Some results on relative L^* –order of analytic function in the unit polydisc

Balram Prajapati[1], Rakesh Kumar[2], Anupma Rastogi[3]
[1]Research Scholar, [2]Research Scholar, [3]Assistant Professor & Department of Mathematics & Astronomy &
University of Lucknow, Lucknow, Uttar Pradesh, India

Abstract *- In this paper we proved some earlier results on the basis of relative L^*-order of analytic function in a unit polydisc*

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Introduction, Definition and Notation

Let f be function in the unit disc $U = \{z : |z| < 1\}$ is said to be of finite Nevanlinna order [8] if their exist a number μ such that the Nevanlinna characteristic function

$$T_f(r) = \frac{1}{2\pi}\int_0^{2\pi} \log\left|f\left(re^{i\theta}\right)\right| d\theta$$

satisfies $T_f(r) < (1-r)^{-\mu}$ for all $0 < r_0(\mu) < r < 1$.
The greatest lower bound if all such number μ is called the Nevanlinna order off. Thus the Nevanlinna order ρ_f off is given by

$$\rho_f = \lim_{r\to 1}\sup \frac{\log T_f(r)}{-\log(1-r)}$$

Similarly, the Nevanlinna lower order λ_f of f are given by

$$\lambda_f = \lim_{r\to 1}\inf \frac{\log T_f(r)}{-\log(1-r)}$$

Banerjee and Datta [5] give the following definition in a unit disc.

***Definition* 1.** [5] of f be analytic in U and g be entire, the relative order of f with respect to g denoted by $\rho_g(f)$ is defined by

$$\rho_g(f) = \inf\left\{\mu > 0 : T_f(r) < T_g\left[\left(\frac{1}{1-r}\right)^{\mu}\right], \text{for all } 0 < r_0(\mu) < r < 1\right\}$$

Similarly, one may define $\lambda_g(f)$, the relative lower order off with respect to g, with $g(z) = \exp z$. The definition coincides with the definition of Nevanlinna order off.

$$\lambda_g(f) = \lim_{r\to 1}\inf \frac{\log T_g^{-1} T_f(r)}{-\log(1-r)}$$

Extending the nation of single variables to several variables, let $f(z_1, z_2, \dots . z_n)$ be a non-constant analytic function of n complex variables $z_1, z_2, \dots . z_n$ in the unit polydisc

$$U = \left\{(z_1, z_2, \dots . z_n) : \left|z_j\right| \le 1, j = 1,2, \dots, n\,;\ r_1 > 0, r_2 > 0, \dots . r_n > 0\right\}$$

generalized n-variables k^{th} Nevanlinna order and the generalized n-variables k^{th} Nevanlinna lower order for functions of n complex variables analytic in a unit polydisc as follows.

$$_{v_n}\rho_f^{[k]} = \lim_{r_1, r_2, \dots, r_n \to 1^-} \frac{\log^{[k]} T_f(r_1, r_2, \dots ., r_n)}{-\log[(1-r_1)(1-r_2) \dots . (1-r_n)]}$$

and

$$_{v_n}\lambda_f^{[k]} = \lim_{r_1, r_2, \dots, r_n \to 1^-} \frac{\log^{[k]} T_f(r_1, r_2, \dots ., r_n)}{-\log[(1-r_1)(1-r_2) \dots . (1-r_n)]}$$

where $\log^{[l]} x = \log\left(\log^{[l-1]} x\right)$ for $l = 1,2, \dots$ and $\log^{[0]} x = x$ when $n = k = 1$, the above definition reduces to the definition of Juneja and Kapoor [8].

***Definition* 2.** Generalized n-variables k^{th}-relative Nevanlinna order and the generalized n-variables k^{th}-relative Nevalinna lower order for functions of n-complex variables analytic in unit polydisc as follows:

$$_{v_n}\rho_g^{[k]}(f) = \lim_{r_1,r_2,\dots,r_n\to 1^-}\sup\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{-\log[(1-r_1)(1-r_2)\dots(1-r_n)]}$$

and

$$_{v_n}\lambda_g^{[k]}(f) = \lim_{r_1,r_2,\dots,r_n\to 1^-}\inf\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{-\log[(1-r_1)(1-r_2)\dots(1-r_n)]}$$

where k and n are any two positive integers. If we consider $k = n = 1$ in definition 2, then it coincides with definition 1.

