The Lattice Structure of The Subgroups of Order 27 In The Subgroup Lattices of 3 X 3 Matrices Over Z_3

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ABSTRACT

Let $\mathcal{G}$ be the set of all 3 X 3 non-singular matrices $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, where a,b,c,d,e,f,g,h,i are integers modulo p. Then $\mathcal{G}$ is a group under matrix multiplication modulo p, of order $(p^n-1)(p^n-p)(p^n-p^2)\dots\dots(p^n-p^{n-1})$. Let G be the subgroup of $\mathcal{G}$ defined by $G=\left\{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1\right\}$. Then G is of order $\frac{(p^n-1)(p^n-p)(p^n-p^2)\dots\dots(p^n-p^{n-1})}{p-1}$. Let L(G) be the lattice formed by all subgroups G. In this paper, we give the structure of the subgroups of order 27 of L(G) in the case when P= 3.

Keywords: Matrix group, subgroups, Lagrange's theorem, Lattice, Atom.

1. Introduction

Let L(G) be the Lattice of Subgroups of G, where G is a group of 3x3 matrices over Z_p having determinant value 1 under matrix multiplication modulo p, where p is a prime number.

Let $\mathcal{G}=\left\{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} : a, b, c, d, e, f, g, h, i \in Z_p, \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0\right\}$

Then $\mathcal{G}$ is a group under matrix multiplication modulo p.

Let $G = \left\{ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \in \mathcal{G} : \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 1 \right\}$

Then G is a subgroup of $\mathcal{G}$.

we have, $o(\mathcal{G}) = (p^n - 1)(p^n - p)(p^n - p^2) \dots \dots (p^n - p^{n-1})$

and $o(G) = \frac{(p^n-1)(p^n-p)(p^n-p^2)\dots\dots(p^n-p^{n-1})}{p-1}$.

In this paper, we give the structure of the subgroups of order 27 of L(G) in the case when P= 3.

2.Preliminaries

In this section we give the definition needed for the development of the paper.

Definition 2.1

A partial order on a non-empty set P is a binary relation≤ on P that is reflexive, anti-symmetric and transitive. The pair (P, ≤) is called a **partially ordered set or poset**. A poset. (P, ≤) is totally ordered if every x, y ∈ P are comparable, that is either x ≤ y or y ≤ x. A non-empty subset S of P is a chain in P if S is totally ordered by ≤.

Definition 2.2

Let (P, ≤) be a poset and let S⊆P. An upper bound of S is an element x∈P for which s≤x for all s∈S. The least upper bound of S is called the **supremum or join** of S.A lower bound for S is an element x ∈ P for which x≤ s for all s∈ S. The greatest lower bound of S is called the **infimum or meet** of S.

Definition 2.3

Poset (P, ≤) is called a **lattice** if every pair x, y elements of P has a supremum and an infimum, which are denoted by x ∨ y and x ∧ y respectively.

Definition 2.4

For two elements a and b in P, a is said to***cover*** b or b is said to be covered by a (in notation, a≻b or b≺a) if and only if b<a and, for no x∈P, b<x<a.

Definition 2.5

An element a∈P is called an ***atom***, if a≻0 and it is a dual atom, if a≺1.

Theorem 2.6

If G is a finite group and H is a subgroup of G, then the order of H is a divisor of the order of G.

Theorem 2.7

If G is a finite group and a ∈ G, then the order of 'a' is a divisor of the order of G.

Theorem 2.8

Let G be a finite group and let p be any prime number that divides the order of G. Then G contains an element of order p.

Theorem 2.9

If p is a prime number and p^{α} |o(G), $p^{\alpha+1}$ ∤o(G), then G has a subgroup of orderp^{α}, called a p-sylow subgroup.

Theorem 2.10

The number of p-sylow subgroups in G, for a given prime p, is of the form 1+kp.

3. Arrangement of elements of G according to their orders :

The number of elements of order 2 is 117. The number of elements of order 3 is 728. The number of elements of order 4 is 702. The number of elements of order 6 is 936. The number of elements of order 8 is 1404. The number of elements of order 13 is 1728.

4. Subgroups of G of different orders :

The number of subgroups of order 2 is 117. The number of subgroups of order 3 is 364. The number of subgroups of order 4 is 351. The number of subgroups of order 6 is 468.

The number of subgroups of order 8 is 468. The number of subgroups of order 9 is 117. The number of subgroups of order 13 is 144. The number of subgroups of order 16 is 351. The number of subgroups of order 27 is 52.

5. Lattice structure of some lower interals of subgroups of order 16 in L(G) over Z_3

Let R be an arbitrary subgroup of order 27. Then the elements of U must have orders 1,3 or 9.

