Hyper Zagreb-K-Banhatti Indices of Graphs

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Abstract: *Gutman and Trinajstic introduced the Zagreb indices in 1972. Kulli put forward the K-Banhatti indices in 2016. These are closely related two types of indices. In this paper, we introduce the first and second hyper Zagreb-K-Banhatti indices of a graph. We obtain some relations between Zagreb, K-Banhatti and first hyper Zagreb-K-Banhatti indices. Also we provide lower and upper bounds for the first hyper Zagreb-K-Banhatti index of a graph in terms of Zagreb and K-Banhatti indices.*

Keywords: *hyper Zagreb index, hyper K-Banhatti index, first hyper Zagreb-K-Banhatti index, second hyper Zagreb-K-Banhatti index, graph.*

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*35.

I. INTRODUCTION

Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. In Chemical Science, physicochemical properties of chemical compounds often modeled by means of molecular based structure descriptors which are also referred to as graph indices or topological indices. For graph indices, see [1]. A molecular graph is a graph such that its vertices correspond to the atoms and edges to the bonds. A single number that can be used to characterize some property of the graph of a molecular is called a topological index for that graph. Several topological indices have found some applications in QSPR/QSAR study [2, 3, 4].

Let G be a simple, connected graph with n vertices and m edges with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge e connecting the vertices u and v will be denoted by uv. If $e = uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in a graph G which is defined by $d_G(e) = d_G(u)+ d_G(v) - 2$ with $e = uv$. The vertices and edges of a graph are called its elements. For undefined concepts and notations, we refer [5].

The first and second Zagreb indices take into account the contributions of pairs of adjacent vertices. The first and second Zagreb indices were introduced by Gutman and Trinajstić in [6] and they are defined as

$$M_1(G) = \sum_{u\in V(G)} d_G(u)^2 \qquad \text{or} \qquad M_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right]$$

and $$M_2(G) = \sum_{uv\in E(G)} d_G(u)\, d_G(v).$$

These indices have been extensively studied. For their history, applications and mathematical properties, see [7, 8, 9, 10, 11, 12, 13, 14].

In [15], Shirdel et al. introduced the first hyper Zagreb index of a graph and defined it as

$$HM_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right]^2.$$

Some properties of the first hyper Zagreb index were obtained [16, 17, 18, 19, 20].

The second hyper Zagreb index is defined [21] as

$$HM_2(G) = \sum_{uv\in E(G)} \left[d_G(u)\, d_G(v)\right]^2.$$

Followed by the first Zagreb index of a graph G, Furtula et al. [22] introduced the forgotten topological index and defined it as

$$F(G) = \sum_{u\in V(G)} d_G(u)^3 = \sum_{uv\in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]$$

In [23], Kulli introduced the first and second K-Banhatti indices, intending to take into account the contributions of pairs of incident elements. The first and second K-Banhatti indices of a graph G are defined as

$$B_1(G)=\sum_{ue}\left[d_G(u)+d_G(e)\right] \qquad B_2(G)=\sum_{ue}d_G(u)d_G(e),$$

where ue means that the vertex u and edge e are incident in G.

The first and second hyper K-Banhatti indices of a graph G are defined by Kulli in [24] as

$$HB_1(G)=\sum_{ue}\left[d_G(u)+d_G(e)\right]^2, \qquad HB_2(G)=\sum_{ue}\left[d_G(u)d_G(e)\right]^2.$$

The K-Banhatti indices have been studied extensively. For their mathematical properties and applications, see [25,26, 27, 28, 29, 30, 31, 32, 33, 34].

In [35], Miličević et al. introduced the first and second reformulated Zagreb indices of a graph G in terms of edge degrees instead of vertex degrees and defined as

$$EM_1(G)=\sum_{e\in E(G)}d_G(e)^2, \qquad EM_2(G)=\sum_{e\sim f}d_G(e)d_G(f),$$

where $e \sim f$ means that the edges e and f are adjacent.

The third reformulated Zagreb index of a graph G is defined as

$$EM_3(G)=\sum_{e\sim f}\left[d_G(e)+d_G(f)\right]$$

The reformulated Zagreb indices were studied, for example, in [36, 37, 38, 39].

