On Minimally Nonouterplanarity of a Semi-Splitting Block Graph of a Graph

V. R. Kulli[1] and K. M. Niranjan[2]

[1]Department of Mathematics,UBDTCE,Davangere, Karnataka, India
[2]Department of Mathematics, Gulbarga University, Gulbarga-585106, Karnataka, India,

Abstract: *Let $G = (V,)$ be a simple connected undirected graph with vertex set V and edge set E. The advent of graph theory has played a prominent role in wide variety of engineering applications and optimizes its use in many applications. In this paper, we present here characterize graphs whose semi-splitting block graphs are minimally nonouterplanar.*

Keywords : *Semi-Splitting; Block; minimally nonouterplanar.*

I. INTRODUCTION

All graphs considered here are finite, undirected and without loops or multiple lines. We use the terminology of [1]. The open-neighbourhood $N(u)$ of a point u in $V(G)$ is the set of points adjacent to u. $N(u) = [v/uv \in E(G)]$.

In 1975, Kulli [2] introduced the idea of a minimally nonouterplanar graph. The inner point $i(G)$ of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any embedding of G in the plane. Obviously G is outerplanar if and only if $i(G)=0$. A graph G is minimally nonouterplanar if $i(G)=1$, and G is n-minimally $(n\geq 2)$ nonouterplanar if $i(G)=n$. A graph is planar if it can be drawn on the plane in such a way that no two of its lines intersect.

If $B = \{u_1, u_2, \ldots, u_r, r \geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two blocks B_1 and B_2 of G are incident with a common cutpoint, then they are adjacent blocks. If $B = \{ e_1, e_2, \ldots e_s, s\geq 1\}$is a block of a graph G, then we say that line e_1 and block B are incident with each other, as are e_2 and B and so on. This idea was introduced by Kulli in [3]. The points, lines and blocks of a graph are called its members.

For each point v_i of G, we take a new point u_i and the resulting set of points is denoted by $V_1(G)$.

The semi-splitting block graph $S_B(G)$ of a graph G is defined as the graph having point set $V(G)\cup V_1(G)\cup b(G)$ with two points are adjacent if they correspond to a adjacent points of G or one corresponds to a point v_i of $V_1(G)$ and the other to a point w_j of G and w_j is in $N(v_i)$ or one corresponds to a point u_i of $V(G)$ and the other to a point b_i of $b(G)$, where $b(G)$ is the set of blocks of G. This concept was introduced by Kulli and Niranjan [4]. Many other graph valued functions in graph theory were studied, [e.g in [5]-[32]].

The splitting graph $S(G)$ of a graph G is defined as the graph having point set $V(G)\cup V_1(G)$ with two points are adjacent if they correspond to a adjacent points of G or one corresponds to a point v_i of $V_1(G)$ and the other to a point w_j of G and w_j is in $N(v_i)$. This concept was introduced by Sampathkumar and Walikar in [34].

A graph G, the semi-splitting block graph $S_B(G)$ and the splitting graph $S(G)$ are shown in Fig. 1.

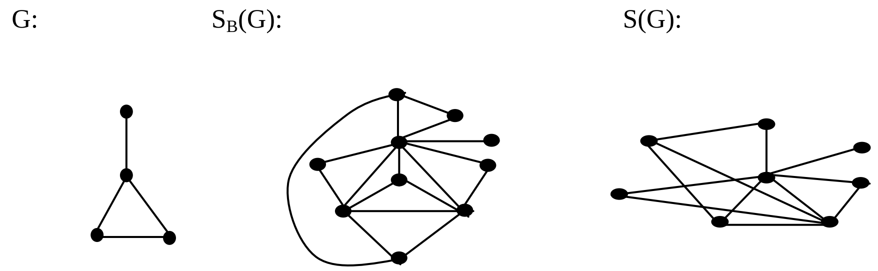


Fig. 1

We make use of the following results to prove our main results.

Theorem A [35]. The splitting graph $S(G)$ of a graph G is planar if and only if every block of G is an even cycle or a line or a triangle.

Theorem B [35]. *If G* has a cycle of odd length $p\geq5$ then $S(G)$ is nonplanar.

Theorem C [35]. The splitting graph $S(G)$ of a graph G is outerplanar if and only if every component of G is a path or a triangle.

Theorem D [4]. If G is a cycle with p≥4 points, then $S_B(G)$ is nonplanar.

Theorem E [4]. The semi-splitting block graph $S_B(G)$ is outerplanar if and only if G is either $K_{1,2}$ or K_2.

