GREGUS TYPE COMMON FIXED POINT THEOREMS in b-METRIC SPACE with APPLICATION

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Abstract:- *The aim of this paper is to establish Gregus Type common fixed point theorems in b-metric spaces. Our results improve and extend some known results. An example is given to show that our results are proper generalizations of the existing ones and we also provide an application.*

2000 Mathematics Subject Classification : 47H10, 54H25

Key Words and Phrases *: Fixed point, Common fixed point, b-metric space, b-compatible.*

I. INTRODUCTION and PRELIMINARIES

Fixed point theory is an important branch of the functional analysis. In 1989 Bakhtin [1] worked on the Contraction mapping principle in metric space. In 1993 Czerwik [3] extended the result of Contraction mappings in b-metric space. We see also the fixed point theory of the contraction principal for single valued and multivalued mappings in b-metric space was used by many authors in [6], Shah [9], Khamsi [13]. The notion of b-metric space has been introduced and generalization of Banach [2] fixed point theorem. Recently, Hussain and Mitrovic [7] proved fixed point results for multivalued weak quasi contractions in a b-metric space with the increased range of the Lipschitzian constants.

Defination 1: Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d: X\times X \to R^+$ is said to be b-metric if and only if for all $x,y,z \in X$ the following conditions are satisfied:

(1) $d(x,y) = 0$ if and only if $x = y$;
(2) $d(x,y) = d(y,x)$;
(3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.
A triplet (X, d, s), is called a b-metric space with coefficient 's'.

The classical spaces l^p R and $L^p[0, 1]$, $p \in (0, 1)$, are examples of b-metric spaces. The concept of convergence in such space is similar to that of the standard metric spaces.

Defination 2

Let (X,d) be a b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then

i. The sequence $\{x_n\}$ is said to be convergent in (X,d) and converges to x, if for every $\varepsilon > 0$ there exist $n_0 \in N$ such that $d(x_n,x) < \epsilon$ for all $n > n_o$. This fact is represented by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n\to\infty$.

ii. The sequence $\{x_n\}$ is said to be a Cauchy sequence in (X,d) if for every $\varepsilon > 0$ there exist $n_0 \in N$ such that $d(x_n, x_{n+p}) < \varepsilon$ for all $n > n_0$, $p > 0$ or equivalently, if $\lim_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.

iii. (X,d) is said to be a complete b-metric space if every cauchy sequence in X converges to some $x \in X$.

Definition 3. Self maps f and g of a b-metric space (X, d, s) are b-compatible if $\lim_{n\to\infty} d(fgx_n\ gfx_n) = 0$, when $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

The following recent result of Miculescu and Mihail [14] is useful in the context of b-metric spaces.

Lemma 1. [15] Every sequence $(x_n)_{n\in N}$ of elements from a b-metric space (X, d, s), having the property that there exist $\gamma \in [0, 1)$ such that

$$d(x_{n+1}, x_n) \leq \gamma\, d(x_n, x_{n-1})$$

for every $n \in N$, is Cauchy.

The aim of this paper is to obtain some Gregus type fixed point theorem for four mappings [5] in b-metric space.

II. MAIN RESULT

Now, we present our result on common fixed point theorem of Gregus type for four mappings in the context of b-metric spaces.

Theorem 1. Let the pairs (S, I) and (T , J) be b-compatible defined on a complete b-metric space (X, d, s) and satisfying

$$d(Sx, Ty) \leq a\,\frac{(d(Ix,\ Jy) + d(Ty,\ Jy) + d(Sx,\ Jy))}{3} + (1-a)\max\left\{d(Ix, Jy), d(Sx, Ix), d(Ty, Jy), \frac{d(Sx,\ Jy) + d(Ty,\ Ix)}{2s}\right\} \quad (1)$$

for all $x, y \in X$ where $0 \leq a < 1$. If $S(X) \subseteq J(X)$, $T(X) \subseteq I(X)$ and if I, J, S and T are continuous, then I, J, S and T have a unique common fixed point.