Somasundaram and Thamizharasi [6] introduced the notion of L-order for entire functions where $L \equiv L(r_1, r_2, \dots, r_n)$ is a positive continuous function increasing slowly i.e., $L(ar_1, ar_2, \dots, ar_n) \sim L(r_1, r_2, \dots, r_n)$ as $r_1, r_2, \dots, r_n$ is a positive constant a.

***Definition* 3.** The generalized L-order $\left[_{v_n}\rho_f^{[k]}\right]^{L^*}$ and $\left[_{v_n}\lambda_f^{[k]}\right]^{L^*}$

$$\left[_{v_n}\rho_f^{[k]}\right]^{L^*} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\sup\frac{\log^{[k]} T_f(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}$$

and

$$\left[_{v_n}\lambda_f^{[k]}\right]^{L^*} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\inf\frac{\log^{[k]} T_f(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}.$$

***Definition* 4.** The generalized L-order (alternatively generalized relative L-order) $\left[_{v_n}\rho_g^{[k]}(f)\right]^{L}$ and relative (alternatively generalized relative L-lower order) $\left[_{v_n}\lambda_g^{[k]}(f)\right]^{L}$ of analytic function f in U (unit polydisc) with respect to another entire function g are defined as

$$\left[_{v_n}\rho_g^{[k]}(f)\right]^{L} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\sup\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}$$

and

$$\left[_{v_n}\lambda_g^{[k]}(f)\right]^{L} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\inf\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}.$$

***Definition* 5.** The relative generalized L^*-order (alternatively generalized relative L^*-order) $\left[_{v_n}\rho_g^{[k]}(f)\right]^{L^*}$ and relative generalized L^*-lower order (alternatively generalized relative L^*-lower order) $\left[_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}$ of analytic function f in U (unit polydisc) with respect to another function g are defined as

$$\left[_{v_n}\rho_g^{[k]}(f)\right]^{L^*} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\sup\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}$$

and

$$\left[_{v_n}\rho_g^{[k]}(f)\right]^{L^*} = \lim_{r_1,r_2,\dots,r_n\to 1^-}\inf\frac{\log^{[k]} T_g^{-1}T_f(r)(r_1,r_2,\dots,r_n)}{\log\left[\frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right]}.$$

The following definition is also well known.

***Definition* 6.** Two entire function f and g are said to be asymptotically equivalent if there exist $0 \le \alpha < \infty$, such that $\frac{F(r)}{G(r)} \to \alpha$ as $\alpha \to \infty$ and in this case we write $f \sim g$. If $f \sim g$ then clearly $g \sim f$

In the paper we establish some results relating to the composition of two non-constant analytic functions of n complex variables in the unit polydisc

$$U = \left\{(z_1, z_2, \dots, z_n) : \left|z_j\right| \le 1, j = 1, 2, \dots, n;\ r_1 > 0, r_2, \dots, r_n > 0\right\}$$

Also we prove a few theorems related to generalized n −variables based k^{th} relative L^*-Nevanlinna order $\left[_{v_n}\rho_g^{[k]}(f)\right]^{L^*}$ (generalized n-variables based k^{th}relative L^*-Nevanlinna lower order $\left[_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}$ of an analytic function f with respect to an entire function g of n complex variables. Which are in fact some entertains of earlier results. We do not explain the standard definitions and notations in the theory of entire functions are available in [7][1][4][2][3].

***Theorem* 1.** Let f and g be any two non constant analytic functions of n-complex variables in the unit polydisc U such that $0 < \left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} \le \left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{\nu_n}\lambda_{g}^{[l]}\right]^{L^*} \le \left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*} < \infty$. Then

$$\frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*}} \le \lim_{r_1,r_2,\dots,r_n\to 1^-} \inf \frac{\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n)}{\log^{[k]} T_g(r_1,r_2,\dots,r_n)}$$

$$\le \frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*}} \le \lim_{r_1,r_2,\dots,r_n\to 1^-} \sup \frac{\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n)}{\log^{[l]} T_g(r_1,r_2,\dots,r_n)} \le \frac{\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_{g}^{[l]}\right]^{L^*}}$$

where k and l are any two positive integers.

