Results on Split, Non-Split, Inverse Split and Inverse Non-Split Domination Number of Some Special Graphs

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Abstract — *Let G=(V, E) be a simple, finite, connected and undirected graph. A non-empty subset $D \subseteq V$ is called a dominating set if every vertex in V-D adjacent to at least one vertex in D. The minimum cardinality taken over all the minimal dominating sets of G is called the domination number of G, denoted by $\gamma(G)$. Let D be the minimum dominating set of G. If V-D contains a dominating set D' is called the inverse dominating set of G with respect to D. The inverse dominating number $\gamma'(G)$ is the minimum cardinality taken over all the minimal inverse dominating set of G. A dominating set $D \subseteq V$ of a graph G is a split(non-split) dominating set if the induced sub graph <V-D> is disconnected(connected). The split (non-split) domination number $\gamma_s(G)$ [$\gamma_{ns}(G)$] is the minimum cardinality of a split(non-split) dominating set.*

Keywords — *Dominating set, inverse dominating set, split and non-split domination number, inverse split domination number and inverse non-split domination number.*

I. INTRODUCTION

Let G=(V,E) be a simple, finite, connected and undirected graph. Any undefined term in this paper can be found in [1,2]. In this paper, we obtained split, non-split and inverse split and non-split domination in graphs. Graphs are used as models for both naturals and human made structures. The formalized and represented by graph rewrite systems. In 1962 O.Ore **i**n his book on "graph theory" used terminology "dominating set" and "domination number". The purpose of this paper is to introduce the concept of split and non-split domination number, inverse split and inverse non-split domination number of some special graphs. Let D' be the minimum inverse dominating set of G. Then D' is called an inverse split (non-split) dominating set of G if the induced sub graph <V-D'> disconnected (connected). The inverse split (non-split) domination number is denoted by$\gamma_s'(G)$[$\gamma_{ns}'(G)$] and it is minimum cardinality taken over all the minimal inverse split (non-split) dominating sets of G. In this paper, many bounds on $\gamma_s'(G)$ and$\gamma_{ns}'(G)$ are obtained and their exact values of some special graphs are found.

II. BASIC DEFINITIONS

2.1 Dominating set

A non-empty subset D⊆V in a graph G=(V,E) is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The **domination number γ(G)** of G is the minimum cardinality of a minimal dominating set of G.

2.2 Split dominating set

A dominating set D⊆V(G) is a split dominating set if the induced subgraph <V-D> is disconnected. The minimum cardinality of a split dominating set is called the split **domination number** of G. It is denoted by **γ_s(G).**

2.3 Inverse split dominating set

Let D' be the minimum inverse dominating set of G with respect to D. Then, D' is called the inverse split dominating set of G if the induced sub graph<V-D'> is disconnected. The inverse split domination number is denoted by $\gamma_s'(G)$ and it is the minimum cardinality taken over all the minimal inverse split dominating set of G.

2.4 Non-split dominating set

A dominating set $D\subseteq V(G)$ is a non-split dominating set if the induced subgraph <V-D> is connected. The minimum cardinality of a non-split dominating set is called the non-split domination number of G. It is denoted by $\gamma_{ns}(G)$.

2.5 Inverse non-split dominating set

Let D' be the minimum inverse dominating set of G with respect to D. Then, D' is called the inverse non-split dominating set of G if the induced sub graph<V-D'> is connected. The inverse non-split domination number is denoted by $\gamma_{ns'}(G)$ and it is the minimum cardinality taken over all the minimal inverse split dominating set of G.

III. DEFINITIONS OF SOME NAMED GRAPHS

3.1 Graceful graph

It is the graph with 6 vertices and 18 edges.

3.2 Ore graph

It is the graph with 6 vertices and 9 edges.

3.3 Bull graph

The bull graph is a planar undirected graph with 5 vertices and 5 edges.

3.4 Lollipop graph

It is a complete graph with 6 vertices and 10 edges.

3.5 Line graph

It is a graph with 6 vertices and 8 edges.

3.6 Classes of perfect graph

It is the induced sub graph with 8 vertices and 12 edges.