Theorem F [4]. The semi-splitting block graph $S_B(G)$ of a graph G is planar if and only if G has no subgraph homeomorphic to C_4 or K_{4-x} or G_1 (see Fig. 3).

Theorem G [4]. The semi-splitting block graph $S_B(G)$ of a graph G is outerplanar if and only if G has no subgraph homeomorphic to $K_{1,3}$ or P_4 or C_3.

Theorem H [4]. The semi-splitting block graph $S_B(G)$ of a graph G is planar if and only if every block of G is a line or a triangle or a triangle together one line is adjoined to some point.

II. Main Results

We now characterize graphs whose semi-splitting block graphs are minimally nonouterplanar.

Theorem 1. The semi-splitting block graph $S_B(G)$ is minimally nonouterplanar if and only if G is C_3.

Proof. Suppose $S_B(G)$ is minimally nonouterplanar. Then by Theorem 2, every block of G is a line or a triangle. Assume every block of G is a line. We consider the following cases.

Case 1. Suppose G is a path. Then by clearly, $i(S_B(G))\geq2$, a contradiction.

Case 2. Suppose G is $K_{1,n}$.Then by Theorem 3, $i(S_B(G))=2n-4$, a contradiction.

Suppose $G=C_3.C_3$.Then $S_B(G)$ has two-inner points (see Fig. 3) , again a contradiction. Then G is a triangle.

Conversely, suppose G is C_3.Then it is easy to see that $S_B(C_3)$ has an inner point (see Fig. 2). Hence $S_B(G)$ is minimally nonouterplanar.

In the following theorem, we establish a characterization of graphs whose semi-splitting block graphs are n-minimally nonouterplanar ($n\geq 2$).

Theorem 2. If G is $K_{1,n}$, $n\geq 3$ then $i(S_B(K_{1,n}))=2n-4$.

Proof. Let G is $K_{1,n}$ with $n\geq 3$ points. Then by Theorem 2, $S_B(G)$ is planar.

We now prove that $S_B(K_{1,n})$ has $2n-4$ inner points by the method of induction on the number of points $n(\geq 3)$ of G. Suppose $n=3$. Then $G=K_{1,3}$ and $S_B(K_{1,3})$ has 2-inner points (see Fig. 4). Hence the result is true for $n=3$.

Now assume the result is true for $n=m$. That is when $G=K_{1,n}$, $S_B(G)$ is m-minimally nonouterplanar.

Now suppose $n=m+1$. Then $G=K_{1,m+1}$. Now we have to prove $S_B(G)$ is $(m+1)$-minimally nonouterplanar. Let $e=v_1v_2$ be the end line. Delete from G the line e, resulting the graph G_1.By inductive hypothesis, $S_B(G_1)$ is m-minimally nonouterplanar. Now rejoin the line $e=v_1v_2$ to the point v_1, resulting the graph G. Formation of $S_B(G)$ is an extension of $S_B(G_1)$ with adding points v_2, v'_2 and b_1.The points v_1,v_2, v'_2 together with b_1 produces a subgraph homeomorphic from K_4 which has an inner points. Therefore $S_B(G)$ has $(m+1)$-inner points. Thus $S_B(G)$ is $(m+1)$-minimally nonouterplanar. Therefore $n=m+1$. Thus $S_B(G)$ is n-minimally nonouterplanar (see Fig. 5).Hence the proof the theorem.

Theorem 3. If G is a path of odd length, then $i(S_B(P_n))=n/2$, where $n\geq 4$.

Proof. Let G is a path of odd length with $n\geq 4$ points. Then by Theorem 2, $S_B(G)$ is planar.We now prove that $S_B(P_n)$ has $n/2$ inner points by the method of induction on the number of points $n(\geq 4)$ of G.

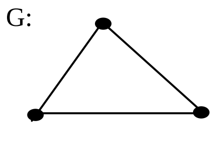
Suppose P_4: $V_1V_2V_3V_4$ is a path of odd length with four points. By Theorem C, $S(G)$ is outerplanar. If the pair of points (V_1,B_1), (V_2,B_1), (V_2,B_2), (V_3,B_2), (V_3,B_3), (V_4,B_3) are adjacent in $S_B(G)$ can not be adjacent in $S(G)$. Thus $S(G)\subseteq S_B(G)$. Since the lines $[(V_1,B_1), (V_2,B_1), (V_2,B_2), (V_2,B_2), (V_3,B_2), (V_3,B_3), (V_4,B_3)]\in(S_B(G))$, then in each case these exists at least one inner point. Thus $i[S_B(G))]=2$ (see Fig. 6), depicts the inner point number of $S_B(G)$ is at most 2. Thus $i[S_B(G)]=2$.