The first and second K indices of a graph G were introduced by Kulli in [40] and they are defined as

$$K^1(G)=\sum_{e\sim f}\left[d_G(e)+d_G(f)\right]^2, \qquad K^2(G)=\sum_{e\sim f}\left[d_G(e)d_G(f)\right]^2.$$

In [41], Kulli introduced the K^*-edge index of a graph G and defined it as

$$K_e^*(G)=\sum_{e\sim f}\left[d_G(e)^2+d_G(f)^2\right]$$

Motivated by the work on Zagreb and K-Banhatti indices, Kulli et al. introduced the Zagreb-K-Banhatti index of a graph G and defined it [42] as

$$MB(G)=\sum_{\substack{a \text{ is either adjacent}\\ \text{or incident to } b}}\left[d_G(a)+d_G(b)\right]^2,$$

where a and b are elements of G.

We now introduce the first and second hyper Zagreb-K-Banhatti indices of a graph G and they are defined as

$$HMB_1(G)=\sum_{\substack{a \text{ is either adjacent}\\ \text{or incident to } b}}\left[d_G(a)+d_G(b)\right]^2,$$

$$HMB_2(G)=\sum_{\substack{a \text{ is either adjacent}\\ \text{or incident to } b}}\left[d_G(a)d_G(b)\right]^2.$$

In this paper, we establish some relations between the first hyper Zagreb-K-Banhatti index, Zagreb, K-Banhatti indices. Also we provide lower and upper bounds for the first hyper Zagreb-K-Banhatti index of a graph in terms of Zagreb, K-Banhatti indices.

II. COMPARISON OF HYPER-*K*-BANHATTI, ZAGREB, *K*-BANHATTI -TYPE INDICES

Theorem 1. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G)=HM_1(G)+K^1(G)+HB_1(G).$$

Proof: Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G)=\sum_{\substack{a \text{ is either adjacent}\\ \text{or incident to } b}}\left[d_G(a)+d_G(b)\right]^2$$

$$= \sum_{ab\in E(G)} \left[d_G(a)+d_G(b)\right]^2 + \sum_{e,f\in E(G), e\sim f} \left[d_G(e)+d_G(f)\right]^2 + \sum_{a(ab)} \left[d_G(a)+d_G(b)\right]^2$$

$$= HM_1(G) + K^1(G) + HB_1(G).$$

We use the following theorem to prove our next result.

Theorem 2 [28]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HB_1(G) = 2HM_1(G) - 4M_1(G) + 24m.$$

Theorem 3. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

Proof: From Theorem 1, we have

$$HMB_1(G) = HM_1(G) + K^1(G) + HB_1(G).$$

$$= 3HM_1(G) + K^1(G) - 4M_1(G) + 24m, \text{ by Theorem 2.}$$

We use the following result to establish our next result.

Theorem 4 [28]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HM_1(G) = EM_1(G) + 4M_1(G) - 4m.$$

Theorem 5. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3HM_1(G) + K^1(G) + 8M_1(G) + 12m.$$

Proof: From Theorem 1, we have

$$HMB_1(G) = HM_1(G) + K^1(G) + HB_1(G).$$

From Theorems 2 and 4, we obtain

$$HMB_1(G) = 3EM_1(G) + K^1(G) + 8M_1(G) + 12m$$

We use the following result to prove our next result.

Theorem 6 [28]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HB_1(G) = 5F(G) + 8M_2(G) - 12M_1(G) + 8m.$$

Theorem 7. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = EM_1(G) + K^1(G) - 8M_1(G) + 8M_2(G) + 4m.$$

Proof: From Theorem 1, we have

$$HMB_1(G) = HM_1(G) + K^1(G) + HB_1(G).$$

From Theorem 4, we obtain

$$HMB_1(G) = EM_1(G) + 4M_1(G) - 4m + K^1(G) + HB_1(G).$$

From Theorem 6, we get

$$HMB_1(G) = EM_1(G) + K^1(G) + 5F(G) - 8M_1(G) + 8M_2(G) + 4m.$$

We use the following result to establish our next result.