3.7 Moser spindle graph

Moser spindle is an undirected graph with 7 vertices and 11 edges.

3.8 Kite graph

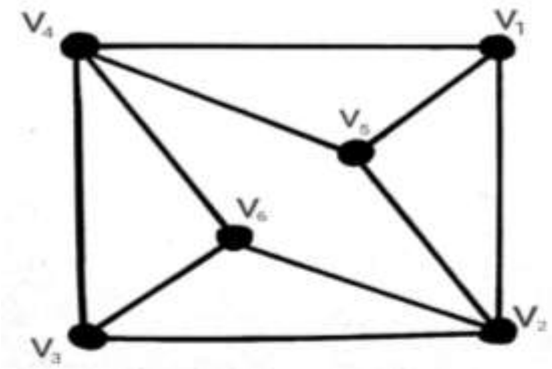
It is a graph with 5 vertices and 6 edges.

3.9 Tricyclic graph

It is a simple and undirected graph with 6 vertices and 9 edges.

IV. SOME RESULTS ON SPLIT, NON-SPLIT, INVERSE SPLIT AND INVERSE NON-SPLIT DOMINATION NUMBER OF SOME SPECIAL GRAPHS

4.1 Graceful graphs



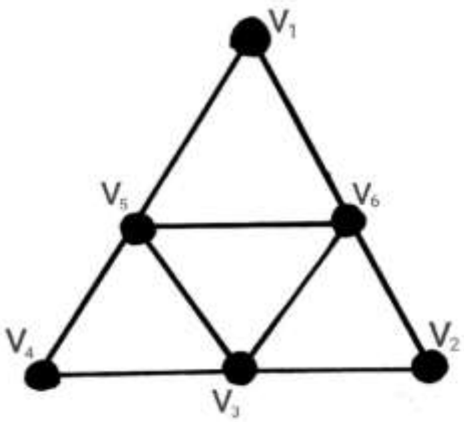
Result

If G is a Graceful graph then it exists split and non-split domination number and inverse split and inverse non-split domination number.

Proof

Let G be the Graceful Graph with 6 vertices and 10 edges.
Let V= $\{v_1,v_2,v_{3.....}v_6\}$ be the vertex set of the graph G.
From this graph,
The minimum dominating set D = $\{v_2,v_4\}$
Here, <V-D> = $\{v_{1,}v_{3,}v_{5,}v_6\}$
Then, the induced sub graph <V-D>is disconnected.
Hence, the split domination number is 2
$\therefore$ γ_s(G) = 2
Let us take the inverse dominating set D$'$= $\{v_1,v_3\}$
Here, <V-D$'$> = $\{v_2,v_{4,}v_{5,}v_6\}$
Then, the induced sub graph <V-D$'$>is connected.
Hence, the graph has inverse non-split dominating set.
The inverse non-split domination number is 2.
$\therefore$ γ_{ns}'(G) = 2
When we take the dominating set D = $\{v_1,v_3\}$
$\therefore$<V-D> = $\{v_2,v_{4,}v_{5,}v_6\}$
Then, the induced sub graph<V-D> is connected.
Hence, the graph has non-split dominating set.
$\therefore$The non-split domination number is 2.
$\therefore$ γ_{ns}(G) = 2
Here, the inverse dominating set D$'$ = $\{v_2,v_4\}$
<V-D$'$> = $\{v_{1,}v_{3,}v_{5,}v_6\}$
Then, the induced sub graph <V-D$'$> is disconnected.
$\therefore$The inverse non-split domination number is 2.
$\therefore$ γ_s'(G) = 2
Hence, this graph exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.2 Ore graph



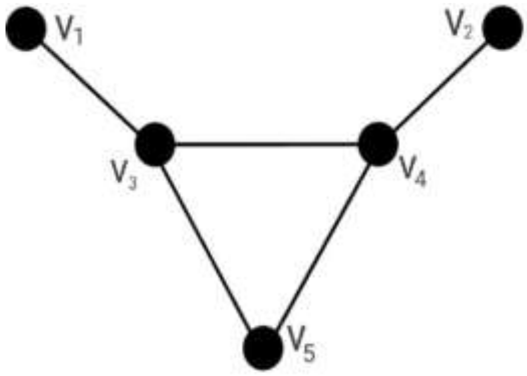
Result

If G is a Ore graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then$\gamma_s^{'}$ (G =$\gamma_{ns}^{'}$(G) = 2