***Proof*.** From the definition of generalized n-variables based k^{th} Nevanlinna L^*- order and generalized n-variables k^{th} Nevanlinna L^*- lower order of analytic functions in the unit polydisc U, we have for arbitrary positive ε and for all sufficiently large values of $\frac{1}{1-r_1}, \frac{1}{1-r_2}, \dots$ and $\frac{1}{1-r_n}$ that

$$\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n) \ge \left(\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)} \exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right] \quad (1)$$

and

$$\log^{[l]} T_g(r_1,r_2,\dots,r_n) \ge \left(\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*} + \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)} \exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right] \quad (2)$$

now from(1)&(2), it follows for all sufficiently large values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \dots$ and $\left(\frac{1}{1-r_n}\right)$ that

$$\frac{\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n)}{\log^{[l]} T_g(r_1,r_2,\dots,r_n)} \ge \frac{\left(\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} - \varepsilon\right)}{\left(\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*} + \varepsilon\right)}$$

as $\varepsilon > 0$ is positive we obtain that

$$\lim_{r_1,r_2,\dots,r_n\to 1^-} \frac{\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n)}{\log^{[l]} T_g(r_1,r_2,\dots,r_n)} \ge \frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*}} \quad (3)$$

again for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \dots$ and $\left(\frac{1}{1-r_n}\right)$, tending to infinity.

$$\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n) \ge \left(\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} + \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)} \exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right] \quad (4)$$

and for all sufficiently large values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \dots$ and $\left(\frac{1}{1-r_n}\right)$

$$\log^{[l]} T_g(r_1,r_2,\dots,r_n) \ge \left(\left[{}_{\nu_n}\lambda_{g}^{[l]}\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\dots(1-r_n)} \exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\dots,\frac{1}{1-r_n}\right)\right] \quad (5)$$

so combining (4)&(5), we get for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \dots$ and $\left(\frac{1}{1-r_n}\right)$, tending to infinity that

$$\frac{\log^{[k]} T_{fog}(r_1,r_2,\dots,r_n)}{\log^{[l]} T_g(r_1,r_2,\dots,r_n)} \le \frac{\left(\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} + \varepsilon\right)}{\left(\left[{}_{\nu_n}\rho_{g}^{[l]}\right]^{L^*} - \varepsilon\right)}$$

since $\varepsilon(> 0)$ is arbitrary, it follows that

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*}} \tag{6}$$

also for a sequence of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots.$ and $\left(\frac{1}{1-r_n}\right)$, tending to infinity, we get

$$\log^{[l]} T_g(r_1,r_2,\ldots,r_n) \le \left(\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*} + \varepsilon\right)\left[\log\frac{1}{(1-r_1)(1-r_2)\ldots.(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\right] \tag{7}$$

now from (1)&(7), we obtain for a sequence of values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots.$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)} \ge \frac{\left(\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} - \varepsilon\right)}{\left(\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*} + \varepsilon\right)}$$

choosing $\varepsilon \to 0$, we get that

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \sup\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)} \ge \frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*}} \tag{8}$$

also for all sufficiently large values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots.$ and $\left(\frac{1}{1-r_n}\right)$,

$$\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n) \le \left(\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*} + \varepsilon\right)\left[\log\frac{1}{(1-r_1)(1-r_2)\ldots.(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\right] \tag{9}$$

so form(5)&(9), it follows for all sufficiently large values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots.$ and $\left(\frac{1}{1-r_n}\right)$, that

$$\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)} \le \frac{\left(\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*} + \varepsilon\right)}{\left(\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*} - \varepsilon\right)}$$

as$\varepsilon(>0)$ is arbitrary, we obtain that

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[l]}\right]^{L^*}} \tag{10}$$

Thus the theorem (3), (6), (8)&(10)the following theorem can be proved in the line of theorem 1 and so its proof is omitted.