Proof. Assume that $a \in (0, 1)$. Let $x_0 \in X$ be arbitrary. Since $S x_0 \in J(X)$, there is any $x_1 \in X$ such that $Jx_1 = Sx_0$, and also as $T x_1 \in I(X)$, let $x_2 \in X$ be such that $Ix_2 = Tx_1$.
In general, $x_{2n+1} \in X$ is chosen such that $J x_{2n+1} = S x_{2n}$ and $x_{2n+2} \in X$ such that $I x_{2n+2} = T x_{2n+1}$, $n = 0, 1, 2, \ldots$. Denote a sequences y_n with

$$y_{2n} = J x_{2n+1} = S x_{2n},$$
$$y_{2n+1} = I x_{2n+2} = T x_{2n+1},$$

$n = 0, 1, 2, \ldots$. We show that sequence y_n is a cauchy sequence. We suppose that $d(y_{2n}, y_{2n+1}) > 0$ for every n. If not then for some k, $y_{2k+1} = y_{2k}$.
And from (1) We obtain

$$\begin{aligned} d(y_{2k+2}, y_{2k+1}) &= d(Sx_{2k+2}, T x_{2k+1}) \\ &\leq a\,\frac{(d(Ix2k+2,\ Jx2k+1) + d(T x2k+1,\ Jx2k+1) + d(Sx2k+2,\ Jx2k+1))}{3} \\ &\quad + (1-a)\max\Big\{ d(Ix_{2k+2}, Jx_{2k+1}), d(Sx_{2k+2}, Ix_{2k+2}), d(T x_{2k+1}, Jx_{2k+1}), \\ &\quad \frac{d(Sx2k+2,\ Jx2k+1) + d(T x2k+1,\ Ix2k+2)}{2s} \Big\} \\ &\leq a\,\frac{d(y2k+1,\ y2k) + d(y2k+1,\ y2k) + d(y2k+2,\ y2k)}{3} \\ &\quad + (1-a)\max\Big\{ d(y_{2k+1}, y_{2k}), d(y_{2k+2}, y_{2k+1}), d(y_{2k+1}, y_{2k}), \\ &\quad \frac{d(y2k+2,\ y2k) + d(y2k+1,\ y2k+1)}{2s}\Big\} \\ &\leq a\,\frac{d(y2k+1,\ y2k) + d(y2k+2,\ y2k)}{3} + (1-a)\max\Big\{ d(y_{2k+1}, y_{2k}), \\ &\quad d(y_{2k+2}, y_{2k+1}), \frac{d(y2k+2,y2k) + 0}{2s} \Big\} \\ &\leq a\,\frac{d(y2k+2,\ y2k)}{3} + (1-a)\max\Big\{d(y_{2k+2}, y_{2k+1}), d(y_{2k+1}, y_{2k}), \frac{d(y2k+2,\ y2k)}{2s}\Big\} \end{aligned}$$

$$\leq a\frac{d(y2k+2,\ y2k)}{3} + (1-a)\max\{d(y_{2k+2}, y_{2k}), d(y_{2k+1}, y_{2k})\}$$
$$\leq a\frac{d(y2k+2,\ y2k)}{3} + (1-a)\, d(y_{2k+2}, y_{2k+1})$$
$$\leq a\frac{d(y2k+2,\ y2k+1) + d(y2k+1,\ y2k)}{3}3 + (1-a)d(y_{2k+2}, y_{2k+1})$$
$$\leq a\frac{d(y2k+2,\ y2k+1)}{3} + (1-a)d(y_{2k+2}, y_{2k+1})$$
$$\leq \frac{(1-2a)}{3}d(y_{2k+2}, y_{2k+1}).$$