Proof

Let G be the Ore graph with 6 vertices and 9 edges.
Let V = $\{v_1,v_2,v_{3.....}v_6\}$be the vertex set of the graph G.
Here, the minimum dominating set D = $\{v_5,v_6\}$
∴<V-D> = $\{v_1,v_{2,}v_{3,}v_4\}$
Then, the induced sub graph <V-D> is disconnected.
Hence, the split domination number is 2
∴ γ_s(G) = 2
Let us take the another dominating set D = $\{v_1,v_3\}$
∴<V-D> = $\{v_2,v_{4,}v_{5,}v_6\}$
Then, the induced sub graph <V-D> is connected.
Hence, the graph has non-split dominating set.
∴The non-split domination number is 2.
∴ γ_{ns}(G) = 2
When the dominating set D = $\{v_5,v_6\}$
The inverse dominating set $D^{'}$ = $\{v_1,v_3\}$
∴<V-$D^{'}$> = $\{v_2,v_{4,}v_{5,}v_6\}$
Then, the induced sub graph <V-$D^{'}$> is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 2.
∴ γ_{ns}'(G) = 2
When we take the dominating set D = $\{v_1,v_3\}$
The inverse dominating set $D^{'}$=$\{v_5,v_6\}$
∴<V-$D^{'}$> = $\{v_1,v_{2,}v_{3,}v_4\}$
Here, the induced sub graph <V-$D^{'}$> is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s^{'}$(G) = 2
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.3 Bull graph



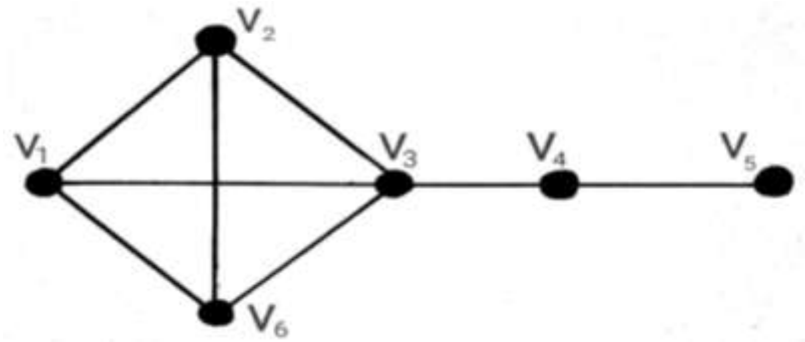
Result

If G is a Bull graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{}^{'}$ (G <$\gamma_{ns}{}^{'}$(G).

Proof

Let G be the Bull graph with 5 vertices and 5 edges.
Let V = $\{v_1,v_2,v_3,v_4,v_5\}$be the vertex set of the graph G.
From this graph,
The minimum dominating set D = $\{v_3,v_4\}$
∴<V-D> = $\{v_1,v_2,v_5\}$
Then, the induced sub graph <V-D> is disconnected.
Hence, the graph has split dominating set.
∴The split domination number is 2.
∴ γ_s(G) = 2
Let us take the inverse dominating set $D^{'}$ = $\{v_1,v_2, v_5\}$
∴<V-D'> = $\{v_3,v_4\}$ be the induced sub graph.
Here, the induced sub graph <V-D'> is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 3.
∴ $\gamma_{ns}{}'$(G) = 3
When we take the dominating set D = $\{v_1,v_2, v_5\}$
∴<V-D> = $\{v_3,v_4\}$
Then, the induced sub graph <V-D> is connected.
Hence, the graph has non-split dominating set.
∴The non-split domination number is 3.
∴ γ_{ns}(G) = 3
Let the inverse dominating set $D^{'}$ = $\{v_3,v_4\}$
∴<V-D'> = $\{v_1,v_2,v_5\}$
Here, the induced sub graph <V-D'> is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s{}^{'}$(G) = 2
Here, **$\gamma_s{}^{'}$ (G<$\gamma_{ns}{}^{'}$(G)**
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.4 Lollipop graph