***Theorem* 2.** Let f and g be any two non constant analytic functions of n-complex variables in the unit polydisc U with $0 < \left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*} \le \left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{\nu_n}\lambda_f^{[s]}\right]^{L^*} \le \left[{}_{\nu_n}\rho_f^{[s]}\right]^{L^*} < \infty$. and k and s are any two positive integers. Then

$$\frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_f^{[s]}\right]^{L^*}} \le \lim_{r_1,r_2,\ldots,r_n\to 1} \inf\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[s]} T_f(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[s]}\right]^{L^*}} \le \lim_{r_1,r_2,\ldots,r_n\to 1} \sup\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[s]} T_f(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_f^{[s]}\right]^{L^*}}.$$

***Theorem* 3.** Let f and g be any two non constant analytic functions of n-complex variables in the unit polydisc U such that $0 < \left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{\nu_n}\rho_g^{[l]}\right]^{L^*} < \infty$. Then

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \inf\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_g^{[l]}\right]^{L^*}} \le \lim_{r_1,r_2,\ldots,r_n\to 1} \sup\frac{\log^{[k]} T_{fog}(r_1,r_2,\ldots,r_n)}{\log^{[l]} T_g(r_1,r_2,\ldots,r_n)}$$

where k and l are any two positive integers.

***Proof*.** From the definition of generalized n-variables based k^{th}-Nevanlinna L^*-order, we get for sequence of values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots.$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\log^{[l]} T_g(r_1, r_2, \ldots, r_n) \le \left(\left[{}_{v_n}\rho_g^{[k]}\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (11)$$

now form (9)&(11), it follows from a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[l]} T_g(r_1, r_2, \ldots, r_n)} \le \frac{\left(\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} + \varepsilon\right)}{\left(\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*} - \varepsilon\right)},$$ as $\varepsilon(>0)$ is arbitrary,then $$\lim_{r_1, r_2, \ldots, r_n \to 1} \inf \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[l]} T_g(r_1, r_2, \ldots, r_n)} \le \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*}} \quad (12)$$

again for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity

$$\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n) \ge \left(\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (13)$$

so combining (2)&(13), we get for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[l]} T_g(r_1, r_2, \ldots, r_n)} \ge \frac{\left(\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} - \varepsilon\right)}{\left(\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*} + \varepsilon\right)}$$

since $\varepsilon(>0)$ is arbitrary, it follows that

$$\lim_{r_1, r_2, \ldots, r_n \to 1} \sup \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[l]} T_g(r_1, r_2, \ldots, r_n)} \ge \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*}} \quad (14)$$

Thus the theorem follows from (12)&(14).

***Theorem* 4.** Let f and g be any two non-constant analytic functions of n-complex variables in the unit polydisc Uwith $0 < \left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{v_n}\rho_f^{[s]}\right]^{L^*} < \infty$. Where k and s are any two positive integer then

$$\lim_{r_1, r_2, \ldots, r_n \to 1} \inf \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[s]} T_f(r_1, r_2, \ldots, r_n)} \le \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_f^{[s]}\right]^{L^*}} \le \lim_{r_1, r_2, \ldots, r_n \to 1} \sup \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[s]} T_f(r_1, r_2, \ldots, r_n)}$$

The following theorem is a natural consequence of theorem 1 and theorem 3.

***Theorem* 5.** Let f and g be any two non constant analytic functions of n-complex variables in the unit polydisc U such that $0 < \left[{}_{v_n}\lambda_{fog}^{[k]}\right]^{L^*} \le \left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{v_n}\lambda_g^{[l]}\right]^{L^*} \le \left[{}_{v_n}\rho_g^{[l]}\right]^{L^*} < \infty$. Then

$$\lim_{r_1, r_2, \ldots, r_n \to 1^-} \inf \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g(r_1, r_2, \ldots, r_n)} \le \min\left\{\frac{\left[{}_{v_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[l]}\right]^{L^*}}, \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*}}\right\}$$
$$\le \max\left\{\frac{\left[{}_{v_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[l]}\right]^{L^*}}, \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[l]}\right]^{L^*}}\right\} \le \lim_{r_1, r_2, \ldots, r_n \to 1} \sup \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[l]} T_g(r_1, r_2, \ldots, r_n)}$$

where k and l are any two positive integers.

