The Lattice Structure of The Subgroups of Order 8 In The Subgroup Lattices of 3 X 3 Matrices Over Z_2

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ABSTRACT

Let $\mathcal{G}$ be the set of all 3 X 3 non-singular matrices $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, where a,b,c,d,e,f,g,h,i are integers modulo p. Then $\mathcal{G}$ is a group under matrix multiplication modulo p, of order $(p^n-1)(p^n-p)(p^n-p^2)\ldots\ldots(p^n-p^{n-1})$. Let G be the subgroup of $\mathcal{G}$ defined by $G=\left\{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1\right\}$. Then G is of order $\frac{(p^n-1)(p^n-p)(p^n-p^2)\ldots\ldots(p^n-p^{n-1})}{p-1}$. Let L(G) be the lattice formed by all subgroups G. In this paper, we give the structure of the subgroups of order 8 of L(G) in the case when P=2.

Keywords: *Matrix group, Subgroups, Poset, Lattice, Atom.*

1. Introduction

Let L(G) be the Lattice of Subgroups of G, where G is a group of 3x3 matrices over Z_p having determinant value 1 under matrix multiplication modulo p, where p is a prime number.

Let $\mathcal{G}=\left\{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} : a,b,c,d,e,f,g,h,i \in Z_p, \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0\right\}$

Then $\mathcal{G}$ is a group under matrix multiplication modulo p.

Let $G=\left\{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1\right\}$

Then G is a subgroup of $\mathcal{G}$.

we have, $o(\mathcal{G}) = (p^n-1)(p^n-p)(p^n-p^2)\ldots\ldots(p^n-p^{n-1})$

and $o(G) = \frac{(p^n-1)(p^n-p)(p^n-p^2)......(p^n-p^{n-1})}{p-1}$.

In this paper, we give the structure of the subgroups of order 8 of L(G) in the case when P=2.

2. Preliminaries

In this section we give the definition needed for the development of the paper.

Definition 2.1(Poset): A partial order on a non-empty set P is a binary relation ≤ on P that is reflexive, anti-symmetric and transitive. The pair (P, ≤) is called a **partially ordered set or poset**. A poset (P, ≤) is totally ordered if every x, y ∈ P are comparable, that is either x ≤ y or y ≤ x. A non-empty subset S of P is a chain in P if S is totally ordered by ≤.

Definition 2.2: Let (P, ≤) be aposet and let S⊆P. An upper bound of S is an element x∈P for which s ≤ x for all s∈S. The least upper bound of S is called the **supremum or join** of S.A lower bound for S is an element x ∈ P for which x≤ s for all s∈ S. The greatest lower bound of S is called the **infimum or meet** of S.

Definition 2.3 (Lattice): Poset (P, ≤) is called a lattice if every pair x, y elements of P has a supremum and an infimum, which are denoted by x ∨ y and x ∧ y respectively.

Definition 2.4 (Atom): An element 'a' is an atom, if a>0 and a dual atom, if a<1.

3. Arrangement of elements of G according to their orders

Let G be the subgroup of $\mathcal{G}$ defined by

Let $G = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1$

Then $|G| = \frac{(p^n-1)\,(p^n-p)\,(p^n-p^2).....(p^n-p^{n-1})}{p-1}$

$= \frac{(2^3-1)(2^3-2)(2^3-2^2)}{2-1}$

$= \frac{(8-1)(8-2)(8-4)}{1} = (7)(6)(4) = 168$

3.1 *Element of order 1(one element)*

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3.2 Elements of order 2 (21 elements)

$$\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix}$$

3.3 Elements of order 3 (56 elements)

$$\begin{pmatrix}1&0&1\\1&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$$

$$\begin{pmatrix}1&1&0\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\1&0&0\end{pmatrix}$$

$$\begin{pmatrix}1&0&1\\0&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&1&0\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&1\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&1&0\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\0&1&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\1&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\0&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