Which gives $d(y_{2k+2}, y_{2k+1}) = 0$ and so $y_{2k+2} = y_{2k+1}$ which further implies that $y_{2k+2} = y_{2k+3}$. Thus y_n becomes a constant sequence and y_{2k} is a common fixed point of S, T, I and J.
Now take $d(y_{2n}, y_{2n+1}) > 0$ for each n. From (1) we have

$$d(y_{2n+1}, y_{2n}) = d(Sx_{2n}, T\,x_{2n+1})$$
$$\leq a\frac{d(Ix2n,\ Jx2n+1) + d(T\,x2n+1,\ Jx2n+1) + d(Sx2n,\ Jx2n+1)}{3}$$
$$+ (1-a)\max\{d(Ix_{2n}, Jx_{2n+1}), d(Sx_{2n}, Ix_{2n}), d(T\,x_{2n+1}, Jx_{2n+1}),$$
$$\frac{d(Sx2n,\ Jx2n+1) + d(T\,x2n+1,\ Ix2n)}{2s}\}$$
$$\leq a\frac{d(y2n-1,y2n) + d(y2n+1,y2n) + d(y2n,y2n)}{3}$$
$$+ (1-a)\max\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n}),$$
$$\frac{d(y2n,y2n) + d(y2n+1,y2n-1)}{2s}\}.$$

So,

$$d(y_{2n+1}, y_{2n}) \leq a\frac{d(y2n-1,y2n) + d(y2n+1,y2n)}{3} + (1-a)$$
$$\max\{d(y_{2n-1}, y_{2n}),\ d(y_{2n+1}, y_{2n}),\ \frac{d(y2n+1,y2n-1)}{2s}\} \quad (2)$$

If $d(y_{2n+1}, y_{2n}) > d(y_{2n-1}, y_{2n})$ for some $n \in N$, then from the (2) we have

$$d(y_{2n+1}, y_{2n}) \leq a\frac{d(y2n+1,\ y2n) + d(y2n+1,\ y2n)}{3} + (1-a)d(y_{2n+1}, y_{2n})$$
$$\leq \frac{2a}{3}\, d(y_{2n+1}, y_{2n}) + (1-a)d(y_{2n+1}, y_{2n})$$
$$\leq (1 - \frac{a}{3})\, d(y_{2n+1}, y_{2n})$$
$$< d(y_{2n+1},\ y_{2n}),$$

which is a contradiction. Hence

$$d(y_{2n+1}, y_{2n}) \leq d(y_{2n-1}, y_{2n}) \quad (3)$$

for all $n \in N$ and from (2) we obtain

$$d(y_{2n+1}, y_{2n}) \leq a\frac{d(y2n-1,\ y2n) + d(y2n-1,\ y2n) + d(y2n,\ y2n)}{3}$$
$$+ (1-a)\max\{d(_{y2n-1}, y_{2n}), \frac{d(y2n+1,\ y2n-1)}{2s}\}$$
$$\leq \frac{2a}{3}\, d(y_{2n-1}, y_{2n}) + (1-a)\max\{d(y_{2n-1}, y_{2n}), \frac{d(y2n+1,\ y2n-1)}{2s}\} \quad (4)$$

by (3) we have

$$\frac{d(y2n+1,\ y2n-1)}{2s} \leq s\frac{(d(y2n+1,\ y2n) + d(y2n,\ y2n-1))}{2s}$$
$$\leq d(y_{2n}, y_{2n-1}).$$

Consequently

$$d(y_{2n+1}, y_{2n}) \le (1 - \tfrac{a}{3})\, d(y_{2n-1}, y_{2n}) \quad (5)$$

for all $n \in N$. Similarly

$$d(y_{2n+1}, y_{2n+2}) \le (1 - \tfrac{a}{3})\, d(y_{2n}, y_{2n+1}) \quad (6)$$

for all $n \in N$. So

$$d(y_{n+1}, y_n) \le (1 - \tfrac{a}{3})\, d(y_{n-1}, y_n) \quad (7)$$

for all $n \in N$. hence $\{y_n\}$ is a Cauchy sequence in (X, d, s). By completeness of (X, d, s) there exist $u \in X_n$ such that

$$\lim_{m\, n\to\infty} Ix_{2n+2} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Jx_{2n+1} = \lim_{n\to\infty} Sx_{2n+1} = u. \quad (8)$$