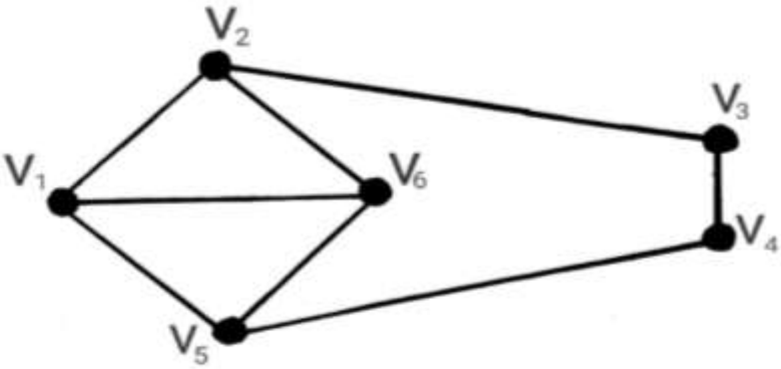
Result

If G is a Lollipop graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{}^{'}$ (G =$\gamma_{ns}{}^{'}$(G) = 2

Proof

Let G be the Lollipop graph with 6 vertices and 8 edges.
Let V= $\{v_1,v_2,v_{3.....}v_6\}$ be the vertex set of the graph G.
From this graph,
The minimum dominating set D = $\{v_3,v_4\}$
Here, <V-D> = $\{v_{1,}v_{2,}v_{5,}v_6\}$
Then, the induced sub graph <V-D> is disconnected.
Hence, the graph G has split dominating set.
∴The split domination number is 2.
∴ γ_s(G) = 2
If the dominating set D = $\{v_3,v_4\}$
Then the inverse dominating set D$^{'}$ = $\{v_1,v_5\}$
∴<V-D$^{'}$>=$\{v_{2,}v_{3,}v_{4,}v_6\}$ be the induced sub graph which is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 2.
∴ γ_{ns}'(G) = 2
If we take the another dominating set D = $\{v_1,v_5\}$
Here, <V-D> = $\{v_{2,}v_{3,}v_{4,}v_6\}$
Then, the induced sub graph <V-D> is connected.
The graph G has non-split dominating set.
∴The non-split domination number is 2.
∴ γ_{ns}(G) = 2
Here, the inverse dominating set D$^{'}$ = $\{v_3,v_4\}$
∴<V-D$^{'}$> = $\{v_{1,}v_{2,}v_{5,}v_6\}$ be the induced sub graph which is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s{}^{'}$(G) = 2
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.5 Line graph



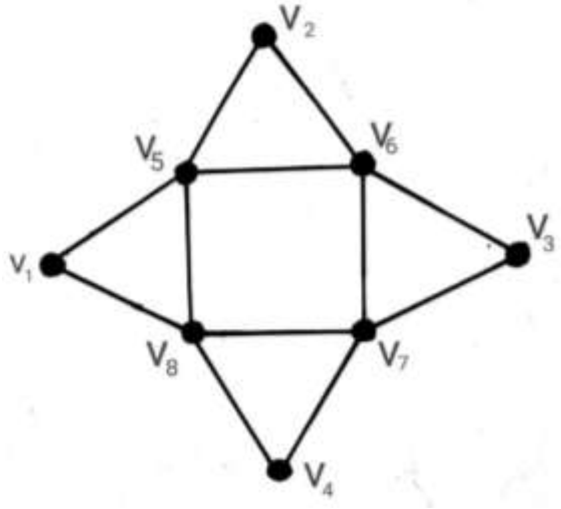
Result

If G is a Line graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{}'$ (G $=\gamma_{ns}{}'$(G).

