Some Properties of Induced and Second Order Induced Intuitionistic Fuzzy Sets-I

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Abstract: *In this paper we have proved some basic theorems on four different types of induced intuitionistic fuzzy sets, namely $A^{\circ}(B), A^{*}(B), A_{\circ}(B)$ and $A_{*}(B)$, corresponding to the intuitionistic fuzzy sets A, B of a set E. We have also established some relations between induced and second order induced intuitionistic fuzzy sets.*

Keywords: *Degree of membership, degree of non-membership, induced Intuitionistic fuzzy set, second order induced intuitionistic fuzzy sets.*

Mathematics Subject Classification: 03B99, 03E72.

I. INTRODUCTION

The notion of Intuitionistic fuzzy set (IFS) was introduced in [1]. Various properties on intuitionistic fuzzy sets were discussed in [2- 4,7]. The concept of induced intuitionistic fuzzy sets was introduced in [5]. Later a few properties of induced intuitionistic fuzzy sets have been discussed in [6]. Some relations of second order induced intuitionistic fuzzy sets have also been established in [6] . Here, in this paper, we have discussed some theorems on induced intuitionistic fuzzy sets and established a few relations between induced intuitionistic fuzzy sets and second order induced intuitionistic fuzzy sets.

The section 2, deals with the definitions , notations and some properties of induced intuitionistic fuzzy set of a set.

In section 3, we have proved some theorem on induced intuitionistic fuzzy sets.

Finally, in section 4, we have established some relations between induced and second order induced intuitionistic fuzzy sets .

II. Preliminaries

This section contains some basic definitions, notations and properties of intuitionistic and induced intuitionistic fuzzy sets on a set which are used through-out the paper.

Definition 2.1 **[1,2]**: Let E be any non-empty set. An intuitionistic fuzzy set A of E is an object of the form $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \}$, where the functions $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denotes the degree of membership and the non - membership of the element x ε E respectively and for every $x \in E, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2 **[1-3]**: If A and B are two intuitionistic fuzzy sets of a non - empty set E, then $A \subseteq B$ if and only if for all $x \in E$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;

$A = B$ if and only if for all $x \in E$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$;

$A \cap B = \{(x, \min(\mu_A(x), \nu_A(x)), \max(\nu_A(x), \nu_B(x))) : x \in E\}$;

$A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) : x \,\varepsilon\, E\}$.

Considering the degree of membership $\mu_A(x), \mu_B(x)$ and the non - membership $\nu_A(x), \nu_B(x)$ of each element $x \in E$ of the intuitionistic fuzzy sets A and B respectively of a non-empty set E, the four types of induced intuitionistic fuzzy sets are defined follows:

Definition 2.3 **[5]:** If A and B are two intuitionistic fuzzy sets of a non - empty set E then

$$A^{\circ}(B) = A^{\circ} = \{(x, \mu_A(x), \min(\nu_A(x), \nu_B(x)) : x \in E\};$$

$$A_{\circ}(B) = A_{\circ} = \{(x, \mu_A(x), \max(\nu_A(x), \nu_B(x))) : x \in E\};$$

$$A^{*}(B) = A^{*} = \{(x, \max(\mu_A(x), \mu_B(x)), \nu_A(x)) : x \in E\};$$

$$A_{*}(B) = A_{*} = \{(x, \min(\mu_A(x), \mu_B(x)), \nu_A(x) : x \in E)\}.$$

Note 2.4 **[5]:** It is to be noted that, in general,

$A^{\circ}(B) \neq B^{\circ}(A), A_{\circ}(B) \neq B_{\circ}(A), A^{*}(B) \neq B^{*}(A), A_{*}(B) \neq B_{*}(A)$.

Property 2.5 **[6]:** $(A^{\circ})^{*}(B) = (A^{*})^{\circ}(B) = A \cup B$

Property 2.6 **[6]:** $(A_{\circ})_{*}(B) = (A_{*})_{\circ}(B) = A \cap B$

Property 2.7 **[6]:** $(A_{\circ})^{*}(B) = (A^{*})_{\circ}(B) = \{(x, max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))) : x \in E\}$

Property 2.8 **[6]:** $(A_{*})^{\circ}(B) = (A^{\circ})_{*}(B) = \{(x, min(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))) : x \in E\}$

III. Some properties of induced intuitionistic fuzzy sets

***Theorem 3.1*:** Let A_1, A_2 and B be three intuitionistic fuzzy sets of E. If $A_1 \subseteq A_2$ then

i) ${A_1}^{*}(B) \subseteq {A_2}^{*}(B)$

ii) ${A_1}^{\circ}(B) \subseteq {A_2}^{\circ}(B)$

iii) ${A_1}_{*}(B) \subseteq {A_2}_{*}(B)$

iv) ${A_1}_{\circ}(B) \subseteq {A_2}_{\circ}(B)$.

