A Flow Model For The Detection of Obstruction Along a Sludge Tunnel

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Abstract
Blockage of a sludge tunnel as a closed drainage is a major problem in transportation processes. Debris may get clogged up by residues as sedimentary built-up at any location along the tunnel. The clogged up process if not detected early, will eventually lead to partial blockage or total blockage which will result to economic waste as well as environmental health hazard; hence the urgency for early blockage detection at any location of the tunnel. We have shown in this paper that the classical wave equation has the potential for capturing partial blockage phenomenon along a sludge tunnel. Also, we explore the dynamics of sonic wave to detect the presence of any obstruction at any point in the tunnel. Furthermore, we constructed a system of eigen-value equation that admits the spectrum of polychromatic waves whose components are high frequency oscillations propagating along both the upstream and downstream of the tunnel and any blockage encountered will cause a reversal of mode and phase that can be detected by any instrument susceptible to waveguides

Keywords: sludge, residue, eigen-value, waveguide

INTRODUCTION

When constructing any new tunnel, the fear usually pertains to the extent for which the residues within the fluid flowing through would be deposited at any location along the tunnel. If in spite of any precautions that may be taken, it does happen, it is necessary to detect, measure and remove such deposition during the course operation of the tunnel to avoid over accumulation, contamination and the ultimate blockage of the channel. It is difficult to apply measuring device directly if the fluid flow begins; the tunnel chamber may not be penetrated as the flow continues. Hence the purpose of analytic method as an alternative to determine the region of sedimentation using sonic waves as a fluid dynamic property. The oil and gas industry for instance now resort to early blockage detection to mitigate blockages for both safety and economic reasons. Several studies have been conducted in this research direction to determine the best approach for early detection of leakages and blockages in natural and artificial tunnels. Among them is the backpressure technique, which is also referred to as a frictional loss method. This technique, anchor on a baseline which proposed a linear relationship between flow rate and the difference of the square of the pressure at the inlet and outlet of a pipe under steady-state conditions. Scott and Satterwhite (1998) applied the back pressure technique to detect blockage in gas pipelines. The difficulty in using this method however, lies in the establishment of the baseline, which requires a time-consuming multi-rate test. Another leak detection method is the negative pressure wave which was reported by Sang *et al (*2006). Leak detection techniques using negative pressure waves (NPWs) are based on the principle that when a leakage occurs, it causes a pressure alteration as well as a decrease in flow speed which results in an instantaneous pressure drop and speed variation along the pipeline. As the instantaneous pressure drop occurs, it generates a negative pressure wave at the leak position and propagates the wave with a certain speed towards the upstream and downstream ends of the pipe *Orukari, M. A* (2015). The wave contains leakage information which can be estimated through visual inspection and signal analysis to determine the leakage location by virtue of the time difference with which the waves reach the pipeline ends. Due to the cost-effective nature of the NPW method, it has been widely used in pipeline monitoring due to its fast response time and leak localisation ability Yu *et al* (2009). However, it is only effective for massive instantaneous leaks and easily leads to false alarms due to the difficulty in differentiating between normal pressure waves and leakages. As a result of these deficiencies in the above leakage detection techniques, an improved mathematical model is required for efficient and effective leakage detection in both natural and man- made tunnels.

MATHEMATICAL MODEL FORMULATION

The use of temperature gradient and the thermodynamical properties of sound in gaseous medium. The motion of a fluid as a dynamical system admits for mass conservation and continuity equation by Isreal- Cookey et.al (2010) and Ngaigai and Orukari (2017)

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{q}) = \frac{\partial \rho}{\partial t} + \vec{q} \cdot \nabla \rho + \rho\, div \vec{q} = 0 \tag{1}$$

Where ρ, $\vec{q}$ are the density and velocity of the fluid respectively. Suppose a point P moves from a position vector $\vec{r}$ at time t to a position vector $\vec{r} + \delta \vec{r}$ at time $t + \delta t$, with velocity increment of $\vec{q}(\vec{r} + \delta \vec{r}, t + \delta t) - \vec{q}(\vec{r}, t)$

$$\therefore \quad P \text{ is given by } \quad \vec{a} = \frac{d\vec{q}}{dt} = \frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} \tag{2}$$

Assumptions:

a) Energy dissipation due to viscosity or heat transfer is neglected. There is no shear stress.

b) The external forces are derived from a scalar field $v(r)$. Hence Euler's equation of motion is written as

$$\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} + \frac{1}{\rho} \nabla \rho + \nabla v = 0 \tag{3}$$

Linearization

The velocity is small compared with the instantaneous local speed of propagation of changes in any property of fluid, we will have $|q| \ll \left|\frac{\partial \rho}{\partial t}\right| |\nabla \rho|$ with the condition that $\left|\frac{\partial \rho}{\partial t}\right| \neq 0$, $|\nabla \rho| \neq 0$

Equation (1) now becomes:

$$\frac{\partial \rho}{\partial t} + \rho \, div \vec{q} = 0 \qquad (4)$$

Equation (1) becomes

$$\frac{\partial q}{\partial t} + \frac{1}{\rho} grad P + grad V = 0 \qquad (5)$$

The deposition built-up along the tunnel is defined by the density profile ρ .