Proof

Let G be the line graph with 6 vertices and 8 edges.
Let V= $\{v_1,v_2,v_{3\ldots\ldots}v_6\}$ be the vertex set of the graph G.
From this graph,
The minimum dominating set D = $\{v_2,v_5\}$
Here, <V-D> = $\{v_{1,}v_{3,}v_{4,}v_6\}$
Then, the induced sub graph <V-D> is disconnected.
Hence, the graph G has split dominating set.
∴The split domination number is 2.
∴ γ_s(G) = 2
Here, the inverse dominating set D$'$= $\{v_1,v_4\}$
∴<V-D$'$> = $\{v_{2,}v_{3,}v_{5,}v_6\}$ be the induced sub graph which is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 2.
∴ γ_{ns}'(G) = 2
If we take the another dominating set D = $\{v_1,v_4\}$
Here, <V-D> = $\{v_{2,}v_{3,}v_{5,}v_6\}$
Then, the induced sub graph <V-D> is connected.
The graph G has non-split dominating set.
∴The non-split domination number is 2.
∴ γ_{ns}(G) = 2
Here, the inverse dominating set D$'$ = $\{v_2,v_5\}$
∴<V-D$'$> = $\{v_{1,}v_{3,}v_{4,}v_6\}$ be the induced sub graph which is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s{}'$(G) = 2
Here,**γ_s'(G)=γ_{ns}'(G)**
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.6 Classes of perfect graph



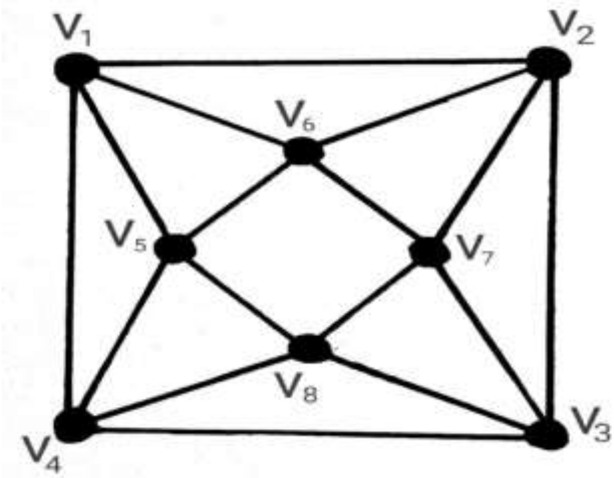
Result

If G is a Classes of perfect graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{}'$ (G <$\gamma_{ns}{}'$(G).

Proof

Let G be the classes of perfect graph with 8 vertices and 12 edges.
Let V= {v_1,v_2,v_3.....v_8} be the vertex set of the graph G.
From this graph,
The minimum dominating set D = {v_5,v_7}
Here, <V-D> = {v_1,v_2,v_3,v_6,v_8}
Then, the induced sub graph <V-D> is disconnected.
Hence, the graph G has split dominating set.
∴The split domination number is 2.
∴ γ_s(G) = 2
Here, the inverse dominating set D'= {v_1,v_2,v_3,v_4}
<V-D'> = {v_5,v_6,v_7,v_8}
Hence the induced sub graph <V-D'> is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 4.
∴ γ_{ns}'(G) = 4
Let us take the dominating set D = {v_1,v_2,v_3,v_4}
Here <V-D> = {v_5,v_6,v_7,v_8} be the induced sub graph which is connected.
The graph G has non-split dominating set.
∴The non-split domination number is 4.
∴ γ_{ns}(G) = 4
If we take the inverse dominating set D' = {v_5,v_7}
Here, <V-D'> = {v_1,v_2,v_3,v_4,v_8,v_6} be the induced sub graph which is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s{}'$(G) = 2
Here,$\gamma_s{}'$ (G)<$\gamma_{ns}{}'$(G)
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.7 Transitive graph



