Zero Accomplishment Method for Finding an Optimal More-For-Less Solution of Transportation Problem with Mixed Constraints

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***Abstract**: A method has been developed for finding an optimal more-for-less solution in transportation problem. Method is presented in the form of an algorithm and illustrated through a numerical example. Approach is simple, easy to understand and apply.*

Keywords — *Transportation problem, mixed constraints, zero accomplishment, optimal solution, more-for-less solution.*

I. INTRODUCTION

A transportation problem (TP) is a particular case of linear programming problem in which the goods are transported from a set of sources (i.e. factories) to a set of destinations by the most economical shipping route.

It is one of the most important and successful application of quantities analysis to solving business problems has been in the physical distribution of products, commonly referred to as TP. The objective to solve TP is to minimize the cost of carrying resources, goods, or people from one location (often known as sources) to another location (often known as destinations) using diverse types of transportation modes (ship, aircraft, truck, train, pipeline, motorcycle and others) by air, water, road, aerospace, tube, and cable with some restrictions as capacity and time windows.

In this work, we have considered a generalization of the standard transportation problem in which the origin and destination constraints consist not only of equality but also of greater than or equal to ($\geq$) or less than or equal to ($\leq$) type constraints. The problem having such type of constraints is said to be a transportation problem with mixed constraints. Because of mixed nature, the problem cannot be solved precisely to obtain an optimal solution by any of available method. From the optimal MFL solution to TPs with mixed constraints, we can observe the nature of supply points and demand points which are very much helpful to the decision makers for taking decisions on the source and destination points. It is used to evaluate economical activities and make self-satisfied managerial decisions when they are handling a variety of logistic problems.

In some cases of TP with mixed constraints, an increase in the supplies and demand or in other words, increase in the flow results a decrease in the optimum transportation cost. This type of behavior which means paradoxical in mathematics is also known as transportation paradox or "More-For-Less" paradox.

More-For-Less paradox means we can supply more 'total goods' for less (or equal) 'total cost' i.e. shipping the same amount or more from origin to destination in the same cost or reduced cost, maintaining all shipping costs positive. It helps in decision making for businessmen. The primary objective of the MFL method is to minimize the total cost. It has been covered from a theoretical stand point by Charnes and Klingman. Robb provided an intuitive explanation of the transportation occurrence. Gupta et al and Ashram obtained the MFL solution for TP with mixed constraints by relaxing the constraints and introducing new slack variables.

Advantages of More-For-Less Paradox: (1) Helps in decision making for businessmen. (2) Helps in minimizing the cost. (3) Help in observing the nature of supply points and demand points. (4) Flexibility in TP.

In this work we solved those realistic situations where More-For-Less Paradox occurs in the Transportation problem with mixed constraints. The transportation models or problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales.

Rest of paper is organized as follows: Section 2 contains literature related to TP. Section 3 contains a mathematical model of Transportation problem with Mixed Constraints followed by tabular form. An algorithm followed by zero accomplishment method is discussed in Section 4. In Section 5 Algorithm is illustrated by numerical example. At last paper ends with conclusion and references.

II. LITERATURE REVIEW

We have gone through from many research papers and concluded that the transportation problem was first formulated mathematically by **F.L. Hitchcock** in 1941**,** and discussed in detail by the Nobel Laureate **T.C. Koopmans** in 1947. The linear programming formulation of the transportation problem and associated solution procedure were first given by **G.B. Dantzig**. In the same year, **Klingman** solved the same problem by considering a standard transportation problem having only one additional origin and one additional destination. Dantzig (1963) used the simplex method to the transportation problems as the primal simplex transportation method. Reena.G. Patel,and P.H.Bhathawala (2017) [7] developed optimal solution of a degenerate Transportation Problem.