3.4 Elements of order 4 (42 elements)

$$\begin{pmatrix}0&0&1\\1&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&1\\1&0&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&0\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&1\\1&0&0\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\0&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&1\\1&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\1&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\1&1&1\end{pmatrix}$$

$$\begin{pmatrix}0&1&1\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\1&0&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\1&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\0&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&1&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\1&1&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&1\\0&1&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}$$

3.5 Elements of order7 (48 elements)

$$\begin{pmatrix}0&1&1\\1&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&0\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&0\\0&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&1\\0&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&1\\1&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&1&1\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&0&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&0\\1&1&1\end{pmatrix}$$

$$\begin{pmatrix}0&1&0\\0&0&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&0&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\0&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\1&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&1&1\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&1\\1&1&1\end{pmatrix}$$

$$\begin{pmatrix}1&1&1\\0&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\1&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&1\\1&1&1\end{pmatrix}$$

4. We find the subgroups of G of different orders namely 2,4 and 8

4.1 Subgroups of order 2

Let H be an arbitrary subgroup of G of order 2. Then the elements of H must have order 1 or 2.

Thus, all the subgroups of G of order 2 are obtained as follows:

$$H_1=\left\{\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},e\right\},H_2=\left\{\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},e\right\},H_3=\left\{\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},e\right\},$$

$$H_4 = \left\{\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix}, e\right\}, H_5 = \left\{\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix}, e\right\}, H_6 = \left\{\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix}, e\right\},$$

$$H_7 = \left\{\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}, e\right\}, H_8 = \left\{\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix}, e\right\}, H_9 = \left\{\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix}, e\right\},$$

$$H_{10} = \left\{\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}, e\right\}, H_{11} = \left\{\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix}, e\right\}, H_{12} = \left\{\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix}, e\right\},$$

$$H_{13} = \left\{\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}, e\right\}, H_{14} = \left\{\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix}, e\right\}, H_{15} = \left\{\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix}, e\right\},$$

$$H_{16} = \left\{\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}, e\right\}, H_{17} = \left\{\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix}, e\right\}, H_{18} = \left\{\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix}, e\right\},$$

$$H_{19} = \left\{\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix}, e\right\}, H_{20} = \left\{\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix}, e\right\}, H_{21} = \left\{\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix}, e\right\}$$

4.2 Subgroups of order 4

Let L be an arbitrary subgroup of G of order 4. Then the elements of L must have orders 1,2 or 4. If L contains an element of order 4, then L is generated by an element of order 4. Thus, all the subgroups of G of order 4 are obtained as follows:

$$L_1 = \left\{\begin{pmatrix}0&0&1\\1&1&0\\1&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&1\\0&1&1\\1&0&0\end{pmatrix}, e\right\}, L_2 = \left\{\begin{pmatrix}0&1&1\\0&1&0\\1&0&0\end{pmatrix}, \begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix}, \begin{pmatrix}0&0&1\\0&1&0\\1&1&0\end{pmatrix} e\right\},$$

$$L_3 = \left\{\begin{pmatrix}1&0&1\\1&0&0\\1&1&0\end{pmatrix}, \begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix}, \begin{pmatrix}0&1&0\\0&1&1\\1&1&0\end{pmatrix}, e\right\}, L_4 = \left\{\begin{pmatrix}1&1&1\\1&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&1\\1&1&1\end{pmatrix} e\right\},$$

$$L_5 = \left\{\begin{pmatrix}0&1&1\\1&1&0\\0&1&0\end{pmatrix}, \begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix}, \begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix}, e\right\}, L_6 = \left\{\begin{pmatrix}1&1&0\\1&0&1\\1&0&0\end{pmatrix}, \begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix}, \begin{pmatrix}0&0&1\\1&0&1\\0&1&1\end{pmatrix} e\right\},$$