Proof: Let A_1, A_2 and B be three intuitionistic fuzzy sets of E and $A_1 \subseteq A_2$ .Then for each $x \in E,\ \mu_{A_1}(x) \leq \mu_{A_2}(x)$ and $\nu_{A_1}(x) \geq \nu_{A_2}(x)$ … … … (3.1.1)

i) For each $x \in E$,

$$\mu_{{A_1}^*(B)}(x) = max\left(\mu_{A_1}(x), \mu_B(x)\right) \leq max\left(\mu_{A_2}(x), \mu_B(x)\right) = \mu_{{A_2}^*(B)}(x) \quad \text{and}$$

$$\nu_{{A_1}^*(B)}(x) = \nu_{A_1}(x) \geq \nu_{A_2}(x) = \nu_{{A_2}^*(B)}(x) \quad \text{(By (3.1.1)).}$$

Hence, ${A_1}^*(B) \subseteq {A_2}^*(B)$.

ii) For each $x \in E$,

$$\mu_{{A_1}^\circ(B)}(x) = \mu_{A_1}(x) \leq \mu_{A_2}(x) = \mu_{{A_2}^\circ(B)}(x) \quad \text{and}$$

$$\nu_{{A_1}^\circ(B)}(x) = min\left(\nu_{A_1}(x), \nu_B(x)\right) \geq\ min\left(\nu_{A_2}(x), \nu_B(x)\right) = \nu_{{A_2}^\circ(B)}(x) \quad \text{(By (3.1.1)).}$$

Hence, ${A_1}^\circ(B) \subseteq {A_2}^\circ(B)$.

iii) For each $x \in E$,

$$\mu_{{A_1}_*(B)}(x) = min\left(\mu_{A_1}(x), \mu_B(x)\right) \leq min\left(\mu_{A_2}(x), \mu_B(x)\right) = \mu_{{A_2}_*(B)}(x) \quad \text{and}$$

$$\nu_{{A_1}_*(B)}(x) = \nu_{A_1}(x) \geq \nu_{A_2}(x) = \nu_{{A_2}_*(B)}(x) \quad \text{(By (3.1.1)).}$$

Hence, ${A_1}_*(B) \subseteq {A_2}_*(B)$.

iv) For each $x \in E$,

$$\mu_{{A_1}_\circ(B)}(x) = \mu_{A_1}(x) \leq \mu_{A_2}(x) = \mu_{{A_2}_\circ(B)}(x) \quad \text{and}$$

$$\nu_{{A_1}_\circ(B)}(x) = max\left(\nu_{A_1}(x), \nu_B(x)\right) \geq\ max\left(\nu_{A_2}(x), \nu_B(x)\right) = \nu_{{A_2}_\circ(B)}(x) \quad \text{(By (3.1.1)).}$$

Hence, ${A_1}^\circ(B) \subseteq {A_2}^\circ(B)$.

***Theorem 3.2*:** Let A, B_1 and B_2 be three intuitionistic fuzzy sets of E. If $B_1 \subseteq B_2$ then

i) $A^*(B_1) \subseteq A^*(B_2)$

ii) $A^\circ(B_1) \subseteq A^\circ(B_2)$

iii) $A_*(B_1) \subseteq A_*(B_2)$

iv) $A_\circ(B_1) \subseteq A_\circ(B_2)$

Proof: Let A, B_1 and B_2 be three intuitionistic fuzzy sets of E and $B_1 \subseteq B_2$. Then, for each $x \in E$, $\mu_{B_1}(x) \leq \mu_{B_2}(x)$ and $\nu_{B_1}(x) \geq \nu_{B_2}(x)$ … … … (3.2.1)

i) For each $x \in E$,

$$\mu_{A^*(B_1)}(x) = max\left(\mu_A(x), \mu_{B_1}(x)\right) \leq max\left(\mu_A(x), \mu_{B_2}(x)\right) = \mu_{A^*(B_2)}(x) \quad \text{and}$$

$$\nu_{A^*(B_1)}(x) = \nu_{A_1}(x) \geq \nu_{A_2}(x) = \nu_{A^*(B_2)}(x) \quad \text{(By (3.2.1)).}$$

Hence, $A^*(B_1) \subseteq A^*(B_2)$.

ii) For each $x \in E$,

$$\mu_{A^\circ(B_1)}(x) = \mu_A(x) \leq \mu_A(x) = \mu_{A^\circ(B_2).}(x) \quad \text{and}$$

$$\nu_{A^\circ(B_1)}(x) = min\left(\nu_A(x), \nu_{B_1}(x)\right) \geq min\left(\nu_A(x), \nu_{B_2}(x)\right) = \nu_{A^\circ(B_2).}(x) \quad \text{(By (3.2.1)).}$$

Hence, $A^\circ(B_1) \subseteq A^\circ(B_2)$.