Theorem 8 [28]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HM_1(G) = EM_1(G) + M_1(G) + B_1(G).$$

Theorem 9. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3EM_1(G) + K^1(G) + B_1(G) + 5M_1(G) + 16m.$$

Proof: From Theorem 1, we have

$$HMB_1(G) = HM_1(G) + K^1(G) + HB_1(G).$$

From Theorem 8, we get

$$HMB_1(G) = EM_1(G) + M_1(G) + B_1(G) + K^1(G) + HB_1(G).$$

From Theorem 2, we obtain

$$HMB_1(G) = EM_1(G) + M_1(G) + B_1(G) + K^1(G) + 2HM_1(G) - 4M_1(G) + 24m.$$

From Theorem 4, we get

$$HMB_1(G) = 3EM_1(G) + K^1(G) + B_1(G) + 5M_1(G) + 16m.$$

We use the following result to establish our next result.

Theorem 10 [28]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HM_1(G) = B_2(G) + 2M_1(G).$$

Theorem 11. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3B_2(G) + K^1(G) + 2M_1(G) + 24m.$$

Proof: From Theorem 3, we have

$$HMB_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

Using Theorem 10, we obtain

$$HMB_1(G) = 3B_2(G) + K^1(G) + 2M_1(G) + 24m.$$

We use the following result to prove Theorem 13.

Theorem 12 [40]. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$K^1(G) = K_e^*(G) + 2EM_2(G).$$

Theorem 13. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3B_2(G) + K_e^*(G) + 2EM_2(G) + 2M_1(G) + 24m.$$

Proof: From Theorem 11, we have

$$HMB_1(G) = 3B_2(G) + K^1(G) + 2M_1(G) + 24m.$$

Using Theorem 12, we obtain

$$HMB_1(G) = 3B_2(G) + K_e^*(G) + 2EM_2(G) + 2M_1(G) + 24m.$$

We use the following result to prove our next result.

Theorem 14 [39]. Let G be a graph with m edges. Then

$$EM_1(G) = F(G) + 2M_2(G) - 4M_1(G) + 4m.$$

Theorem 15. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 3F(G) + K^1(G) + 6M_2(G) - 4M_1(G) + 24m.$$

Proof: From Theorem 5, we have

$$HMB_1(G) = 3EM_1(G) + K^1(G) + 8M_1(G) + 12m.$$

Using Theorem 14, we obtain

$$HMB_1(G) = 3F(G) + K^1(G) + 6M_2(G) - 4M_1(G) + 24m.$$

Theorem 16. Let G be a graph with $n \geq 3$ vertices and m edges. Then

$$HMB_1(G) = 6F(G) + K^1(G) + 10M_2(G) - 12M_1(G) + 8m.$$

Proof: From Theorem 7, we have

$$HMB_1(G) = EM_1(G) + K^1(G) + 5F(G) - 8M_1(G) + 8M_2(G) + 4m.$$

Using Theorem 14, we obtain

$$HMB_1(G) = 6F(G) + K^1(G) + 10M_2(G) - 12M_1(G) + 8m.$$

III. BOUNDS OF HYPER ZAGREB-*K*-BANHATTI, ZAGREB, *K*-BANHATTI -TYPE INDICES

Theorem 17. For any graph G with $n \geq 3$ vertices and m edges,

$$12m\delta^2(G) + K^1(G) - 4M_1(G) + 24m \leq HMB_1(G) \leq 12m\Delta^2(G) + K^1(G) - 4M_1(G) + 24m$$

with equality if and only if G is regular.

Proof: We have $2\delta(G) \leq d_G(u) + d_G(v) \leq 2\Delta(G)$ for any edge of G. Thus

$$\sum_{uv\in E(G)} [2\delta(G)]^2 \leq \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right]^2 \leq \sum_{uv\in E(G)} [2\Delta(G)]^2.$$

Therefore $\quad 4m\delta^2(G) \leq HM_1(G) \leq 4m\Delta^2(G).$

Further, equality in both lower and upper bounds is attained if and if G is regular.

From Theorem 3, we have

$$HMB_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

Using inequality (1), we obtain

$$12m\delta^2(G) + K^1(G) - 4M_1(G) + 24m \leq HMB_1(G) \leq 12m\Delta^2(G) + K^1(G) - 4M_1(G) + 24m.$$

Second part is obvious.