Since J and T are continues and b-compatible we obtain

$$\frac{1}{s} d(Ju, T u) \le d(Ju, JT x_{2n+1}) + d(JT x_{2n+1}, T u)$$
$$\le d(Ju, JT x_{2n+1}) + s(d(JT x_{2n+1}, T Jx_{2n+1}) + d(T Jx_{2n+1}, T u))$$
$$\to 0$$

because $T x_{2n+1} \to u$, it implies $JT x_{2n+1} \to Ju$ and $d(JT x_{2n+1}, TJx_{2n+1}) \to 0$ because Tx_{2n+1} and Jx_{2n+1} converge same u so because b-compatible implies that $d(JT x_{2n+1}, TJx_{2n+1}) \to 0$ and finally $TJx_{2n+1} \to T u$. So, $Ju = T u$. Similarly, we have $Iu = Su$.

Really,

$$\frac{1}{s} d(Iu, Su) \le d(Iu, ISx_{2n}) + d(ISx_{2n}, Su)$$
$$\le d(Iu, ISx_{2n}) + s(d(ISx_{2n}, SIx_{2n}) + d(SIx_{2n}, Su))$$
$$\to 0$$

If $Su \ne T u$ From (1) we obtain

$$d(Su, T u) \le a\,\frac{d(Iu, Ju) + d(T u, Ju) + d(Su, Ju)}{3} + (1 - a) \max\{d(Iu, Ju),$$
$$d(Su, Iu), d(T u, Ju), \frac{d(Su, Ju) + d(T u, Iu)}{2s}\}$$
$$\le a\,\frac{d(Su, T u) + d(T u, T u) + d(Su, Tu)}{3} + (1 - a) \max\{d(Su, T u),$$
$$d(Su, Su), d(T u, T u), \frac{d(Su, T u) + d(T u, Su)}{2s}\}$$
$$\le \frac{2a}{3} d(Su, T u) + (1 - a)d(Su, T u)$$
$$\le (1 - \tfrac{a}{3})\, d(Su, T u)$$
$$< d(Su, T u),$$

which is a contradiction. Therefore $Su = T u$, this implies that $Iu = Su = Ju = T u$.

Let $v = Iu = Su = Ju = Tu$, we get

$$Sv = SIu = ISu = Iv \quad (9)$$

and

$$T v = TJu = JTu = Jv \quad (10)$$

If $Su \ne T v$ from (1) we obtain

$$d(Su, T v) \le a\,\frac{d(Iu, Jv) + d(T v, Jv) + d(Su, Jv)}{3} + (1 - a) \max\{d(Iu, Jv),$$
$$d(Su, Iu),\; d(T v, Jv), \frac{d(Su, Jv) + d(T v, Iu)}{2s}\}$$

$$\leq a \frac{d(Su, Tv) + d(Tv, Tv) + d(Su, Tu)}{3} + (1-a) \max \{ d(Su, Tv),$$

$$d(Su, Su), d(Tv, Tv), \frac{d(Su,Tv) + d(Tv,Su)}{2s}\}$$

$$\leq \frac{2a}{3} d(Su, Tv) + (1-a) d(Su, Tv)$$

$$\leq (1 - \frac{a}{3})\, d(Su, Tv)$$

$$< d(Su, Tv)$$

which is a contradiction. So we have Su = T v, hence we obtain T v = v so, from (10) we have v is a common fixed point for T and J.

Similarly, if $Sv \neq Tu$ from (1) we obtain

$$d(Sv, Tu) \leq (1 - \frac{a}{3})\, d(Sv, Tu)$$

$$< d(Sv, Tu)$$

It is contradiction. So we have Sv = T u. Therefore Sv = v from (9). We have v is a common fixed point for S and J. Therefore v is a common fixed point for S, T, I and J.

Theorem 2. Let the pairs (S, I) and (T , J) be b-compatible on a complete b-metric space (X, d, s) and satisfying

$$d(Sx, Ty) \leq (1 - \frac{a}{3})\, d(Ix, Jy) \tag{11}$$

for all $x, y \in X$ where $0 < a < 1$. If $S(X) \subseteq J(X)$ and $T(X) \subseteq I(X)$ and if I, J, S and T are continues, then I, J, S and T *have a unique common fixed point.*

***Remark* 3**. We note that Theorem 1 improves the main result Theorem 2.1 in [8].

Now we present example and application of our main result.