iii) For each $x \in E$,

$$\mu_{A_*(B_1)}(x) = min\left(\mu_A(x), \mu_{B_1}(x)\right) \leq min\left(\mu_A(x), \mu_{B_2}(x)\right) = \mu_{A_*(B_2)}(x) \quad \text{and}$$

$$\nu_{A_*(B_1)}(x) = \nu_A(x) \geq \nu_A(x) = \nu_{A_*(B_2)}(x) \quad \text{(By (3.2.1)).}$$

Hence, $A_*(B_1) \subseteq A_*(B_2)$.

iv) For each $x \in E$,

$$\mu_{A_\circ(B_1)}(x) = \mu_A(x) \leq \mu_A(x) = \mu_{A_\circ(B_2)}(x) \quad \text{and}$$

$$\nu_{A_\circ(B_1)}(x) = max\left(\nu_A(x), \nu_{B_1}(x)\right) \geq max\left(\nu_A(x), \nu_{B_2}(x)\right) = \nu_{A_\circ(B_2)}(x) \quad \text{(By (3.2.1)).}$$

Hence, $A_\circ(B_1) \subseteq A_\circ(B_2)$.

IV. Some relations between induced and second order induced intuitionistic fuzzy sets

Here we have established some relations between induced intuitionistic fuzzy sets and second order induced intuitionistic fuzzy sets.

***Property 4.1*:** $(A_*)^\circ(B) = (A^\circ)_*(B) = A_*(B) \cup B_*(A) = A^\circ(B) \cap B^\circ(A)$.

Proof: By Property 2.8 we have,

$$(A_*)^\circ(B) = (A^\circ)_*(B) = \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\} \quad \ldots \ldots (4.1.1)$$

Now, $A_*(B) \cup B_*(A)$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\} \cup \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), \nu_B(x)\right) : x \in E\right\}$$

$$= \left\{\left(x, \max\left(min\left(\mu_A(x), \mu_B(x)\right), min\left(\mu_A(x), \mu_B(x)\right)\right), \min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\} \quad \ldots\ldots\ldots (4.1.2)$$

and $A^{\circ}(B) \cap B^{\circ}(A)$

$$= \{(x, \mu_A(x), \min(\nu_A(x), \nu_B(x))): x \in E\} \cap \{(x, \mu_B(x), \min(\nu_A(x), \nu_B(x))): x \in E\}$$

$$= \{(x, \min(\mu_A(x), \mu_B(x)), \max(\min(\nu_A(x), \nu_B(x)), \min(\nu_A(x), \nu_B(x)))): x \in E\}$$

$$= \left\{\left(x, min\left(\mu_A(x), \mu_B(x)\right), min\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\} \quad \ldots\ldots\ldots (4.1.3)$$

So from (4.1.1), (4.1.2) and (4.1.3), we have,

$$(A_*)^{\circ}(B) = (A^{\circ})_*(B) = A_*(B) \cup B_*(A) = A^{\circ}(B) \cap B^{\circ}(A).$$

***Property 4.2*:** $(A_{\circ})^*(B) = (A^*)_{\circ}(B) = A^*(B) \cap B^*(A) = A_{\circ}(B) \cup B_{\circ}(A)$.

Proof: By Property 2.7 we have,

$$(A_{\circ})^*(B) = (A^*)_{\circ}(B) = \{(x, max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))): x \in E\} \quad \ldots\ldots\ldots (4.2.1)$$

Now, $A^*(B) \cap B^*(A)$

$$= \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), \nu_A(x)\right) : x \in E\right\} \cap \left\{\left(x, max\left(\mu_A(x), \mu_B(x)\right), \nu_B(x)\right) : x \in E\right\}$$

$$= \left\{\left(x, \min\left(max\left(\mu_A(x), \mu_B(x)\right), max\left(\mu_A(x), \mu_B(x)\right)\right), \max\left(\nu_A(x), \nu_B(x)\right)\right) : x \in E\right\}$$

$$= \{(x, max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))): x \in E\} \quad \ldots\ldots\ldots (4.2.2)$$

and $A_{\circ}(B) \cup B_{\circ}(A)$

$$= \{(x, \mu_A(x), \max(\nu_A(x), \nu_B(x))): x \in E\} \cup \{(x, \mu_B(x), \max(\nu_A(x), \nu_B(x))): x \in E\}$$

$$= \{(x, \max(\mu_A(x), \mu_B(x)), \min(\max(\nu_A(x), \nu_B(x)), \max(\nu_A(x), \nu_B(x)))): x \in E\}$$

$$= \{(x, max(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))): x \in E\} \quad \ldots\ldots\ldots (4.2.3)$$

Hence from (4.2.1), (4.2.2) and (4.2.3), we have,

$(A_{\circ})^{*}(B) = (A^{*})_{\circ}(B) = A^{*}(B) \cap B^{*}(A) = A_{\circ}(B) \cup B_{\circ}(A).$

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