We use the following result to prove our next result.

Theorem 18 [17]. For any graph with $n \geq 3$ vertices and m edges,

$$\delta(G)M_1(G) + 2M_2(G) \leq HM_1(G) \leq \Delta(G)M_1(G) + 2M_2(G).$$

with equality if and only if G is regular.

Theorem 19. For any graph G with $n \geq 3$ vertices and m edges,

$$(3\delta(G) - 4)M_1(G) + 6M_2(G) + K^1(G) + 24m \leq HMB_1(G) \leq (3\Delta(G) - 4)M_1(G) + 6M_2(G) + K^1(G) + 24m,$$

with equality if and only if G is regular.

Proof: From Theorem 3, we have

$$HMB_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

Using Theorem 18, we obtain

$$3\left[\delta(G)M_1(G) + 2M_2(G)\right] + K^1(G) - 4M_1(G) + 24m \leq HMB_1(G) \leq$$

$$3\left[\Delta(G)M_1(G) + 2M_2(G)\right] + K^1(G) - 4M_1(G) + 24m.$$

Thus $\quad (3\delta(G) - 4)M_1(G) + 6M_2(G) + K^1(G) + 24m \leq HMB_1(G) \leq$

$$(3\Delta(G) - 4)M_1(G) + 6M_2(G) + K^1(G) + 24m.$$

Further, equality in both lower and upper bounds will hold if and only if $d_G(u) + d_G(v) = 2\delta(G) = 2\Delta(G)$ for each $uv\in E(G)$. This implies that G is regular.

We use the following result to prove our next result.

Theorem 20 [28]. For any graph G with $n \geq 3$ vertices,

$$[\delta(G) - 2]M_1(G) + 2M_2(G) \leq B_2(G) \leq [\Delta(G) - 2]M_1(G) + 2M_2(G).$$

Further, equality in both lower and upper bounds will hold if and only if G is regular.

Theorem 21. For any graph G with $n \geq 3$ vertices and m edges,

$$3\delta(G) - 4M_1(G) + 6M_2(G) + K^1(G) + 24m \leq HMB_1(G) \leq$$

$$3\Delta(G) - 4M_1(G) + 6M_2(G) + K^1(G) + 24m,$$

with equality if and only if G is regular.

Proof: From Theorem 11, we have

$$HMB_1(G) = 3B_2(G) + K^1(G) + 2M_1(G) + 24m.$$

From Theorem 20, we obtain

$$3[\delta(G)-2]M_1(G)+6M_2(G)+K^1(G)+2M_1(G)+24m \le HMB_1(G) \le$$
$$3[\Delta(G)-2]M_1(G)+6M_2(G)+K^1(G)+2M_1(G)+24m.$$

Thus
$$3\delta(G)-4M_1(G)+6M_2(G)+K^1(G)+24m \le HMB_1(G) \le$$
$$3\Delta(G)-4M_1(G)+6M_2(G)+K^1(G)+24m.$$

Further, equality in both lower and upper bounds will hold if and only if $d_G(u)=d_G(v)=2\delta(G)=2\Delta(G)$ for each $uv\in E(G)$, which implies that G is regular.

We use the following result to prove our next result.

Theorem 22 [28]. For any connected graph G with $n \ge 3$ vertices and m edges,

$$4(\delta(G)-1)^2+2M_1(G)-4m \le B_2(G) \le [2M_1(G)-4m]\Delta(G).$$

Theorem 23. For any graph G with $n \ge 3$ vertices and m edges,

$$12(\delta(G)-1)^2+K^1(G)+8M_1(G)+12m \le HMB_1(G) \le$$
$$6[M_1(G)-2m]\Delta(G)+K^1(G)+2M_1(G)+24m.$$

Proof: From Theorem 11, we have

$$HMB_1(G)=3B_2(G)+K^1(G)+2M_1(G)+24m.$$

Using Theorem 22, we get

$$12(\delta(G)-1)^2+6M_1(G)-12m+K^1(G)+2M_1(G)+24m \le HMB_1(G) \le$$
$$3[2M_1(G)-4m]\Delta(G)+K^1(G)+2M_1(G)+24m.$$

Therefore

$$12(\delta(G)-1)^2+K^1(G)+8M_1(G)+12m \le HMB_1(G) \le$$
$$6[M_1(G)-2m]\Delta(G)+K^1(G)+2M_1(G)+24m.$$

We use the following existing result to prove our next result.