III. EXAMPLE and APPLICATION

Let us consider the metric space X = [0, 1] defined by $d(x, y) = (x - y)^2$, so (X, d, 2) is a b-compatible b-metric space. We see that b-metric d is continuous. Also consider the self mappings on X as follows:

$S, T, I, J : X \to X$

$$S(x) = (\frac{x}{3})^4,\ T(x) = (\frac{x}{2})^2,\ J(x) = (\frac{x}{3})^2 \text{ and } I(x) = (\frac{x}{2}) \text{ for all } x \in X$$

From the Figure 1, it is clear that each function satisfy the required condition. Clearly, S, T, I, J are self mappings complying with $S(x) \subseteq J(x)$, $T(x) \subseteq I(x)$ then the pairs (S, I) and (T , J) are b-compatible. If $\{x_n\}$ is a sequence in S satisfying

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ix_n = x \text{ for some } x \in X,$$

then by the continuity of S and I we have

$$\lim_{n\to\infty} d(S(I(xn)), I(S(xn))) = \lim_{n\to\infty} (S(I(xn)), I(S(xn))^2$$

$$= (S(x) - I(x))^2$$

$$= ((\frac{x}{3})^4 - (\frac{x}{3})^2)^2$$

$$= 0$$

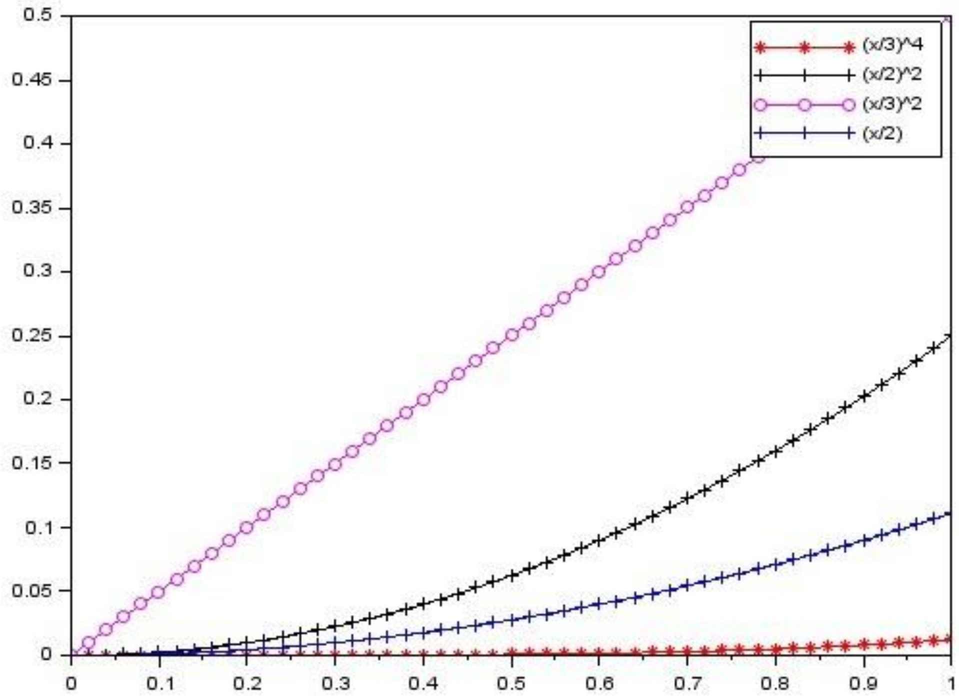

Figure 1: Graph of mappings S, T, I and J.

only for x = 0. Similarly, the pair (T , J) is b-compatible. It is easy to see that both pairs are not commuting.