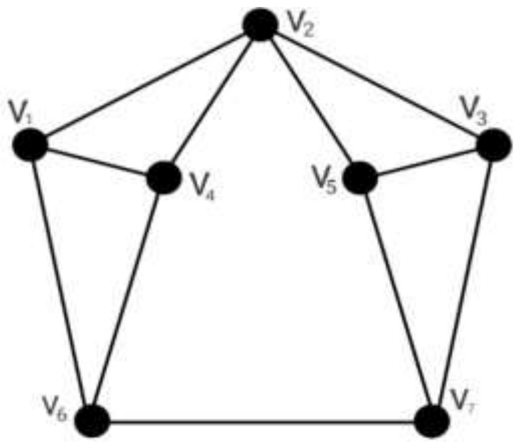
Result

If G is a Transitive graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{'}$ (G =$\gamma_{ns}{'}$(G) = 2

Proof

Let G be the Transitive Graph with 8 vertices and 16 edges.
Let V= $\{v_1,v_2,v_{3.....}v_8\}$ be the vertex set of the graph G.
From this graph,
The minimum dominating set D = $\{v_1,v_3\}$
Here, <V-D> = $\{v_{2,}v_{4,}v_{5,}v_{6,}v_{7,}v_8\}$
Then, the induced sub graph <V-D> is connected.
Hence, the graph G has non-split dominating set.
∴The non-split domination number is 2.
∴ γ_{ns}(G) = 2
If we take the inverse dominating set D$'$ = $\{v_5,v_7\}$
Here, <V-D$'$> =$\{v_{1,}v_{2,}v_{3,}v_{4,}v_{8,}v_6\}$ be the induced sub graph which is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 2.
∴ $\gamma_{ns}{'}$(G) = 2
Hence, this graph only exist non-split and inverse non-split dominating set.
∴ It does not exit split dominating set and inverse split dominating set.

4.8 Moser spindle



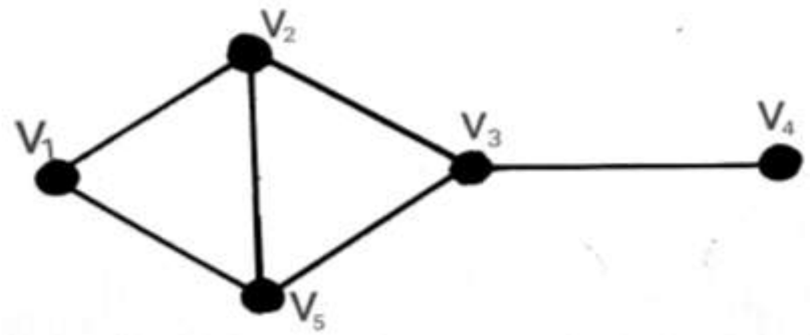
Result

If G is a Moser Spindle graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{'}$ (G =$\gamma_{ns}{'}$(G) = 2

Proof

Let G be the Moser Spindle graph with 7 vertices and 11 edges.
Let V= {v_1,v_2,$v_{3.....}v_7$} be the vertex set of the graph G.
From this graph,
The minimum dominating set D = {v_2,v_7}.
Here, <V-D> = {$v_{1,}v_{3,}v_{4,}v_{5,}v_6$}
Then, the induced sub graph<V-D> is disconnected.
Hence, the graph G has split dominating set.
∴The split domination number is 2.
∴ γ_s(G) = 2
Here, the inverse dominating set D$'$= {$v_{4,}v_5$}
<V-D'> = {$v_{1,}v_{2,}v_{3,}v_{6,}v_7$} be the induced sub graph which is connected.
Hence, the graph G has inverse non-split dominating set.
The inverse non-split domination number is 2.
∴ γ_{ns}'(G) = 2
Let us take the dominating set D = {$v_{5,}v_4$}
Here <V-D> = {$v_{1,}v_{2,}v_{3,}v_{6,}v_7$} be the induced sub graph which is connected.
The graph G has non-split dominating set.
∴The non-split domination number is 2.
∴ γ_{ns}(G) = 2
If we take the inverse dominating set D$'$ = {v_2,v_7}
Here, <V-D$'$> = {$v_{1,}v_{3,}v_{4,}v_{5,}v_6$} be the induced sub graph which is disconnected.
Hence, the graph G has inverse split dominating set.
The inverse split domination number is 2.
∴ $\gamma_s{'}$(G) = 2
Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.9 Kite graph



Result

If G is a Kite graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then $\gamma_s{'}$ (G =$\gamma_{ns}{'}$(G) = 2

Proof

Let G be the Kite Graph with 5 vertices and 6 edges.
Let V= {v_1,v_2,$v_{3.....}v_5$} be the vertex set of the graph G.
From this graph,
The minimum dominating set D = {v_1,v_3}.
Here, <V-D> = {$v_{2,}v_{4,}v_5$}
Then, the induced sub graph<V-D> is disconnected.

