On fuzzy contra GPR - open maps and fuzzy contra GPR - closed maps in fuzzy topological spaces

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Abstract

The aim of this paper is to introduced and investigate the fuzzy contra *gpr*- continuous and fuzzy contra *gpr*- irresolute maps and study their properties.Furthermore introduced the concept of fuzzy contra *gpr* - open maps and fuzzy contra *gpr* - closed maps in fuzzy topological spaces and obtain charterizations of theses maps.

Key words: fuzzy contra *gpr*- continuous,fuzzy contra *gpr*- irresolute maps,fuzzy contra *gpr* - open maps and fuzzy contra *gpr* - closed maps.

1.Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [8][9] in the year 1965.The subsequent research activities in this area and related areas have found applications in many branches of science and engineering.C.L.Chang[1] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers have contributed to development of fuzzy topological spaces.

2. preliminaries

Throughout this paper by $(X, \tau), (Y, \sigma)$ and (Z, η) as simply by X, Y, Z respectively.We mean fuzzy topological spaces (briefly fts)due to chang[1].Let A be a fuzzy subset of X. The fuzzy closure of A denoted by $cl(A)$ and defined as $cl(A) = \wedge\{B|B \geq A, B$ is a fuzzy closure subset of $(X, \tau)\}$ The fuzzy interior of A denoted by $Int(A)$ and defined as $int(A) = \vee\{B|B \leq A$,B is a fuzzy closure subset of $(X, \tau)\}$.

Definition:2.1 Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a function from a fts (Y, σ).The

function f is called (i)fuzzy θ- open[2] if $f(\lambda)$ is a fuzzy θ-open set of Y for each fuzzy open set λ of Y.
(ii)fuzzy θ-irresolute if the inverse image of each fuzzy θ-open in Y is fuzzy θ-open in Y is fuzzy θ-open set in X.

3. Fuzzy contra *gpr*-continuous maps and fuzzy *gpr*-irresolute maps in fuzzy topological spaces

In the section,the concepts of fuzzy contra *gpr*-continuous maps and fuzzy contra *gpr*-irresolute maps in fuzzy topological spaces is introduced and study some of their properties.

Definition:3.1 Let X and Y be fuzzy topological spaces. A map $f : X \longrightarrow Y$ is said to be fuzzy contra *gpr*-continuous if the inverse image of every fuzzy open set in Y is fuzzy *gpr*-closed in X.

Theorem:3.2 If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is fuzzy contra continuous.Then f is fuzzy contra *gpr*-continuous.

Proof: Let μ be a fuzzy open set in fuzzy topological spaces Y. Since f is fuzzy contra continuous $f^{-1}(\mu)$ is fuzzy closed set in fuzzy topological spaces X. As every fuzzy closed set is fuzzy *gpr*-closed.We have set in $f^{-1}(\mu)$ is fuzzy *gpr*-closed in fuzzy topological spaces X.Therefore f is fuzzy contra *gpr*-continuous.

The converse of the above theorem need not be true in general as seen from the following example

Theorem:3.3 If a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is fuzzy contra almost continuous.then it is fuzzy contra *gpr*-continuous.

Proof: Let a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a fuzzy contra almost continuous and μ be a fuzzy open set in fuzzy topological spaces Y. Then $f^{-1}(\mu)$ is fuzzy regular closed set in fuzzy topological spaces x. Now $f^{-1}(\mu)$ is fuzzy *grp*-closed in X, as every closed set is fuzzy *gpr*-closed.Therefore f is fuzzy contra *gpr*-closed. Therefore f is fuzzy contra *grp*-continuous.

The converse of the above theorem need not be true in general as seen from the following example

Theorem:3.4 If a function $f : (X, \tau) \longrightarrow (y, \sigma)$ is fuzzy contra *gpr*-continuous and fuzzy completely semi-continuous, then it is contra fuzzy continuous.