Now for x, y ∈ X, we have

$$\begin{aligned} d(S(x), T(y)) &= (S(x) - T(y))^2 \\ &= ((\tfrac{x}{3})^4 - (\tfrac{y}{2})^2)^2 \\ &= ((\tfrac{x}{3})^2 - (\tfrac{y}{2}))^2 \, ((\tfrac{x}{3})^2 - (\tfrac{y}{2}))^2 \\ &\leq (\tfrac{1}{9} + \tfrac{1}{2})^2 \, d(J(x), I(y)) \\ &= (\tfrac{11}{18})^2 \, d(I(x), J(y)) \\ &= (1 - \tfrac{a}{3}) \, d(I(x), J(y)), \end{aligned}$$

where $(1 - \frac{a}{3}) = (\frac{11}{18})^2 \in (0, 1)$. Thus, the contractive condition (11) satisfied for all x, y ∈ X. Hence all the hypothesis of the Theorem (1) are satisfied, note that S, T, I, J have a unique common fixed point x = 0.

The main theorem of this paper has a unique common fixed point in X, which is ’0’. Now we will see it graphically as follows:

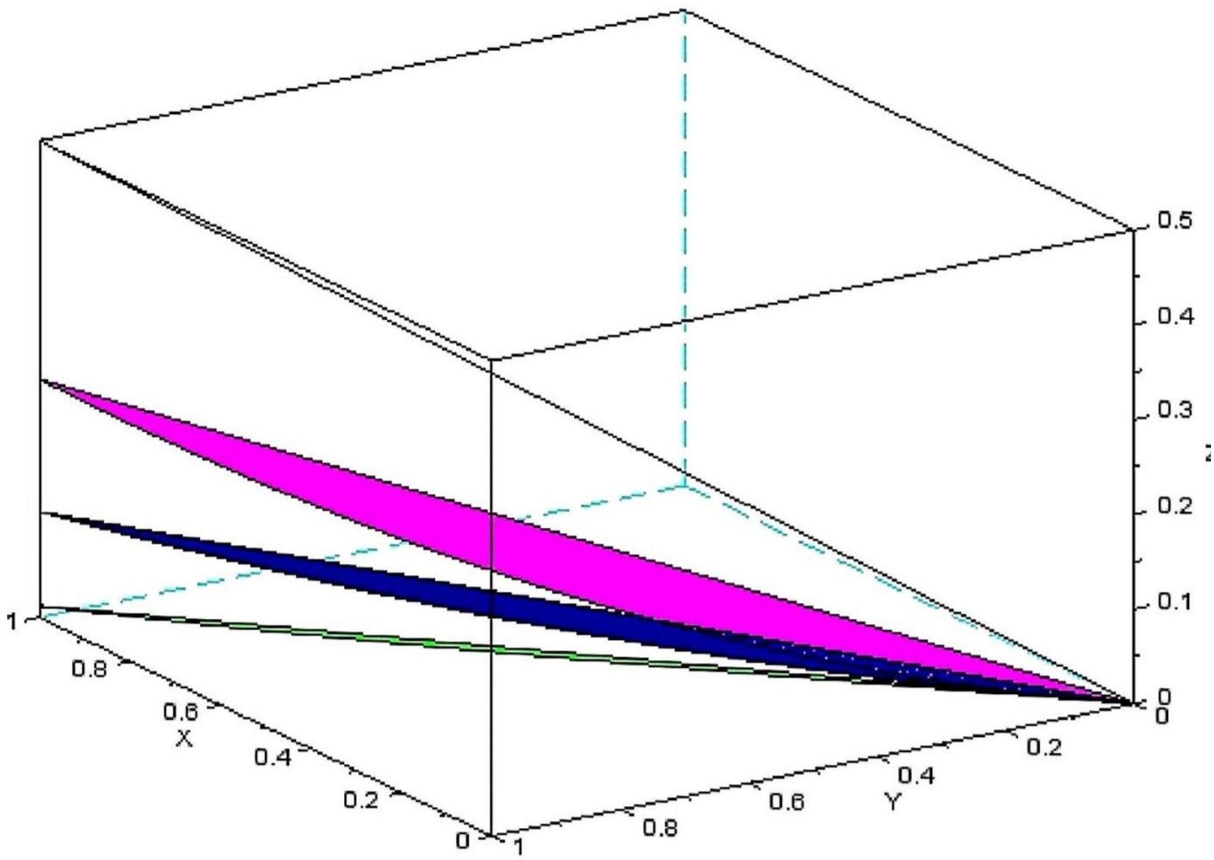

Figure 2: Unique fixed point '0' is shown graphically in 3D for the function S, T, I, J.