$$L_7 = \left\{\begin{pmatrix}0&1&0\\1&1&1\\1&0&0\end{pmatrix}, \begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&1\\1&0&0\\1&1&1\end{pmatrix}, e\right\}, L_8 = \left\{\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&1\\1&1&0\end{pmatrix} e\right\},$$

$$L_9 = \left\{\begin{pmatrix}1&1&0\\0&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix}, \begin{pmatrix}1&0&1\\0&0&1\\0&1&0\end{pmatrix}, e\right\}, L_{10} = \left\{\begin{pmatrix}1&1&1\\0&0&1\\1&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix}, \begin{pmatrix}0&0&1\\1&1&1\\0&1&0\end{pmatrix} e\right\},$$

$$L_{11}=\left\{\begin{pmatrix}1&1&1\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\1&0&1\end{pmatrix},e\right\},L_{12}=\left\{\begin{pmatrix}0&1&1\\1&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&0\\1&1&1\\0&1&1\end{pmatrix}e\right\},$$

$$L_{13}=\left\{\begin{pmatrix}0&1&0\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&1&1\end{pmatrix},e\right\},L_{14}=\left\{\begin{pmatrix}1&1&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\1&1&1\end{pmatrix}e\right\},$$

$$L_{15}=\left\{\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix},e\right\},L_{16}=\left\{\begin{pmatrix}0&1&1\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&1\\0&0&1\end{pmatrix}e\right\},$$

$$L_{17}=\left\{\begin{pmatrix}1&0&1\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\0&0&1\end{pmatrix},e\right\},L_{18}=\left\{\begin{pmatrix}1&0&1\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&1&1\end{pmatrix}e\right\},$$

$$L_{19}=\left\{\begin{pmatrix}1&0&0\\0&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\1&0&1\end{pmatrix},e\right\},L_{20}=\left\{\begin{pmatrix}0&1&1\\1&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\1&1&1\end{pmatrix}e\right\},$$

$$L_{21}=\left\{\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix},e\right\},L_{22}=\left\{\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix}e\right\},$$

$$L_{23}=\left\{\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},e\right\},L_{24}=\left\{\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix}e\right\},$$

$$L_{25}=\left\{\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},e\right\},L_{26}=\left\{\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix}e\right\},$$

$$L_{27}=\left\{\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},e\right\},L_{28}=\left\{\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix}e\right\},$$

$$L_{29}=\left\{\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},e\right\},L_{30}=\left\{\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix}e\right\},$$

$$L_{31}=\left\{\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},e\right\},L_{32}=\left\{\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix}e\right\},$$

$$L_{33}=\left\{\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},e\right\},L_{34}=\left\{\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix}e\right\},$$

$$L_{35}=\left\{\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},e\right\}.$$

Since first twenty one subgroups of order 4 contains exactly two elements of order 4 and we have only forty two elements of order 4 and first twenty one sub groups of order 2 and next fourteen subgroups of order 4 contains exactly 3 elements of order 2, there will be no other subgroups of order 4 except the above thirty five.

Here,

$$H_{19} \subset L_1;\ H_{15} \subset L_2;\ H_{20} \subset L_3;\ H_5 \subset L_4;\ H_4 \subset L_5;\ H_2 \subset L_6;\ H_6 \subset L_7;\ H_{10} \subset L_8;\ H_{17} \subset L_9;$$

$$H_{12} \subset L_{10};\ H_3 \subset L_{11};\ H_1 \subset L_{12};\ H_{14} \subset L_{13};\ H_{13} \subset L_{14};\ H_9 \subset L_{15};\ H_{21} \subset L_{16};\ H_{18} \subset L_{17};\ H_{11} \subset L_{18};$$

$$H_8 \subset L_{19};\ H_7 \subset L_{20};\ H_{16} \subset L_{21};\ H_1,\ H_2,\ H_{15} \subset L_{22};\ H_1,\ H_5,\ H_{19} \subset L_{23};\ H_2,\ H_4,\ H_{20} \subset L_{24};$$