As the inductive hypothesis, let the semi-splitting block graph of a odd length with $n\geq 4$ points has $n/2$ inner points. We now show that the semi-splitting block graph of a path of odd length with $n+2$ points has $(n+2)/2$ inner points.

Let $G'=P_{n+2}$ be a path of odd length with $V_1,V_2,V_3,V_4,\ldots, V_n,V_{n+1},V_{n+2}$ points see Fig.7 . The points V_{n+2}, V'_{n+2} ,V_{n+1},V'_{n+1} ,B_n,B_{n+1} are more points in $S_B(P_{n+2})$ than in $S_B(P_n)$. The lines V_{n+1} B_{n+1} or B_{n+1} V_{n+2} or V_n B_n or B_n V_{n+1} gives one more inner point in $S_B(P_{n+2})$, than that of $S_B(P_n)$. Since $S_B(P_n)$ is nonouterplanar with n points, it has $n/2$ inner points. Thus $S_B(P_{n+2})$ with $n+2$ points has $(n+2)/2$ inner points. Hence $i(S_B(P_{n+2}))=(n+2)/2$. This completes the proof.

Theorem 4. If G is a path of even length, then $i(S_B(P_n))=(n-1)/2$, where $n\geq 5$

Proof. We omit the proof.



$S_B(G)$:

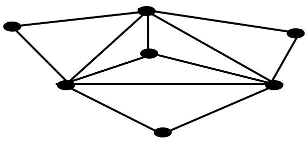
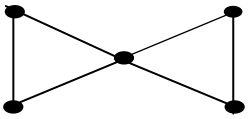


Fig. 2

G:



$S_B(G)$:

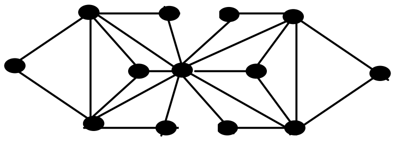
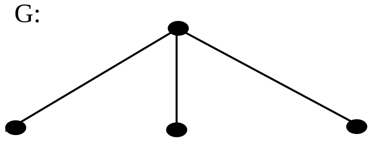


Fig. 3

G:



$S_B(G)$:

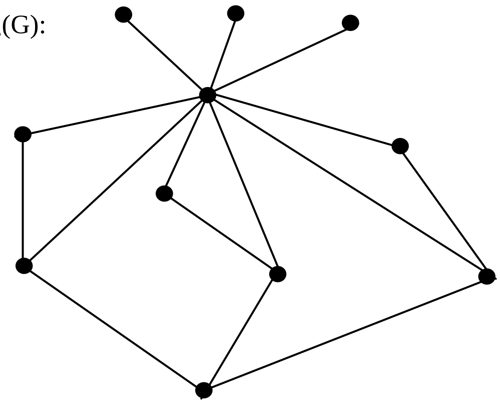


Fig. 4

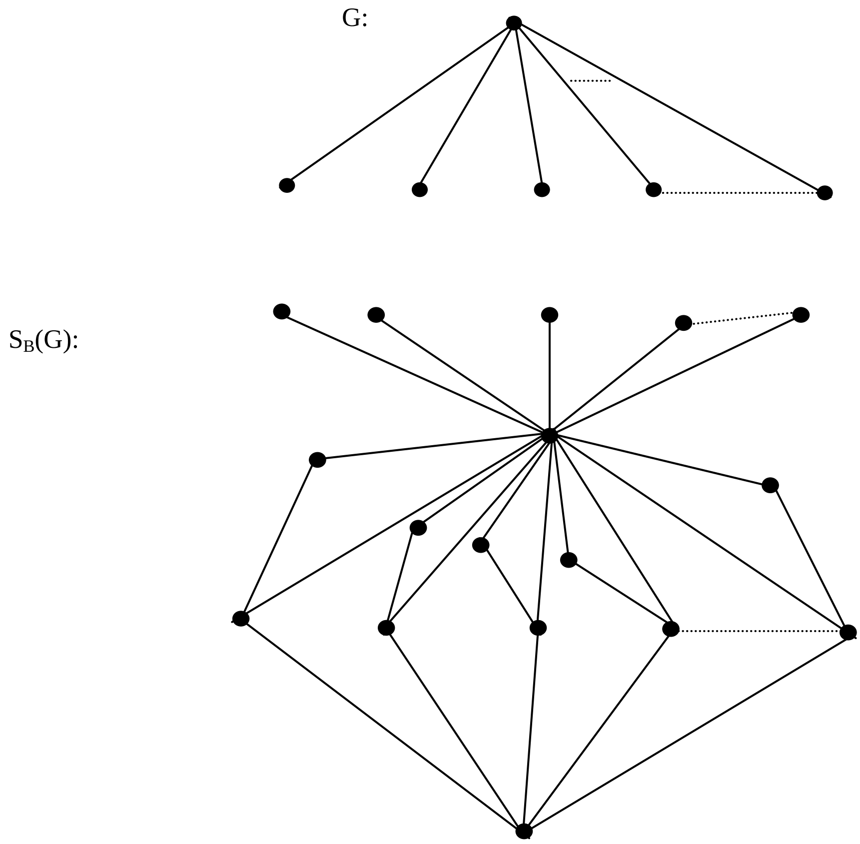


Fig. 5

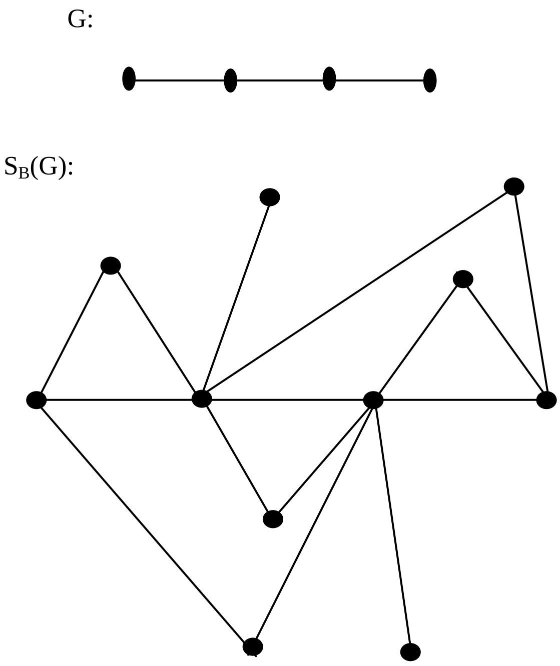
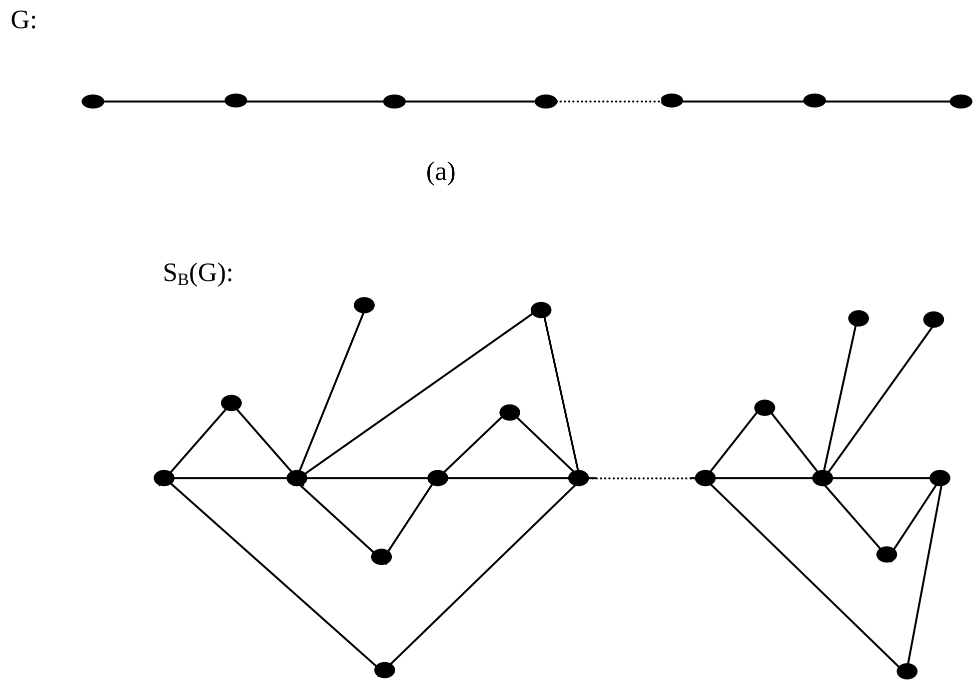


Fig. 6



(b)

Fig. 7

III. Conclusion

We present here semi-splitting block graphs are minimally nonouterplanar

We further to find a characterizations of graphs whose semi-splitting block graphs are planar, outerplanar, minimally nonouterplanar in terms of forbidden subgraphs.

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