Hence, the graph G has split dominating set.

∴The split domination number is 2.

∴ $\gamma_s(G) = 2$

Here, the inverse dominating set $D' = \{v_2, v_4\}$

<V-D'> = $\{v_1, v_3, v_5\}$ be the induced sub graph which is connected.

Hence, the graph G has inverse non-split dominating set.

The inverse non-split domination number is 2.

∴ $\gamma_{ns}'(G) = 2$

Let us take the dominating set D = $\{v_2, v_4\}$

Here <V-D> = $\{v_1, v_5, v_3\}$ be the induced sub graph which is connected.

The graph G has non-split dominating set.

∴The non-split domination number is 2.

∴ $\gamma_{ns}(G) = 2$

If we take the inverse dominating set $D' = \{v_1, v_3\}$

Here, <V-D'> = $\{v_2, v_4, v_5\}$ be the induced sub graph which is disconnected.

Hence, the graph G has inverse split dominating set.

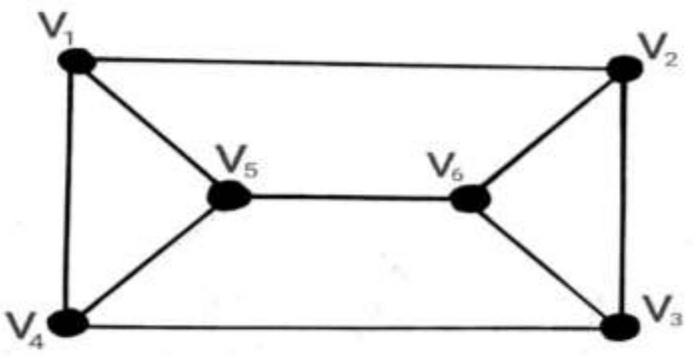
The inverse split domination number is 2.

∴ $\gamma_s'(G) = 2$

Here, **$\gamma_s'(G) = \gamma_{ns}'(G) = 2$**

Hence, this graph G exist split and non-split dominating set and also inverse split and inverse non-split dominating set.

4.10 Tricyclic graph



Result

If G is a Tricyclic graph then it exist split and non-split domination number and inverse split and inverse non-split domination number then γ_s' (G =$\gamma_{ns}'(G) = 2$

Proof

Let G be the Tricyclic Graph with 6 vertices and 9 edges.

Let V= $\{v_1, v_2, v_3 \ldots v_6\}$ be the vertex set of the graph G.

Then $\{v_1, v_3, v_4, v_2\}$ be the outer vertices and $\{v_5, v_6\}$ be the inner vertices.

From this graph,

The minimum dominating set D = $\{v_1, v_3\}$.

Here, <V-D> = $\{v_2, v_4, v_5, v_6\}$

Then, the induced sub graph <V-D> is connected.

Hence, the graph G has non-split dominating set.

∴The non-split domination number is 2.

∴ $\gamma_{ns}(G) = 2$

Here, the inverse dominating set $D' = \{v_5, v_6\}$

<V-D'> = $\{v_1, v_2, v_3, v_4\}$ be the induced sub graph which is connected.

Hence, the graph G has inverse non-split dominating set.

The inverse non-split domination number is 2.

$\therefore \gamma_{ns}'(G) = 2$

Hence this Graph exist only non-split and inverse non-split dominating set.

It does not exist split and inverse split dominating set.

$\therefore \gamma_{ns}(G) = \gamma_{ns}'(G) = 2$

V. CONCLUSION

In this paper, we obtained the split; non-split, inverse split and inverse non-split domination number some special graphs. The relationships between some domination parameters are also verified.

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