Transportation problem with mixed constraints was rigorously studied by **Bridgen** (1974). He solved this problem by considering a related standard transportation problem having two additional sources and two additional destinations. Adlakha et al. (2006) [1] proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an MFL solution, Vogel Approximation Method (VAM) and MODI (Modified Distribution) method was used. Pandian and Natarajan (2010) [3] proposed a method in which form the LBP which is obtained from TP with mixed constraints by changing all inequalities to equalities with the lowest possible feasible right-hand side values. Balance LBP and find an optimal solution of the balanced LBP using the transportation algorithm. Later on, their another paper (2010) [6] developed Fourier method for solving transportation problems with mixed constraints which is based on modified Fourier elimination method.

Pandian and Natarajan (2010) [4] proposed a new method for finding an optimal More-For-Less Solution of transportation problems with mixed constraints in single stage. They develop a new process based on zero-point method to find an optimal MFL. Pandian and Anuradha (2013) [5] gave new method namely path method for finding a more-for-less optimal solution to TPs with mixed constraints by directed path connecting allotted cells is used for modifying the allotment of allotted cells. Rabindra Nath Mondal (2015) [2] introduce a modified VAM method for solving TPs with mixed constraints in MFL paradoxical situation by finding the best way to fulfil the demands of n demand points using the capacities of m supply points.

A literature search revealed that much effort has been concentrated on classical transportation problem with equality constraint as well as with mixed constraint. But no systematic method has been developed for finding an optimal solution or addressing more-for-less situations in transportation problems with mixed constraints.

III. MATHEMATICAL FORMULATION

All Transportation problem can be seen mathematically by introducing the following terminologies:

$O_i = i^{th}$ supply point, $i = 1, 2, \dots, m$

$D_j = j^{th}$ demand point, $j = 1, 2, \dots, n$

a_i = amount of commodity available at supply point O_i

b_j =amount of commodity required at demand point D_j

C_{ij} = transporting cost per unit commodity from supply point O_i to demand point D_j

x_{ij} = amount of commodity transporting from supply point O_i to demand point D_j

m = number of supply points and n = number of demand points

Transportation Problem with mixed constraints in tabular form is as follows:

Table I: Transportation Problem with Mixed Constraints

	D_1	D_2	…	D_j	…	D_n	Supply
O_1	x_{11} c_{11}	x_{12} c_{12}	…	x_{1j} c_{1j}	…	x_{1n} x_{1n}	$(\geq / = / \leq)$
O_2	x_{21} c_{21}	x_{22} c_{22}	…	x_{2j} c_{2j}	…	x_{1n} c_{1n}	$(\geq / = / \leq)$
…	…	…	…	…	…	…	…
O_i	x_{i1} c_{i1}	x_{i2} c_{i2}	…	x_{ij} c_{ij}	…	x_{1n} c_{1n}	$(\geq / = / \leq)$
…	…	…	…	…	…	…	…
O_m	x_{m1} c_{m1}	x_{m2} c_{m2}	…	x_{mj} c_{mj}	…	x_{mn} c_{mn}	$(\geq / = / \leq)$

Demand $(\geq / = / \leq)$ $(\geq / = / \leq)$ … $(\geq / = / \leq)$ … $(\geq / = / \leq)$

The mathematical model for transportation problem with mixed constraints is to find x_{ij} which

$$Minimize\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} \leq a_i; \qquad i \in \alpha_1$$
$$\sum_{j=1}^{n} x_{ij} = a_i\ ; \qquad i \in \alpha_2$$
$$\sum_{j=1}^{n} x_{ij} \geq a_i, \qquad i \in \alpha_3$$
$$\sum_{i=1}^{m} x_{ij} \leq b_j; \qquad j \in \beta_1$$
$$\sum_{i=1}^{m} x_{ij} = b_j; \qquad j \in \beta_2$$
$$\sum_{i=1}^{m} x_{ij} \geq b_j; \qquad j \in \beta_3$$
$$x_{ij} \geq 0$$

$\alpha_1 \cup \alpha_2 \cup\ \alpha_3 = \{1, 2, \dots, m\}$ is set of indices of supply points.
$\beta_1 \cup \beta_2 \cup\ \beta_3 = \{1, 2, \dots, n\}$ is set of indices of demand points.

