The Photo gravitational Restricted Three Body Problem In Source of Radition & Oblate Spheroid

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ABSTRACT
The present paper deals with the equations of motion of photo gravitational restricted three body problems in which the bigger primary is the source of radiation and the smaller one is an oblate spheroid. The idealized model of restricted three body problem is one of the most celebrated problems of celestial mechanics. The restricted problem specifies the motion of a body of infinitesimal mass under the gravitational attraction of two massive bodies moving about their centre of mass in circular orbit.

KEYWORDS: *photo gravitational restricted three body problem, Libration, Oblatness, Primary, Bigger, Triangular points.*

INTRODUCTION

The photo gravitational restricted three body problem is a special case of restricted three body problem in which at least one of the interacting bodies is an intense emitter of radiation. In a model of restricted three body problem sun – planet – particle, the motion of material particle is subjected to the solar radiation pressure. There are several examples of the restricted problems in which either or both the primaries are sources of radiation. The effect of radiation of either or both the primaries on the motion and stability of the system was studied in considerable detail by Radzievsky (1950), Chernikov (1970), Simmons et. Al. (1985), Chaudhary (1985), Szzebehely (1967), and many others.

Locations of the Equilibrium points:

The locations of the equilibrium points of the system are determined by the equations. .

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1}{\left(1 + \frac{3A}{2}\right)} \left\{ \frac{(1-\mu)(1-q)}{r_1} + \frac{\mu}{r_2} + \frac{\mu A}{2 r_2^3} \right\} \quad \ldots \ldots (1)$$

Now, differentiating (1) partially with respect to x, we get

$$\Omega_x = x + \frac{1}{1 + \frac{3A}{2}} \left\{ - \frac{(1-\mu)(1-q)}{r_1^2} \frac{\partial r_1}{\partial x} - \frac{\mu}{r_2^2} . \frac{\partial r_2}{\partial x} - \frac{3\mu A}{2 r_2^4} . \frac{\partial r_2}{\partial x} \right\}$$

After using , it takes the form

$$\Omega_x = x + \frac{1}{1+\frac{3A}{2}}\left\{-\frac{(1-\mu)(1-q)}{r_1^{\,2}}\frac{x+\mu}{r_1}\right.$$

$$\left.-\frac{\mu}{r_2^{\,2}}\cdot\frac{x-1+\mu}{r_2}-\frac{3\mu A}{2r_2^{\,4}}\cdot\frac{x-1+\mu}{r_2}\right\}$$

$$= x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^{\,3}}\right.$$

$$\left.+\frac{\mu(x-1+\mu)}{r_2^{\,3}}(x+\mu)+\frac{3\mu A(x-1+\mu)}{2r_2^{\,5}}\right\}$$

Again differentiating (1) partially with respect to y, we obtain

$$\Omega_y = y + \frac{1}{1+\frac{3A}{2}}\left\{-\frac{(1-\mu)(1-q)}{r_1^{\,2}}\frac{\partial r_1}{\partial y}\right.$$

$$\left.-\frac{\mu}{r_2^{\,2}}\cdot\frac{\partial r_2}{\partial y}-\frac{3\mu A}{2r_2^{\,4}}\cdot\frac{\partial r_2}{\partial y}\right\}$$

which takes the form with the values substituted in from

$$\Omega_y = y + \frac{1}{1+\frac{3A}{2}}\left\{-\frac{(1-\mu)(1-q)}{r_1^{\,2}}\frac{y}{r_1}-\frac{\mu}{r_2^{\,2}}\cdot\frac{y}{r_2}-\frac{3\mu A}{2r_2^{\,4}}\cdot\frac{y}{r_2}\right\}$$

$$= y - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)y}{r_1^{\,3}}+\frac{\mu y}{r_2^{\,3}}-\frac{3\mu Ay}{2r_2^{\,5}}\right\}$$

Substituting these values and , we have the equations.

$$x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(x+\mu)}{r_1^{\,3}} + \right.$$

and

$$\left.\frac{\mu(x-1+\mu)}{r_2^{\,3}} - \frac{3\mu A(x-1+\mu)}{2r_2^{\,5}}\right\} = 0$$

$$y - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^{\,3}} + \frac{\mu}{r_2^{\,3}} + \frac{3\mu A}{2r_2^{\,5}}\right\}y = 0 \quad \dots \dots (2)$$

From second equation of (2), we get

$$y\left[1 - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^{\,3}} + \frac{\mu}{r_2^{\,3}} + \frac{3\mu A}{2r_2^{\,5}}\right\}\right] = 0 \quad \dots \dots (3)$$

Collinear Equilibrium Points:

The collinear equilibrium points are the solutions of the equations $\Omega_x = 0$ and $y = 0$

i.e., from (1.3.5), these points are the solutions of the equations:

$$x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(x+\mu)}{r_1^{\,3}}\right.$$

$$\left. + \frac{\mu(x-1+\mu)}{r_2^{\,3}} - \frac{3\mu A(x-1+\mu)}{2r_2^{\,5}}\right\} = 0$$

and $y = 0$ … … (4) Since the collinear points lie on the x-axis i.e., the line joining the primaries. Hence in order to get the collinear solutions we put y = 0 and obtain

$$r_1^{\,2} = (x+\mu)^2 \quad \text{and} \quad r_2^{\,2} = (x-1+\mu)^2$$

With these values substituted in, (4) takes the form

$$x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(x+\mu)}{\left\{(x+\mu)^2\right\}^{3/2}}\right.$$

$$\left. + \frac{\mu(x-1+\mu)}{\left\{(x-1+\mu)^2\right\}^{\frac{3}{2}}} + \frac{3\mu A(x-1+\mu)}{2\left\{(x-1+\mu)^2\right\}^{\frac{5}{2}}}\right\} = 0$$

To obtain the position of the collinear points on the line joining the primaries, we consider the function:

$$f(x) = x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(x+\mu)}{\left\{(x+\mu)^2\right\}^{\frac{3}{2}}}\right.$$

$$\left. + \frac{\mu(x-1+\mu)}{\left\{(x-1+\mu)^2\right\}^{\frac{3}{2}}} + \frac{3\mu A(x-1+\mu)}{2\left\{(x-1+\mu)^2\right\}^{\frac{5}{2}}}\right\} = 0$$

… … (5)

Let h be an arbitrary number which is positive and very very small. We know that A>0, 1-q>0 and 0<μ<½. Hence we have

$$f\left(\frac{1}{h}\right) = \frac{1}{h} - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)\left(\frac{1}{h}+\mu\right)}{\left\{\left(\frac{1}{h}+\mu\right)^2\right\}^{\frac{3}{2}}}\right.$$

$$\left. + \frac{\mu\left(\frac{1}{h}-1+\mu\right)}{\left\{\left(\frac{1}{h}-1+\mu\right)^2\right\}^{\frac{3}{2}}} + \frac{3\mu A\left(\frac{1}{h}-1+\mu\right)}{2\left\{\left(\frac{1}{h}-1+\mu\right)^2\right\}^{\frac{5}{2}}}\right\} > 0$$

$$f(1-\mu+h)=(1-\mu+h)-\frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(1+h)}{\left\{(1+h)^2\right\}^{\frac{3}{2}}}+\frac{\mu h}{\left\{(h)^2\right\}^{\frac{3}{2}}}+\frac{3\mu Ah}{2\left\{(h)^2\right\}^{\frac{5}{2}}}\right\}<0$$

$$f(-\mu+h)=-\mu+h-\frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(-h)}{\left\{(-h)^2\right\}^{\frac{3}{2}}}+\frac{\mu(-h)}{\left\{(-h-1)^2\right\}^{\frac{3}{2}}}+\frac{3\mu A(-h)}{2\left\{(-h-1)^2\right\}^{\frac{5}{2}}}\right\}<0$$

$$f(-\mu-h)=-\mu-h-\frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(-h)}{\left\{(-h)^2\right\}^{\frac{3}{2}}}+\frac{\mu(-h)}{\left\{(-h-1)^2\right\}^{\frac{3}{2}}}+\frac{3\mu A(-h)}{2\left\{(-h-1)^2\right\}^{\frac{5}{2}}}\right\}>0$$

$$f\left(-\frac{1}{h}\right)=-\frac{1}{h}-\frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)\left(-\frac{1}{h}+\mu\right)}{\left\{\left(-\frac{1}{h}+\mu\right)^{2}\right\}^{\frac{3}{2}}}+\frac{\mu\left(-\frac{1}{h}-1+\mu\right)}{\left\{\left(-\frac{1}{h}-1+\mu\right)^{2}\right\}^{\frac{3}{2}}}+\frac{3\mu A\left(-\frac{1}{h}-1+\mu\right)}{2\left\{\left(-\frac{1}{h}-1+\mu\right)^{2}\right\}^{\frac{5}{2}}}\right\}<0$$

Plotting the graph of f(x) as in the classical case we conclude that there are three solutions of the equations f(x) = 0 which correspond to three collinear solutions of the problem and these collinear points are denoted by L1, L2 and L3.

CONCLUSION

Thus we observe that the photo gravitational restricted three body problem in which the smaller primary is an oblate spheroid and the bigger one is a source of radiation possesses five equilibrium points -two triangular which form nearly equilateral triangles with the primaries and three collinear which lie on the line joining the primaries. The oblateness of the smaller primary and the radiation of the bigger primary affect significantly the location of triangular as well as collinear equilibrium points. The result obtained are in accordance with those of szebehely when q = 0 and A = 0.

REFRENCE

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