)\}$$

$$A_{*} = \{(a, 0.6, 0.2), (b, 0.3, 0.4), (c, 0.1, 0.3)\}$$

$$B^{\circ} = \{(a, 0.7, 0.1), (b, 0.4, 0.4), (c, 0.1, 0.3)\}$$

$$B^{*} = \{(a, 0.7, 0.1), (b, 0.4, 0.5), (c, 0.2, 0.4)\}$$

$$B_{\circ} = \{(a, 0.7, 0.2), (b, 0.4, 0.5), (c, 0.1, 0.4)\}$$

$$B_{*} = \{(a, 0.6, 0.1), (b, 0.3, 0.5), (c, 0.1, 0.4)\}$$

$$A \cup B = \{(a, 0.7, 0.1), (b, 0.4, 0.4), (c, 0.2, 0.3)\}$$

$$A \cap B = \{(a, 0.6, 0.2), (b, 0.3, 0.5), (c, 0.1, 0.4)\}$$

Note2.5 [5]: From the above example it follows that

$A^{\circ}(B) \neq B^{\circ}(A), A_{\circ}(B) \neq B_{\circ}(A), A^{*}(B) \neq B^{*}(A), A_{*}(B) \neq B_{*}(A)$.

Property2.6 [5]: $A^{\circ} \cup B^{\circ} = A^{*} \cup B^{*} = A^{\circ} \cup B^{*} = AUB$.

Property2.7 [5]: $A^{\circ} \cup B_{\circ} = A^{*} \cup B_{*} = AUB$.

Property 2.8 [5]: $A_{\circ} \cap B_{\circ} = A_{\circ} \cap B_{*} = A_{*} \cap B_{*} = A \cap B$.

3. Some Properties of Induced Intuitionistic Fuzzy Sets.

Property 3.1: $(A \cup B)^{*}(B) = A \cup B = A^{*}(B) \cup B^{*}(A)$.

Proof: $(A \cup B)^{*}(B) = \left\{\left(x, max\left(\mu_{A\cup B}(x), \mu_B(x)\right), \nu_{A\cup B}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= A \cup B = A^{*}(B) \cup B^{*}(A) \quad \text{(by Property 2.6)}.$$

Note 3.2: $(A \cap B)^{*}(B) \neq A^{*}(B) \cap B^{*}(A)$ *follows from the Example2.4.*

In fact,

Property 3.3: $(A \cap B)^{*}(B) = B_{\circ}(A)$.

Proof: $(A \cap B)^{*}(B) = \left\{\left(x, max\left(\mu_{A\cap B}(x), \mu_B(x)\right), \nu_{A\cap B}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(min\left(\mu_A(x), \mu_B(x)\right), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, \mu_B(x), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= B_{\circ}(A).$$

Property 3.4: $(A \cup B)^{\circ}(B) = A \cup B = A^{\circ}(B) \cup B^{\circ}(A)$.

Proof: $(A \cup B)^{\circ}(B) = \left\{\left(x, \mu_{A\cup B}(x), min\left(\nu_{A\cup B}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), min\left(min\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$= A \cup B = A^{\circ}(B) \cup B^{\circ}(A)$ (by Property2.6).

Note3.5: $(A \cap B)^{\circ}(B) \neq A^{\circ}(B) \cap B^{\circ}(A)$ *follows from the Example2.4.*

In fact,

Property3.6: $(A \cap B)^{\circ}(B) = B_*(A)$.

Proof: $(A \cap B)^{\circ}(B) = \left\{\left(x, \mu_{A\cap B}(x), min\left(\nu_{A\cap B}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), min\left(max\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), \nu_B(x)\right) : x \in E\right\}$

$= B_*(A)$.

Property3.7: $(A \cup B)_*(B) = B^{\circ}(A)$.

Proof: $(A \cup B)_*(B) = \left\{\left(x, min\left(\mu_{A\cup B}(x), \mu_B(x)\right), \nu_{A\cup B}(x)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(max\left(\mu_A(x), \mu_B(x)\right), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, \mu_B(x), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= B^{\circ}(A)$.

Property 3.8: $(A \cap B)_*(B) = A \cap B = (A \cap B)_{\circ}(B)$.

Proof: $(A \cap B)_*(B) = \left\{\left(x, min\left(\mu_{A\cap B}(x), \mu_B(x)\right), \nu_{A\cap B}(x)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(min\left(\mu_A(x), \mu_B(x)\right), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\} = A \cap B$.