$$H_3,\ H_4,\ H_{10} \subset L_{25};\ H_5,\ H_{12},\ H_6 \subset L_{26};\ H_6,\ H_{17},\ H_3 \subset L_{27};\ H_7,\ H_{21},\ H_{20} \subset L_{28};\ H_8,\ H_{10},\ H_9 \subset L_{29};$$

$$H_9,\ H_{13},\ H_{14} \subset L_{30};\ H_{11},\ H_{15},\ H_{13} \subset L_{31};\ H_{11},\ H_{16},\ H_{17} \subset L_{32};\ H_{12},\ H_{14},\ H_7 \subset L_{33};$$

$$H_{18},\ H_{19},\ H_8 \subset L_{34};\ H_{21},\ H_{16},\ H_{18} \subset L_{35}.$$

4.3 Subgroups of order 8

Since$|G| = 2^3 X 3 X 7, 2^3 || G| but 2^4 \nmid |G|$.

Therefore, G has a 2 – sylow subgroups of order 8.

The number of 2-sylow subgroups is of the form 1+2k and 1+2k | $|G|$

Therefore, $1 + 2K | 3X7$.

The possible values for K are 0, 1 and 10.

Therefore, the maximum number of 2-sylow subgroups of G of order 8 is 21 when k=10.

Since G has no element of order 8, the elements of subgroups of order 8 must have order 1,2 or 4.

Thus, all the subgroups of G of order 8 are obtained as follows:

$$P_1 = \left\{ \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, e \right\},$$

$$P_2=\left\{\begin{pmatrix}0&1&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},e\right\},$$

$$P_3=\left\{\begin{pmatrix}1&0&1\\1&0&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\0&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_4=\left\{\begin{pmatrix}1&1&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\0&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},e\right\},$$

$$P_5=\left\{\begin{pmatrix}0&1&1\\1&1&0\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&0&1\\1&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},e\right\},$$

$$P_6=\left\{\begin{pmatrix}1&1&0\\1&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_7=\left\{\begin{pmatrix}0&1&0\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},e\right\},$$

$$P_8=\left\{\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},e\right\},$$

$$P_9=\left\{\begin{pmatrix}1&1&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},e\right\},$$

$$P_{10}=\left\{\begin{pmatrix}1&1&1\\0&0&1\\1&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},e\right\},$$

$$P_{11}=\left\{\begin{pmatrix}1&1&1\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},e\right\},$$

$$P_{12}=\left\{\begin{pmatrix}0&1&1\\1&1&1\\1&1&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}1&1&0\\1&1&1\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&1\\0&1&0\\1&1&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&0&1\\1&1&1\\1&0&0\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_{13}=\left\{\begin{pmatrix}0&1&0\\1&0&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},e\right\},$$

$$P_{14}=\left\{\begin{pmatrix}1&1&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},e\right\},$$

$$P_{15}=\left\{\begin{pmatrix}1&0&0\\1&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},e\right\},$$

$$P_{16}=\left\{\begin{pmatrix}0&1&1\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_{17}=\left\{\begin{pmatrix}1&0&1\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},e\right\},$$

$$P_{18}=\left\{\begin{pmatrix}1&0&1\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},e\right\},$$

$$P_{19}=\left\{\begin{pmatrix}1&0&0\\0&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\1&1&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_{20}=\left\{\begin{pmatrix}0&1&1\\1&0&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\1&1&1\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&0\\1&1&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}0&1&1\\1&0&1\\0&0&1\end{pmatrix},e\right\},$$

$$P_{21}=\left\{\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&1&1\\0&1&0\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&1\\0&1&1\\0&0&1\end{pmatrix},\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix},e\right\},$$