We tabulate the subgroups of order 27 in L (G)

Table 5.1: Intervals [{e}, V_i] in L(G), i = 1,2…..52

Order	Subgroups
27	V_1
9	O_2, O_{26}, O_{36}
3	K_{232}, K_{234}, K_{236}
1	{e}

Order	Subgroups
27	V_2
9	O_{21}, O_{107}, O_{112}
3	K_{353}, K_{359}, K_{363}
1	{e}

Order	Subgroups
27	V_3
9	O_6, O_{67}, O_{68}
3	$K_{353}, K_{359}, K_{364}$
1	{e}

Order	Subgroups
27	V_4
9	O_{10}, O_{103}, O_{104}
3	$K_{353}, K_{363}, K_{364}$
1	{e}

Order	Subgroups
27	V_5
9	O_{20}, O_{45}, O_{46}
3	$K_{359}, K_{363}, K_{364}$
1	{e}

Order	Subgroups
27	V_6
9	O_{24}, O_{49}, O_{50}
3	K_{70}, K_{161}, K_{164}
1	{e}

Order	Subgroups
27	V_7
9	O_{26}, O_{51}, O_{52}
3	$K_{120}, K_{198}, K_{243}$
1	{e}

Order	Subgroups
27	V_8
9	O_{29}, O_{59}, O_{60}
3	$K_{176}, K_{186}, K_{286}$
1	{e}

Order	Subgroups
27	V_9
9	O_9, O_{55}, O_{56}
3	$K_{247}, K_{252}, K_{254}$
1	{e}

Order	Subgroups
27	V_{10}
9	O_9, O_{47}, O_{48}
3	$K_{134}, K_{138}, K_{142}$
1	{e}

Order	Subgroups
27	V_{11}
9	O_9, O_{39}, O_{40}
3	K_{33}, K_{342}, K_{345}
1	{e}

Order	Subgroups
27	V_{12}
9	O_9, O_{13}, O_{14}
3	K_{36}, K_{37}, K_{38}
1	{e}

Order	Subgroups
27	V_{13}
9	O_1, O_{24}, O_{25}
3	K_{99}, K_{102}, K_{105}
1	{e}

Order	Subgroups
27	V_{15}
9	O_{24}, O_{38}, O_{42}
3	$K_{324}, K_{339}, K_{342}$
1	{e}

Order	Subgroups
27	V_{17}
9	O_{26}, O_{37}, O_{41}
3	$K_{324}, K_{342}, K_{345}$
1	{e}

Order	Subgroups
27	V_{19}
9	O_{20}, O_{57}, O_{58}
3	$K_{212}, K_{276}, K_{278}$
1	{e}

Order	Subgroups
27	V_{14}
9	O_3, O_4, O_{20}
3	K_7, K_{11}, K_{17}
1	{e}

Order	Subgroups
27	V_{16}
9	O_{20}, O_{43}, O_{44}
3	$K_{324}, K_{339}, K_{345}$
1	{e}

Order	Subgroups
27	V_{18}
9	O_{24}, O_{53}, O_{54}
3	$K_{260}, K_{264}, K_{271}$
1	{e}

Order	Subgroups
27	V_{20}
9	O_{16}, O_{93}, O_{94}
3	$K_{176}, K_{186}, K_{348}$
1	{e}

Order	Subgroups
27	V_{21}
9	O_{34}, O_{108}, O_{111}
3	$K_{260}, K_{271}, K_{338}$
1	{e}

Order	Subgroups
27	V_{23}
9	O_{28}, O_{79}, O_{80}
3	$K_{247}, K_{252}, K_{326}$
1	{e}

Order	Subgroups
27	V_{25}
9	O_{19}, O_{113}, O_{115}
3	$K_{212}, K_{276}, K_{336}$
1	{e}

Order	Subgroups
27	V_{27}
9	O_{31}, O_{105}, O_{106}
3	$K_{260}, K_{264}, K_{338}$
1	{e}

Order	Subgroups
27	V_{29}
9	O_{28}, O_{114}, O_{116}
3	$K_{120}, K_{198}, K_{351}$
1	{e}

Order	Subgroups
27	V_{22}
9	O_{10}, O_{101}, O_{102}
3	$K_{264}, K_{271}, K_{338}$
1	{e}

Order	Subgroups
27	V_{24}
9	O_{21}, O_{77}, O_{78}
3	$K_{247}, K_{254}, K_{326}$
1	{e}

Order	Subgroups
27	V_{26}
9	O_{16}, O_{99}, O_{100}
3	$K_{212}, K_{278}, K_{336}$
1	{e}

Order	Subgroups
27	V_{28}
9	O_{31}, O_{63}, O_{64}
3	$K_{252}, K_{254}, K_{326}$
1	{e}

Order	Subgroups
27	V_{30}
9	O_{30}, O_{31}, O_{32}
3	$K_{232}, K_{234}, K_{327}$
1	{e}

Order	Subgroups
27	V_{31}
9	O_6, O_7, O_8
3	$K_{232}, K_{236}, K_{327}$
1	{e}

Order	Subgroups
27	V_{32}
9	O_{10}, O_{89}, O_{90}
3	$K_{120}, K_{243}, K_{351}$
1	{e}