Proof: Let a function $f : (X, \tau) \longrightarrow (y, \sigma)$ be a fuzzy contra *gpr*-continuous and fuzzy completely semi-continuous. Let μ be a fuzzy open in fuzzy topological spaces Y. Then $f^{-1}(\mu)$ is both fuzzy regular semi open and fuzzy *gpr*-closed set in fuzzy topological spaces X.By the theorem [4],$f^{-1}(\mu)$ is is fuzzy closed in fuzzy topological spaces X. Therefore f is fuzzy contra continuous.

Theorem:3.5 If a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is fuzzy contra gpr- continuous and $g : (Y, \sigma) \longrightarrow (Z, \eta)$ is fuzzy continuous, then their composition $gof : (X, \tau) \longrightarrow (Z, \eta)$ is fuzzy contra gpr-continuous.

Proof: Let μ be a fuzzy open set in fuzzy topological spaces Z. Since g is fuzzy continuous $g^{-1}(\mu)$ is fuzzy open set in fuzzy topological spaces Y. Since f is fuzzy contra gpr-continuous. $f^{-1}(g^{-1}(\mu))$ is fuzzy gpr-closed set in fuzzy topological spaces X. But $(gof)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$.Thus gof is fuzzy contra gpr-continuous.

Definition:3.6 Let X and Y be fuzzy topological spaces. A map $f : X \longrightarrow Y$ is said to be fuzzy contra gpr-irresolute map if the inverse image of every fuzzy gpr-open set in Y is fuzzy gpr-closed in X.

Theorem:3.7 if a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is fuzzy contra gpr-irresolute, then it is fuzzy contra gpr-continuous.

Proof: Let β be a fuzzy open set in fuzzy topological spaces Y. Since every fuzzy open set is fuzzy gpr-open ,β is fuzzy gpr-open set in Y. Since f is fuzzy contra gpr- irresolute. $f^{-1}(\beta)$ is fuzzy gpr-closed set in fuzzy topological spaces X. Thus f is fuzzy contra gpr- continuous.

Theorem:3.8 Let X, Y and Z be the fuzzy topological spaces. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is fuzzy contra gpr-irresolute and $g : (Y, \sigma) \longrightarrow (Z, \eta)$ is fuzzy gpr- continuous, Then their composition $gof : (X, \tau) \longrightarrow (Z, \eta)$ is fuzzy contra gpr-continuous.

Proof: Let α be any fuzzy gpr-open set in fuzzy topological spaces Z. Since g is fuzzy contra gpr- irresolute, $g^{-1}(\alpha)$ is fuzzy gpr- closed set in fuzzy topological spaces Y. Since f is fuzzy gpr-irresolute. $f^{-1}(g^{-1}(\alpha))$ is fuzzy gpr- closed set in fuzzy topological spaces X. But $(gof)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$. Thus gof is fuzzy contra gpr- irresolute.

4. Fuzzy contra gpr - open maps and fuzzy contra gpr- closed maps

In this section we introduce fuzzy contra gpr- open maps and fuzzy contra gpr-closed maps in fuzzy topological spaces and obtain certain characterizations of these maps.

Definition:4.1 Let X and y be two fuzzy topological spaces. A map $f : (X, T_1) \longrightarrow (Y, T_2)$ is called fuzzy contra gpr- open if the image of every fuzzy open set in X is fuzzy gpr- closed in Y.

Theorem:4.2 Every fuzzy contra open map is a fuzzy contra gpr- open map.

Proof: Let $f : (X, T_1) \longrightarrow (Y, T_2)$ be a fuzzy contra open map and μ be a fuzzy open set in fuzzy topological spaces X. Then $f(\mu)$ is fuzzy closed set in fuzzy topological spaces Y. Since every fuzzy closed set is fuzzy *gpr*- closed, $f(\mu)$ is fuzzy *gpr*- closed set in fuzzy topological spaces Y. Hence f is fuzzy contra *gpr*- open map.
The converse of the above theorem need not to be true in general.

Theorem:4.3 If $f : (X, T_1) \longrightarrow (Y, T_2)$ is fuzzy open map and $g : (Y, T_2) \longrightarrow (Z, T_3)$ is fuzzy contra *gpr*- open map. then their composition $gof : (X, T_1) \longrightarrow (Z, T_3)$ is fuzzy contra *gpr*- open map.