Here,

$$L_1,\ H_1,\ H_5,\ H_{18},\ H_8 \subset P_1;\ L_2,\ H_{11},\ H_{13},\ H_1,\ H_2 \subset P_2;\ L_3,\ H_2,\ H_4, H_{7,}, H_{21} \subset P_3;$$

$$L_4,\ H_1,\ H_{19},\ H_{12},\ H_6 \subset P_4;\ L_5,\ H_2,\ H_{20},\ H_3,\ H_{10} \subset P_5;\ L_6,\ H_1,\ H_{15}, H_{4,}, H_{20} \subset P_6;$$

$$L_7,\ H_{17},\ H_3,\ H_5,\ H_{12} \subset P_7;\ L_8,\ H_3,\ H_4,\ H_8,\ H_9 \subset P_8;\ L_9,\ H_{11},\ H_{16}, H_{6,}, H_3 \subset P_9;$$

$$L_{10},\ H_5,\ H_6,\ H_{14},\ H_7 \subset P_{10};\ L_{11},\ H_4,\ H_{10},\ H_6,\ H_{17} \subset P_{11};\ L_{12},\ H_2,\ H_{15}, H_{5,}, H_{19} \subset P_{12};$$

$$L_{13},\ H_{12},\ H_7,\ H_9,\ H_{13} \subset P_{13};\ L_{14},\ H_9,\ H_{14},\ H_{11},\ H_{15} \subset P_{14};\ L_{15},\ H_8,\ H_{10}, H_{13}, H_{14} \subset P_{15};$$

$$L_{16},\ H_7,\ H_{20},\ H_{16},\ H_{18} \subset P_{16};\ L_{17},\ H_{19},\ H_8,\ H_{21},\ H_{16} \subset P_{17};\ L_{18},\ H_{15},\ H_{13}, H_{16,}, H_{17} \subset P_{18};$$

$$L_{19},\ H_{10},\ H_9,\ H_{18},\ H_{19} \subset P_{19};\ L_{20},\ H_{12},\ H_{14},\ H_{21},\ H_{20} \subset P_{20};\ L_{21},\ H_{11},\ H_{17}, H_{21,}, H_{18} \subset P_{21};$$

5. Lattice Structure of some lower interval of subgroups of order 8 in L(G) over Z_2

Let P be an arbitrary subgroup of G of order 8. We observe that each subgroup of order 8 contains two elements of order 4 and five elements of order 2 and we have only 21 elements of order 2.

We name all the subgroups of order 8, by the symbols P_K, $1\leq k \leq 21$. We observe that P_K's are of only one type. For example we take P_1 and it contains one subgroup of order 4 and four subgroups of order 2.

Table 5.1: Subgroups of P_1

Order	Subgroups
4	L_1
2	H_1, H_5, H_8, H_{18}
1	$\{e\}$

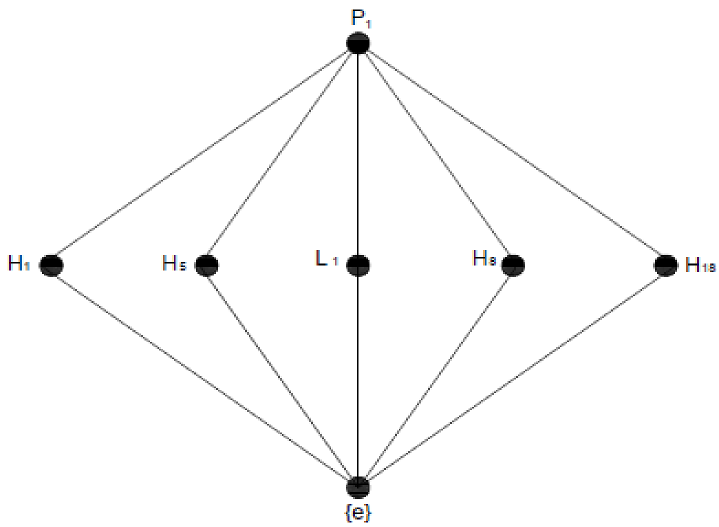

Fig.5.1: The Interval [{e}, P_1]

6. Conclusion

In this paper, we produced the lattice structure of the subgroups of order 8 in the subgroup lattices of 3x3 matrices over Z_2.

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