Order	Subgroups
27	V_{33}
9	O_{31}, O_{75}, O_{76}
3	$K_{276}, K_{278}, K_{336}$
1	{e}

Order	Subgroups
27	V_{34}
9	O_{21}, O_{91}, O_{92}
3	$K_{176}, K_{286}, K_{348}$
1	{e}

Order	Subgroups
27	V_{35}
9	O_{34}, O_{85}, O_{86}
3	$K_{186}, K_{286}, K_{348}$
1	{e}

Order	Subgroups
27	V_{36}
9	O_6, O_{69}, O_{70}
3	K_{70}, K_{161}, K_{352}
1	{e}

Order	Subgroups
27	V_{37}
9	O_{34}, O_{81}, O_{82}
3	$K_{134}, K_{138}, K_{325}$
1	{e}

Order	Subgroups
27	V_{38}
9	O_{19}, O_{83}, O_{84}
3	$K_{134}, K_{142}, K_{325}$
1	{e}

Order	Subgroups
27	V_{39}
9	O_6, O_{65}, O_{66}
3	$K_{138}, K_{142}, K_{325}$
1	{e}

Order	Subgroups
27	V_{40}
9	O_{28}, O_{109}, O_{110}
3	K_{70}, K_{164}, K_{352}
1	{e}

Order	Subgroups
27	V_{41}
9	O_{19}, O_{87}, O_{88}
3	K_{198}, K_{243}, K_{351}
1	{e}

Order	Subgroups
27	V_{43}
9	O_{05}, O_{18}, O_{19}
3	K_{102}, K_{105}, K_{320}
1	{e}

Order	Subgroups
27	V_{45}
9	O_{10}, O_{11}, O_{12}
3	K_{36}, K_{37}, K_{323}
1	{e}

Order	Subgroups
27	V_{47}
9	O_{27}, O_{28}, O_{29}
3	K_{7}, K_{11}, K_{317}
1	{e}

Order	Subgroups
27	V_{49}
9	O_{71}, O_{72}, O_{117}
3	K_{7}, K_{17}, K_{317}
1	{e}

Order	Subgroups
27	V_{42}
9	O_{21}, O_{22}, O_{23}
3	K_{99}, K_{102}, K_{320}
1	{e}

Order	Subgroups
27	V_{44}
9	O_{16}, O_{97}, O_{98}
3	K_{161}, K_{164}, K_{352}
1	{e}

Order	Subgroups
27	V_{46}
9	O_{15}, O_{16}, O_{17}
3	K_{36}, K_{38}, K_{323}
1	{e}

Order	Subgroups
27	V_{48}
9	O_{33}, O_{34}, O_{35}
3	K_{11}, K_{17}, K_{317}
1	{e}

Order	Subgroups
27	V_{50}
9	O_{61}, O_{62}, O_{117}
3	K_{37}, K_{38}, K_{323}
1	{e}

Order	Subgroups
27	V_{51}
9	O_{73}, O_{74}, O_{117}
3	K_{99}, K_{105}, K_{320}
1	{e}

Order	Subgroups
27	V_{52}
9	O_{95}, O_{96}, O_{117}
3	$K_{234}, K_{236}, K_{327}$
1	{e}

We display one typical interval [{e} , V_1] of L (G) in the following diagram.

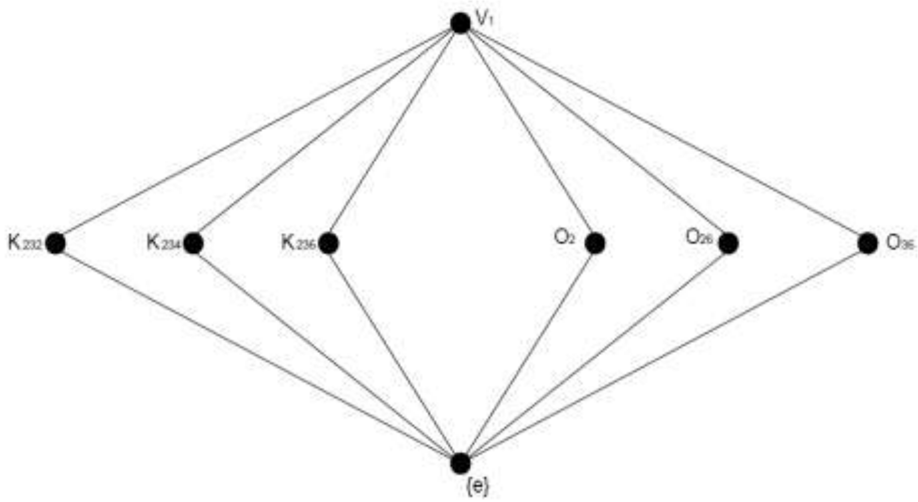

Fig. 5.1: The Interval[{e}, V_1]

CONCLUSION

In this paper, we produced the lattice structure of subgroups of order 27 in the subgroup lattices of 3x3 matrices over Z_3.

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