***Theorem* 6.** Let f and g be any two non constant analytic functions of n-complex variables in the unit polydisc U with$0 < \left[{}_{v_n}\lambda_{fog}^{[k]}\right]^{L^*} \le \left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*} < \infty$ and $0 < \left[{}_{v_n}\lambda_f^{[s]}\right]^{L^*} \le \left[{}_{v_n}\rho_f^{[s]}\right]^{L^*} < \infty$. where k and s are any two positive integers.then

$$\lim_{r_1, r_2, \ldots, r_n \to 1^-} \inf \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[s]} T_f(r_1, r_2, \ldots, r_n)} \le \min\left\{\frac{\left[{}_{v_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\lambda_f^{[s]}\right]^{L^*}}, \frac{\left[{}_{v_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{v_n}\rho_f^{[s]}\right]^{L^*}}\right\}$$

$$\leq \max\left\{\frac{\left[{}_{\nu_n}\lambda_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\lambda_f^{[s]}\right]^{L^*}}, \frac{\left[{}_{\nu_n}\rho_{fog}^{[k]}\right]^{L^*}}{\left[{}_{\nu_n}\rho_f^{[s]}\right]^{L^*}}\right\} \leq \lim_{r_1,r_2,\ldots,r_n\to 1} \sup \frac{\log^{[k]} T_{fog}(r_1, r_2, \ldots, r_n)}{\log^{[s]} T_f(r_1, r_2, \ldots, r_n)}$$

we establish some comparative growth properties related to generalized n-variables based k^{th} relative L^* Nevanlinna order (generalized n-variables based k^{th} relative L^*-Nevanlinna lower order) of an analytic function with respect to an entire function in the unit polydisc U.

***Theorem* 7.** Let f, h be any two non-constant analytic functions of n-complex variables inU and g be entire in n complex varibales such $0 < \left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*} \leq \left[{}_{\nu_n}\rho_g^{[k]}(f)\right]^{L^*} < \infty$ and $0 < \left[{}_{\nu_n}\lambda_g^{[k]}(h)\right]^{L^*} \leq \left[{}_{\nu_n}\rho_g^{[k]}(h)\right]^{L^*} < \infty$. Then

$$\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*} \leq \lim_{r_1,r_2,\ldots,r_n\to 1} \inf \frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)}$$

$$\leq \frac{\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[k]}(h)\right]^{L^*}} \leq \lim_{r_1,r_2,\ldots,r_n\to 1} \sup \frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)} \leq \frac{\left[{}_{\nu_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{\nu_n}\lambda_g^{[k]}(h)\right]^{L^*}}$$

where k is any positive integer.

***Proof*.** From the definition of generalized n-variables based k^{th} relative L^* Nevanlinna order and generalized n-variables based k^{th} relative L^*-Nevanlinna lower order of an analytic function with respect to an entire functions in an unit polydisc U, we have for arbitrary positive ε and for all sufficiently large values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ that

$$\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)$$
$$\geq \left(\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (15)$$

$$\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)$$
$$\leq \left(\left[{}_{\nu_n}\rho_g^{[k]}(h)\right]^{L^*} + \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (16)$$

now from(15)&(16), it follows for all sufficiently large values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)} \geq \frac{\left(\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*} - \varepsilon\right)}{\left(\left[{}_{\nu_n}\rho_g^{[k]}(h)\right]^{L^*} + \varepsilon\right)}$$

as $\varepsilon > 0$ is arbitrary, we obtain that

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \inf \frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)} \geq \frac{\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{\nu_n}\rho_g^{[k]}(h)\right]^{L^*}} \quad (17)$$

again we have for a sequence values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)$$
$$\leq \left(\left[{}_{\nu_n}\lambda_g^{[k]}(f)\right]^{L^*} + \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (18)$$

and for all sufficiently large values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$,

$$\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)$$
$$\geq \left(\left[{}_{\nu_n}\lambda_g^{[k]}(h)\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \quad (19)$$

so combining (18)&(19), we get for sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)} \le \frac{\left(\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}+\varepsilon\right)}{\left(\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}-\varepsilon\right)}$$

since $\varepsilon(>0)$ is arbitrary, it follows that

$$\lim_{r_1,r_2,\ldots,r_n\to 1}\inf\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)} \ge \frac{\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}} \tag{20}$$

also for sequence of values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity,

$$\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n) \ge \left(\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}+\varepsilon\right)\left[\log\frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\right] \tag{21}$$