Also, $(A \cap B)_{\circ}(B) = \left\{\left(x, \mu_{A\cap B}(x), max\left(\nu_{A\cap B}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), max\left(max\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\} = A \cap B$.

Property 3.9: $(A \cup B)_{\circ}(B) = B^*(A)$

Proof: $(A \cup B)_{\circ}(B) = \left\{\left(x, \mu_{A\cup B}(x), max\left(\nu_{A\cup B}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), max\left(min\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), \nu_B(x)\right) : x \in E\right\} = \mathrm{B}^*.$$

4. Some Properties of Second Order Induced Intuitionistic Fuzzy Sets.

In this section we describe some properties of induced intuitionistic fuzzy sets of an induced intuitionistic fuzzy sets on a set E, called second order induced intuitionistic fuzzy sets.

Property 4.1 (Idempotent law):

i) $(A^*)^*(B) = A^*(B)$

ii) $(A^{\circ})^{\circ}(B) = A^{\circ}(B)$

iii) $(A_*)_*(B) = A_*(B)$

iv) $(A_{\circ})_{\circ}(B) = A_{\circ}(B)$

Proof: i) $(A^*)^*(B) = \left\{\left(x, max\left(\mu_{A^*}(x), \mu_B(x)\right), \nu_{A^*}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, max\left(max\left(\mu_A(x), \mu_B(x)\right), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\}$$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\}$$

$$= A^*(B).$$

ii) $(A_*)_*(B) = \left\{\left(x, min\left(\mu_{A_*}(x), \mu_B(x)\right), \nu_{A_*}(x)\right) : x \in E\right\}$

$$= \left\{\left(x, min\left(min\left(\mu_A(x), \mu_B(x)\right), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\}$$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\}$$

$$= A_*(B).$$

iii) $(A^{\circ})^{\circ}(B) = \left\{\left(x, \mu_{A^{\circ}}(x), min\left(\nu_{A^{\circ}}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$$= \left\{\left(x, \mu_A(x), min\left(min\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right): x \in E\right\}$$

$$= \left\{\left(x, \mu_A(x), min\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$$= A^{\circ}(B).$$

iv) $(A_{\circ})_{\circ}(B) = \left\{\left(x, \mu_{A_{\circ}}(x), max\left(\nu_{A_{\circ}}(x), \nu_B(x)\right)\right): x \in E\right\}$

$$= \left\{\left(x, \mu_A(x), max\left(max\left(\nu_A(x), \nu_B(x)\right), \nu_B(x)\right)\right): x \in E\right\}$$

$$= \left\{\left(x, \mu_A(x), max\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$$= A_{\circ}(B).$$

Property 4.2: $(A^{\circ})^*(B) = (A^*)^{\circ}(B) = A \cup B$

Proof: $(A^{\circ})^*(B) = \left\{\left(x, \mu_A(x), min\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}^* (B)$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$$= A \cup B.$$

Also, $(A^*)^{\circ}(B) = \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right): x \in E\right\}^{\circ} (B)$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$$= A \cup B.$$

Property 4.3: $(A_{\circ})_*(B) = (A_*)_{\circ}(B) = A \cap B$

Proof: $(A_{\circ})_*(B) = \left\{\left(x, \mu_A(x), max\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}_* (B)$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$$= A \cap B .$$

Also, $(A_*)_{\circ}(B) = \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right): x \in E\right\}_{\circ} (B)$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right): x \in E\right\}$$

$= A \cap B.$

Property 4.4: $(A_\circ)^*(B) = (A^*)_\circ(B)$

Proof: $(A_\circ)^*(B) = \left\{\left(x, max\left(\mu_{A_\circ}(x), \mu_B(x)\right), \nu_{A_\circ}(x)\right) : x \in E\right\}$

$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, \mu_{A^*}(x), max\left(\nu_{A^*}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= (A^*)_\circ(B).$

Property 4.5: $(A_*)^\circ(B) = (A^\circ)_*(B)$

Proof: $(A_*)^\circ(B) = \left\{\left(x, \mu_{A_*}(x), min\left(\nu_{A_*}(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$

$= \left\{\left(x, min\left(\mu_{A^\circ}(x), \mu_B(x)\right), \nu_{A^\circ}(x)\right) : x \in E\right\}$

$= (A^\circ)_*(B).$

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