Proof: Let α be fuzzy open set in X, T_1. Since f is fuzzy open map. $f(\alpha)$ is fuzzy open set in (Y, T_2) . Since g is fuzzy contra *gpr*- open map. $g(f(\alpha))$ is fuzzy *gpr*- closed set in Z, T_3. But $g(f(\alpha)) = (gof)(\alpha)$. Thus gof is fuzzy contra *gpr*- open map.

Definition:4.4 Let X and Y be two fuzzy topological spaces. A map $f : (X, T_1) \longrightarrow (Y, T_2)$is called fuzzy contra *gpr*- closed. if the image of every fuzzy closed set in X is fuzzy *gpr*- open in Y.

Theorem:4.5 Let $f : (X, T_1) \longrightarrow (Y, T_2)$ is fuzzy contra closed map. then f is fuzzy contra *gpr*- closed map.

Proof: Let α be fuzzy closed set in (X, T_1). Since f is fuzzy contra closed map. $f(\alpha)$ is fuzzy open set in (Y, T_2) . Since every fuzzy open set is fuzzy *gpr*- open. $f(\alpha)$ is fuzzy *gpr*- open set in (Y, T_2). Hence f is fuzzy contra *gpr*- closed map.
The converse of the above theorem need not to be true in general.

Theorem:4.6 If $f : (X, T_1) \longrightarrow (Y, T_2)$ and $g : (Y, T_2) \longrightarrow (Z, T_3)$ be two maps. then their composition $gof : (X, T_1) \longrightarrow (Z, T_3)$ is fuzzy contra *gpr*- closed map if f is fuzzy closed and g is fuzzy contra *gpr*- closed.

Proof: Let α be fuzzy closed set in (X, T_1). Since f is fuzzy closed map. $f(\alpha)$ is fuzzy closed in (Y, T_2) . Since g is fuzzy contra *gpr*- closed map. $g(f(\alpha))$ is fuzzy *gpr*- open set in (Z, T_3). But $g(f(\alpha)) = (gof)(\alpha)$. Thus gof is fuzzy contra *gpr*- closed map.

Remark: The composition of two fuzzy contra *gpr*- closed maps need not be fuzzy contra *gpr*- closed.

Theorem:4.7 A map $f : X \longrightarrow Y$ is fuzzy contra *gpr*- closed if for each fuzzy set δ of Y, and for each fuzzy open set μ of X such that $\mu \geq f^{-1}(\delta)$. There is fuzzy *gpr*- closed set α of Y such that $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$.

Proof: Suppose that f is fuzzy contra *gpr*- closed. Let δ be a fuzzy subset of

Y and μ is a fuzzy open set of X such that $f^{-1}(\delta) \leq \mu$. Let $\alpha = 1 - f(1-\mu)$ is fuzzy *gpr*- closed set in fuzzy topological spaces Y.We have note that $f^{-1}(\delta) \leq \mu$ which implies $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$.
For the converse, suppose that μ is a fuzzy closed set in X. Then $f^{-1}(1-f(\mu) \leq 1-\mu$ and $1-\mu$ is fuzzy open. By hypothesis, there is a fuzzy *gpr*-closed set α of Y such that $1-f(\mu) \leq \alpha$ and $f^{-1}(\alpha) \leq 1-\mu$. Therefore $\mu \leq 1-f^{-1}(\alpha)$. Hence $1-\alpha \leq f(\mu), f(1-f^{-1}(\alpha)) \leq 1-\alpha$ which implies $f(\mu) = 1-\alpha$. since $1-\alpha$ is fuzzy *gpr*-open. $f(\mu)$ is fuzzy *gpr*-open and thus f is fuzzy contra *gpr*-closed.

Lemma:4.8 Let $f : (X, T_1) \longrightarrow (Y, T_2)$ is fuzzy contra irresolute and α be a fuzzy regular semiopen in Y. Then $f^{-1}(\alpha)$ is fuzzy regular semiopen in X.