$\rho = \rho_o(\xi + 1)$ where $|\xi| \ll 1$ over the entire length and breadth of the tunnel. Therefore pressure distribution with respect to density fluctuation will be

$P = P(\rho)$ (where pressure is a known function of density).

$$\Rightarrow \rho^{-1} \nabla P = \rho^{-1} P'(\rho) \nabla P$$

$$= \rho^{-1} P'(\rho) \rho_o \nabla P \qquad (6)$$

$$\Rightarrow \rho^{-1} \nabla P \approx c^2 \nabla \xi \qquad (7)$$

$$c^2 = P'(\xi) \text{ and } \frac{\rho_o}{\rho} \approx \frac{1}{1+\xi} \approx 1 \qquad (8)$$

Equations (4) and (5) can be written in the form

$$\frac{\partial \xi}{\partial t} + div \vec{q} = 0 \qquad (9)$$

$$\frac{\partial \vec{q}}{\partial t} + grad(c^2 \xi + V) = 0 \qquad (10)$$

Where $\vec{q} = grad f(\vec{r}, t) + \overline{\Lambda}(\vec{r})$ (11)

$\overline{\Lambda}(\vec{r})$ is a vector representing the steady flow and also restricting the time varying velocity field $f(\vec{r}, t)$ is said to be irrotational

$\therefore$ equation (10) reduces to

$$\frac{\partial f}{\partial t} + c^2 \xi + V = 0 \tag{12}$$

Considering Helmoltz's theorem

$$\overline{\Lambda}(\vec{r}) = grad\ g + curl\ \vec{B}(\vec{r}) \tag{a}$$

If $f + g = \phi$ (b)

Substituting (a) and (b) in (11) we will have

$$\vec{q}(\vec{r}, t) = grad\ \phi + curl\ \vec{B}(\vec{r}) \tag{13}$$

$$\Rightarrow \vec{q}(\vec{r}, t) = \vec{V}(\vec{r}, t) + curl\ \vec{B}(\vec{r}) \tag{14}$$

where $grad\ \phi = \vec{V}(\vec{r}, t)$ and $div\ curl\ \vec{B}(\vec{r}) = 0$

$$\therefore \frac{\partial \xi}{\partial t} + div \vec{V} = 0 \tag{15}$$

also

$$\frac{\partial \vec{r}}{\partial t} + \nabla (c^2 \xi + V) = 0 \tag{16}$$

Equations (15) and (16) governs the velocity field $\vec{V}(\vec{r}, t)$ of the flow along the tunnel. The steady flow field $curl\ \vec{R}(\vec{r})$ contributes nothing to the velocity profile in the tunnel and so it is disregarded. Thus $\vec{V}(\vec{r}, t)$ is the transporting velocity and $\phi(\vec{r}, t)$ is the velocity potential. To obtain the potential flow

$$\frac{\partial}{\partial t} grad\ \phi + grad (c^2 \xi + V) = 0 \tag{17}$$

Integrating, we will have

$$\frac{\partial \phi}{\partial t} + c^2 \xi + V = k(t) \tag{18}$$

If $k(t)$ is either zero or part of time dependent of ϕ, then we will have

$$\frac{\partial \phi}{\partial t} + c^2 \xi + V = 0 \tag{19}$$

Differentiating equation (19) w.r.t. t we obtain

$$\frac{\partial^2 \phi}{\partial t^2} + c^2 \frac{\partial \xi}{\partial t} = 0 \tag{20}$$

From equation (15), we have

$$\frac{\partial \xi}{\partial t} = -div \vec{V} \Rightarrow \frac{\partial \xi}{\partial t} = -div\ grad\ \phi$$

$$\therefore \frac{\partial \xi}{\partial t} = -\nabla^2 \phi \tag{20a}$$

Equation (20) can be written as

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = 0 \tag{21}$$

Here we see that the velocity potential describing the tunnel flow with in-built condition for residue deposition, satisfies the classical wave equation as in equation (21). The external force field $-\nabla V$ controlled by gravity is weak compared to the internal pressure forces within the tunnel and hence may be neglected. Equation (19) takes the form

$$\frac{\partial \phi}{\partial t} + c^2 \xi = 0 \tag{22}$$

Note:

All the equations derived are valid base on the following linear scale approximations assumed for the variables:

$$\left.\begin{aligned} |\xi| &\ll 1 \\ \vec{V}\cdot\nabla\xi &\ll \xi_t = \frac{\partial \xi}{\partial t} \\ \vec{V}\cdot\nabla\vec{V} &\ll \vec{V}_t = \frac{\partial \vec{V}}{\partial t} \end{aligned}\right\} \tag{23}$$

3 Model Example

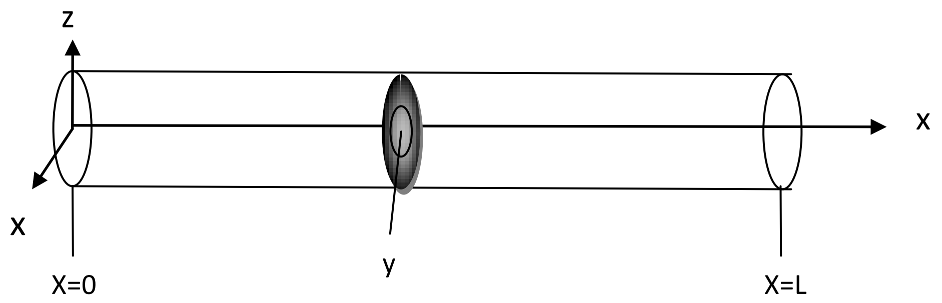

Figure 1: Geometry of the tunnel

The cross-section of the tunnel is a region D bounded by a closed curve Γ in the xy-plane. The velocity potential $\phi(x, y, z, t)$ satisfies the wave equation

$$c^2\nabla^2\phi = \phi_t^2; \quad \forall (x,t)\in D \tag{24}$$

On the surface of the enclosed curve Γ, we have

$$\frac{\partial \phi}{\partial n} = \hat{n}\cdot\nabla\phi = 0, \forall\ (x, y)\in\Gamma \tag{25}$$

where $\hat{n}$ is a unit outward drawn vector normal to Γ. Now equation (24) in terms of components in the form

$$c^2\left[\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right] = \frac{\partial^2\phi}{\partial t^2} \tag{26}$$

The x and y components of ϕ stipulate the orientation of the flow with respect to the tunnel walls and so describe the cross-sectional flow over the tunnel wall. The z-component of ϕ describes the vertical profile of the flow and so stipulate the elevation of the fluid in the tunnel. The time dependent component describes the dynamical history of the fluid as it flows along the tunnel. We assume the solution of equation (26) in the form

$$\phi(x, y, z, t) = u(x, y)e^{i(Az-wt)} \qquad (27)$$

Substituting equation (27) into equation (26), we obtain an explicit equation in u as

$$u_{xx} + u_{yy} + \left[\frac{\omega^2}{c^2} - k^2\right]u = 0 \qquad (28)$$

and

$$\frac{\partial u}{\partial n} = 0 \text{ on } \Gamma \qquad (29)$$

Equation (28) is the resulting eigen-value equation for the problem for which all the eigen values are non-negative with zero as one of them and with

$u(x, y)$ =constant

$$c^{-2}\omega^2 - k^2 = \beta_j^2 \qquad (30)$$

where

$$0 = \beta < \beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq \ldots$$

is the poly chromatic packet determining the various mode of wave propagations Each eigen function $u_j(x, y)$ provides a mode of propagation which travels along the tunnel with a phase velocity V given by

$$V = \frac{\omega}{k} \qquad (31)$$

and which is determined from the dispersion relation

$$\omega^2 = c^2k^2 + c^2\beta_j^2 \ \ldots \qquad (32)$$

From equation (32), the group velocity is obtained by differentiating ω w.r.t. k

$$V_0 = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} \tag{33}$$

Substituting equation (31) into (33) we will have the speed as

$$V_0 = \frac{d\omega}{dk} = c^2 (\frac{1}{V}) = c(\frac{c}{V}) < \varepsilon \tag{34}$$

$$\frac{c}{V} = \frac{c}{\frac{c\sqrt{1+\beta_j^2}}{k^2}} = \frac{1}{\frac{\sqrt{1+\beta_j^2}}{k^2}} < 1 \tag{35}$$

where c is the maximum signal speed generated as sonic wave.

If $\beta = 0$, then we will have $\omega = ck$

hence $\phi = e^{ik(z-ct)}$ (36)

Conclusion

We have shown that the classical wave equation has the potential for capturing partial blockage phenomenon along a sludge tunnel. Also, we explore the dynamics of sonic wave to detect the presence of any obstruction at any point in the tunnel. Furthermore, we constructed a system of eigen-value equation that admits the spectrum of polychromatic waves whose components are high frequency oscillations propagating along both the upstream and downstream of the tunnel and any blockage encountered will cause a reversal of mode and phase that can be detected by any instrument susceptible to waveguides

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