IV. ZERO ACCOMPLISHMENT METHOD

In this chapter we discuss zero accomplishment method which is a method used for finding an optimal More-For-Less solution for Transportation problem with mixed constraints. The idea behind this method comes from the zero point method which is developed by Pandian and Natarajan in their research paper. Zero accomplishment method is a method in which we try to make zeros in the cost matrix by subtracting row minima and column minima from the transportation table to get at least one zero cell in each row and each column. Then check accomplishment condition corresponding to the zero containing cells. Accomplishment conditions are those condition in which we check that demand and supply constraints will satisfy each other at zero cost containing cell. Here are the possible conditions where accomplishment conditions are satisfied or not:

Supply constraints a	Demand constraints b	accomplishment conditions
=	=	$(a > b)$
≥	=	$(a = b)$
≤	=	$(a < b)$
=	≤	$(a = b)$
≥	≤	$(a > b)$
≤	≤	$(a = b)$
=	≥	$(a = b)$
≥	≥	$(a \geq b)$
≤	≥	$(a \leq b)$

After obtaining optimal solution from this proposed algorithm we check the occurrence of More-for-Less paradox occurs using Modi Indices. This is the most important section in which the objective of dissertation is achieved. The proposed algorithm is quite easy to understand and apply. Now we are going to discuss the algorithm in detail.

Here are the steps of algorithm which is helpful in understanding the zero-accomplishment method:

A. ***Algorithm:*** Here are the steps of algorithm which is helpful in understanding the zero-accomplishment method:

1. Construct the transportation problem for the given TP with mixed constraints.
2. From the Transportation table subtract the row minima from the rows element and subtract the column minima from the columns elements to get the transformed table.
3. In transformed table check the accomplishment conditions at the zero containing cell corresponding to the row or column from the given demand and supply constraints in the transformed table.
4. If accomplishment conditions are satisfied for all zero cells then assign zero cells with the help of original transportation table.
5. Firstly, assign that zero cell which has least cost according to the original table and so on. Then we get optimal solution.
6. If conditions are not satisfied then draw minimum no. of horizontal and vertical lines to cover all the zeros of the reduced transportation table.
7. Subtract the smallest uncovered element from all uncovered elements and add the same to all entries lying at the intersection of any two lines. Then go to step 3.
8. Finally, allotment yields an optimal solution to the given TP with mixed constraints.

B. Assignment Procedure:

1. If the demand constraints (b) are of ‘ $\leq$’ and the supply constraints (a) are of ‘ = ’ type then we assign = min(a,b).
2. If the demand constraints (b) are of ‘ $\geq$’ and the supply constraints (a) are of ‘= ’type then we assign = a
3. If the demand and supply are of ‘= ’type then we assign =max (a,b).
4. If the demand and supply constraints are of ‘ $\leq$’ type then we assign = 0.
5. If the demand and supply constraints are of ‘= ’ type then we assign = min (a,b).
6. If the demand (b) and supply constraints (a) are of ‘$\geq$’ type then we assign= max(a,b).
7. If the demand constraints (b) are of ‘$\geq$’ type and supply constraints (a) are of ‘$\leq$’ type then we assign =min(a,b).

The MFL solution is obtained from the optimal solution distribution by increasing and decreasing the shipping quantities while maintaining the minimum requirements for both supply and demand. The plant-to-market shipping shadow price (also called Modi index) at a cell (i, j) is $u_i + v_j$ where u_i and v_j are shadow prices corresponding to the cell (i, j). The negative Modi index at a cell (i, j) indicates that we can increase the ith plant
capacity / the demand of the jth market at the maximum possible level

C. To check MFL using MODI Index:

Step1: Make Modi index matrix from the optimal solution of the TP with mixed constraints obtained by the zero-accomplishment method.

Step2: Now identify the negative Modi indices in rows and columns. If none exist, this is an optimal solution to the TP with mixed constraints. There is no MFL then STOP.