Proof: Let α be fuzzy regular semiopen in Y. To prove $f^{-1}(\alpha)$ is a fuzzy regular semiopen in X. That is to prove $f^{-1}(\alpha)$ is both fuzzy semiopen and fuzzy semiclosed in X. Now α is fuzzy semiopen in Y, Since f is fuzzy contra irresolute, $f^{-1}(\alpha)$ is fuzzy semiclosed in X.

Now α is fuzzy semiopen in Y as fuzzy regular semiopen set is fuzzy semiclosed, Then $1-\alpha$ is fuzzy semiclosed in Y. Since f is fuzzy contra irresolute,$f^{-1}(1-\alpha)$ is fuzzy semiopen in X. But $f^{-1}(1-\alpha) = 1 - f^{-1}(\alpha)$ is fuzzy semiclosed in X, and so $f^{-1}(\alpha)$ is semiopen in X. Thus $f^{-1}(\alpha)$ is both fuzzy semiopen and fuzzy semiclosed in X and hance $f^{-1}(\alpha)$ is a fuzzy regular semiopen in X.

Theorem:4.9 Let $f : (X, T_1) \longrightarrow (Y, T_2)$ is fuzzy contra irresolute and fuzzy *gpr*- closed and α is fuzzy *gpr*- closed set of X, then $f(\alpha)$ is a fuzzy *gpr*- closed set in Y.

Proof: Let α be a fuzzy closed of X. Let $f(\alpha) \leq \mu$, where μ is a fuzzy regular semiopen in Y. Since f is fuzzy contra irresolute. $f^{-1}(\mu)$ is a fuzzy regular semiopen in X.By lemma 4.8 and $\alpha \leq f^{-1}(\mu)$. Since α is fuzzy *gpr*-closed set in X. $cl(\alpha) \leq f^{-1}(\mu)$. Since f is fuzzy *gpr*- closed.$f(cl(\alpha))$ is fuzzy *gpr*- closed set contained in the fuzzy regular semiopen set μ, which implies $cl(f(cl(\alpha))) \leq \mu$ and hence $cl(f(\alpha)) \leq \mu$. Therefore $f(\alpha)$ is a fuzzy *gpr*-closed set in Y.

Corollary:4.10 If a map $f : (X, T_1) \longrightarrow (Y, T_2)$ is fuzzy contra irresolute and fuzzy closed and α is fuzzy *gpr*-closed set in fuzzy topological spaces in X, then $f(\alpha)$ is a fuzzy *gpr* - closed set in fuzzy topological spaces in Y.

Proof: The proof follows from the theorem 4.9 and fact that every fuzzy closed map is fuzzy *gpr*- closed map.

Theorem:4.11 Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be two mappings such that $gof : X \longrightarrow Z$ is fuzzy contra *gpr* -closed map, then
(i) if f is fuzzy continuous and surjective, then g is fuzzy contra *gpr*- closed and
(ii) if g is fuzzy *gpr*- irresolute and injective. then f is fuzzy contra *gpr*- closed

Proof: (i) Let μ be fuzzy closed set in Y. Since f is fuzzy continuous.$f^{-1}(\mu)$ is fuzzy closed set in X. Since gof is fuzzy contra *gpr*- closed map. $(gof)(f^{-1}(\mu))$ is fuzzy *gpr*- open set in Z. But $(gof)(f^{-1}(\mu))= g(\mu)$, as f is surjective. Thus g is fuzzy contra *gpr*-closed.

(ii) Let β be a fuzzy closed set of X.Then $(gof)(\beta)$ is fuzzy contra *gpr*- closed set in Z. Since gof is fuzzy contra *gpr*- closed map. Since g is fuzzy *gpr*- irresolute. $g^{-1}(gof)(\beta)$ is fuzzy contra *gpr*- closed in y. But $g^{-1}(gof)(\beta) = f(\beta)$, as g is injective. thus f is fuzzy contra *gpr*- closed map.

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