Theorem 24 [28]. For any connected graph G,

$$4M_2(G)-2M_1(G) \le B_2(G).$$

Equality is attained if and only if G is regular.

We now obtain lower bound on $HMB_1(G)$.

Theorem 25. Let G be a graph with $n \ge 3$ vertices and m edges. Then

$$12M_2(G)-4M_1(G)+K^1(G)+24m \le HMB_1(G).$$

Proof: From Theorem 11, we have

$$HMB_1(G)=3B_2(G)+K^1(G)+2M_1(G)+24m.$$

From Theorem 24, we obtain

$$12M_2(G)-4M_1(G)+K^1(G)+24m \le HMB_1(G).$$

We use the following result to establish our next result.

Theorem 26 [28]. For any connected graph G with $n \ge 3$ vertices and m edges,

$$B_2(G) \le \frac{[\delta(G)+\Delta(G)]^2}{4m\delta(G)\Delta(G)}M_1(G)^2-2M_1(G).$$

We now establish upper bound on $HMB_1(G)$ in terms of $\delta(G)$ and $\Delta(G)$ of G.

Theorem 27. Let G be a connected graph with $n \ge 3$ vertices and m edges. Then

$$HMB_1(G) \leq \frac{[\delta(G)+\Delta(G)]^2}{4m\delta(G)\Delta(G)} 3M_1(G)^2 - 4M_1(G) + K^1(G) + 24m.$$

Proof: From Theorem 11, we have

$$HMB_1(G) = 3B_2(G) + K^1(G) + 2M_1(G) + 24m.$$

Using Theorem 26, we get

$$HMB_1(G) \leq \frac{[\delta(G)+\Delta(G)]^2}{4m\delta(G)\Delta(G)} 3M_1(G)^2 - 4M_1(G) + K^1(G) + 24m.$$

We use the following result to prove next result.

Theorem 28 [17]. For any connected graph *G*,

$$4M_2(G) \leq HM_1(G)$$

with equality if and only if *G* is regular.

We now obtain lower bound on $HMB_1(G)$.

Theorem 29. For a graph *G* with $n \geq 3$ vertices and *m* edges,

$$12M_2(G) - 4M_1(G) + K^1(G) + 24m \leq HMB_1(G).$$

Proof: From Theorem 3, we have

$$HBM_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

From Theorem 28, we obtain

$$12M_2(G) - 4M_1(G) + K^1(G) + 24m \leq HMB_1(G).$$

We use the following result to establish our next result.

Theorem 30 [17]. For any graph *G*,

$$HM_1(G) \leq 2(\delta(G)+\Delta(G))M_1(G) - 4m\delta(G)\Delta(G)$$

with equality if and only if *G* is regular.

Theorem 31. Let *G* be a graph with $n \geq 3$ vertices and *m* edges. Then

$$HMB_1(G) \leq 6(\delta(G)+\Delta(G))M_1(G) - 4M_1(G) - 4m\delta(G)\Delta(G) + K^1(G) + 24m.$$

Proof: From Theorem 3, we have

$$HMB_1(G) = 3HM_1(G) + K^1(G) - 4M_1(G) + 24m.$$

Then from Theorem 30, we obtain

$$HMB_1(G) \leq 6(\delta(G)+\Delta(G))M_1(G) - 4M_1(G) - 4m\delta(G)\Delta(G) + K^1(G) + 24m.$$

CONCLUSION

In this paper, we have introduced the hyper Zagreb-*K*-Banhatti indices of a graph. We have established some relations between first hyper Zagreb-*K*-Banhatti index, Zagreb and *K*-Banhatti indices. Furthermore, we have found some lower and upper bounds for the first hyper Zagreb-*K*-Banhatti index of a graph in terms of other topological indices.

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