Step3: From a new TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $\leq$ to = and = to $\geq$ in the given problem.

Step4: Again, solve the TP with mixed constraints obtained from the above Step using zero accomplishment method.

Step5: Then the solution obtained in Step 4 is an optimal solution for the TP with mixed constraints.

V. NUMERICAL EXAMPLE

Consider there are three car manufacturing factories which is consider to be sources and three car warehouses which is destinations where these cars are supplied.
Now we have TP with mixed constraints in tabular form:

	D1	D2	D3	SUPPLY
O1	2	5	4	=5
O2	6	3	1	≥ 6
O3	8	9	2	≤ 9
DEMAND	=8	≥ 10	≤ 5	

STEP 1: Subtract row minima from rows element.

0	3	2
5	2	0
6	7	0

STEP 2: Subtract column minima from columns element.

0	1	2
5	0	0
6	5	0

STEP 3: Check the accomplishment condition at the zero containing cell.

0	1	2	=5
5	0	0	≥ 6
6	5	0	≤ 9
=8	≥ 10	≤ 5	

Step 4 : In row 1 and column 1 condition are not satisfied, then draw min. no of horizontal or vertical lines to cover all zeros .

0	1	2	=5
5	0	0	≥ 6
6	5	0	≤ 9
=8	≥ 10	$\leq$ 5	

Step5 : Now select the least cost from the uncovered cost then add this to that cost which is at intersection point of lines and subtract the same from the uncovered cost.

0	6	7
0	0	0
1	5	0

Step 6 : Again check the accomplishment condition at the zeros containing cell corresponding to the given demand and supply constraints are satisfying or not.
Step 7 : Conditions are satisfied, then allotment will be done according to the original transportation table.
Step 8: Zeros containing cell are the place where allotment will be done.
Step 9: Firstly, assign that cell which had least cost according to the original transportation table.

5 0	6	7	= 5
3 0	**10** 0	**5** 0	≥ 6
1	5 1	**0** 0	≤ 9
= 8	≥ 10	≤ 5	

Transportation cost = 5*2+3*6+10*3+5*1+0*2= 63
Step 10: Optimal solution is $x_{11} = 5, x_{21} = 3, x_{22} = 10, x_{23} = 5, x_{33} = 0$
For flow of 23 units with in. TC= 63
Now check MFL, using Modi Index

2	**-1**	**-3**	U_1= -4
-6	3	1	U_2= 0
7	4	2	U_3=1

V_1=6V_2=3V_3=1

Negative sign indicate there is existence of MFL. Solve it by changing '≤' to '=' or '≥'.

0	1	2	≥ 5
5	0	0	≥ 6
6	5	0	≤ 9
=8	≥ 10	=5	

Check accomplishment conditions at zero containing cell corresponding to demand and supply constraints. All accomplishment conditions are satisfied at zero containing cells. Now assignment will be done according to the original transportation cost matrix.

8 0	1	2	≥ 5
5	**10** 0	**5** 0	≥ 6
6	5	**0** 0	≤ 9
=8	≥ 10	=5	

Transportation cost= 8*2+10*3+5*1 = 51, with total flow of 23 units.

VI. CONCLUSIONS

There is increase in the flow as well as there is reduction in the transportation cost. From this point of view proposed algorithm is better and efficient. The proposed algorithm helps in obtaining optimal solution for TPs with mixed constraints. In TPs with mixed constraints we will be very careful about constraints inequalities before solving it. These constraints play vital role in the solution. In this work, we discuss a paradox (so called "More-For-Less") in transportation problem with mixed constraints. Thereby, we develop an algorithm for finding an Optimal More-Fore –Less solution of transportation Problems with mixed constraints. Today, calculation is very simple and not time taking if one solves this type of problem using mathematical software. The managers in decision such as increasing warehouses/plant capacity and/or advertising efforts to increase demand at some destinations and/or for some types of products may use this paradoxical analysis to increase his business under the same environment. Hence, in practically it is an important part of transportation problems with mixed constraints.

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