now from (15)&(21), we obtain for sequence of values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)} \ge \frac{\left(\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}-\varepsilon\right)}{\left(\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}+\varepsilon\right)}$$

choosing $\varepsilon>0$ is arbitrary that

$$\lim_{r_1,r_2,\ldots,r_n\to 1}\sup\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)} \ge \frac{\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}} \tag{22}$$

also for all sufficiently large values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$,

$$\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n) \le \left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}+\varepsilon\right)\left[\log\frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\right] \tag{23}$$

so for (19)&(23), it follows for all sufficiently large values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$ that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)} \le \frac{\left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}+\varepsilon\right)}{\left(\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}-\varepsilon\right)}$$

as $\varepsilon(>0)$ is arbitrary, we obtain from above that

$$\lim_{r_1,r_2,\ldots,r_n\to 1}\sup\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}} \tag{24}$$

Thus the theorem follows from (17), (20), (22)&(24).

***Theorem* 8.** Let f, h be any two analytic functions of n-complex variables in U and g be entire in n complex variables with$0<\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}<\infty$ and $0<\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}<\infty$. where P is any positive integer then

$$\lim_{r_1,r_2,\ldots,r_n\to 1}\inf\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}} \le \lim_{r_1,r_2,\ldots,r_n\to 1}\sup\frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)}$$

***Proof*.** From the definition of generalized n-variables based k^{th}- relative L^*-Nevanlinna order we get for a sequence values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity, that

$$\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n) \ge \left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}-\varepsilon\right)\left[\log\frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)}\exp L\left(\frac{1}{1-r_1},\frac{1}{1-r_2},\ldots,\frac{1}{1-r_n}\right)\right] \tag{25}$$

now from (23)&(25), it follows for a sequence of values of $\left(\frac{1}{1-r_1}\right),\left(\frac{1}{1-r_2}\right),\ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity, that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)} \le \frac{\left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*} + \varepsilon\right)}{\left(\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*} - \varepsilon\right)}$$

as $\varepsilon(> 0)$ is arbitrary, we get

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \inf \frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)} \le \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}} \tag{26}$$

again for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity,

$$\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n) \ge \left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*} - \varepsilon\right)\left[\log \frac{1}{(1-r_1)(1-r_2)\ldots(1-r_n)} \exp L\left(\frac{1}{1-r_1}, \frac{1}{1-r_2}, \ldots, \frac{1}{1-r_n}\right)\right] \tag{27}$$

so combining (16)&(27), we get for a sequence of values of $\left(\frac{1}{1-r_1}\right), \left(\frac{1}{1-r_2}\right), \ldots$ and $\left(\frac{1}{1-r_n}\right)$ tending to infinity, that

$$\frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)} \ge \frac{\left(\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*} - \varepsilon\right)}{\left(\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*} + \varepsilon\right)}$$

since $\varepsilon(> 0)$ is arbitrary, it follows that

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \sup \frac{\log^{[k]} T_g^{-1}T_f(r_1,r_2,\ldots,r_n)}{\log^{[k]} T_g^{-1}T_h(r_1,r_2,\ldots,r_n)} \ge \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}} \tag{28}$$

thus the theorem from (26)&(28) from theorem 7& theorem 8 we may state the following theorem without proof.

***Theorem* 9.** Let f, h be any two analytic functions of n complex variables in U and g be entire in n-complex variables such that $0 < \left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*} \le \left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*} < \infty$ and $0 < \left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*} \le \left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*} < \infty$. Then

$$\lim_{r_1,r_2,\ldots,r_n\to 1} \inf \frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)} \le \min\left\{\frac{\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}}, \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}}\right\}$$

$$\le \max\left\{\frac{\left[{}_{v_n}\lambda_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\lambda_g^{[k]}(h)\right]^{L^*}}, \frac{\left[{}_{v_n}\rho_g^{[k]}(f)\right]^{L^*}}{\left[{}_{v_n}\rho_g^{[k]}(h)\right]^{L^*}}\right\} \le \lim_{r_1,r_2,\ldots,r_n\to 1} \sup \frac{\log^{[k]} T_g^{-1}T_f(r_1, r_2, \ldots, r_n)}{\log^{[k]} T_g^{-1}T_h(r_1, r_2, \ldots, r_n)}$$

where k is any positive integer.

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