From the figure we observe that by means of certain transformation through a fixed point we can have different structures from original structure or vice versa. The result of the paper can be applied to other branch of applied sciences with this aim. The results or their extended form (which of course need further research with this specific aim) may also be used to construct fixed point in Euclidean geometry ,which generally require the use of a compass and ruler. These can be achieved by replacing geometrical figures by suitable (approximate) functions.

Theorem 4. Let T and I be a commuting mappings of a complete b-metric space (X, d, s) into itself satisfying

$$d(T x, T y) \leq (1 - \frac{a}{3}) d(Ix, Iy) \tag{12}$$

for all x, y ∈ X, where $0 \leq a < 1$. If the range of I contains the range of T and if I is continuous, then T and I have a unique common fixed point.

Theorem 5. Let (X, d, s) be a complete b-metric space and let T : X → X be a map such that for all x, y ∈ X and some a ∈ [0, 1),

$$d(T x, T y) \leq (1 - \frac{a}{3}) d(x, y) \tag{13}$$

Then T has a unique fixed point u and $\lim_{n\to\infty} T^n x = u$ for all x ∈ X.

For this we give some example of b-metric space (X, d,$\frac{3}{2}$) with self mapping T which satisfy (5) where b-metric d is not continuous.

Example. Let X = N ∪ ∞ and let d : X × X → [0, ∞) be defined by

$$d(x, y) = \begin{cases} 0, & x = y; \\ \left|\frac{1}{x^2} - \frac{1}{y^2}\right|, \text{if one of } x, y \text{ is even and the other is even or } \infty; \\ 3, \text{if one of } x, y \text{ is odd and the other is odd (and } x \, 6 = y) \text{or } \infty; \\ 2, & \text{other}: \end{cases}$$

then considering all possible cases, it can be checked that for all x, y, p ∈ X,

we have $d(x, p) \leq \frac{3}{2}(d(x, y) + d(y, p))$.

Thus, (X, d) is b-metric space (with $s = \frac{3}{2}$). Let $x_n = 2n$ for each n ∈ N.

Then

$$d(2n, \infty) = \left|\frac{1}{2n} - \frac{1}{\infty}\right| = \frac{1}{2n} \to 0 \text{ as } n \to \infty,$$

that is $x_n \to \infty$, but $d(x_n, 1) = 2 \neq 3 = d(\infty, 1)$ as $n \to \infty$.

Now, define the mapping $T : X \to X$ as

$$T(x) = \begin{cases} 5x, & \text{if } x \in N, \\ \infty, & \text{if } x = \infty \end{cases}$$

In order to check the contractive condition of Theorem 5, consider the following cases.
If x, y ∈ N then

$$d(T(x), T(y)) = \frac{1}{5}\left|\frac{1}{x^2} - \frac{1}{y^2}\right|,$$

while

$$d(x, y) = \begin{cases} 0, & x = y; \\ \left|\frac{1}{x^2} - \frac{1}{y^2}\right|, \text{if one of } x, y \text{ is even and the other is even or } \infty; \\ 3, \text{if one of } x, y \text{ is odd and the other is odd (and } x \, 6 = y) \text{or } \infty; \\ 2, & \text{other}: \end{cases}$$

In all cases there exist λ ∈ (0, 1) such that for all x, y ∈ X the inequality

$$d(T(x), T(y)) \leq \lambda d(x, y),$$

holds.

IV. CONCLUSION

From our investigations we conclude that the self mappings defined on a b-metric space satisfying Gregus type contraction and b-weak compatible conditions have a unique common fixed point. Our investigations and results obtained were supported by the suitable example with graphs which provides new path for researchers in the concerned field.

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