Rough Hyperideals in Meet hyperlattice

[1]S.Ramalakshmi, [2]B.Nanthini [3]M.Iswarya, [4]A.Remis,
[1][2][3][4]*Student, Department of Mathematics, Thassim Beevi Abdul Kader College for Women, Kilakarai.*

Abstract:

In this paper, we consider a rough hyperideals in meet hyperlattice. Moreover, we investigate some theorems and properties for rough hyperideals in meet hyperlattice.

Keywords:

Rough hyperideals, Hyper congruences, Hyperlattices

Introduction:

In this section, we introduce the notion of rough hyperideals in meet hyperlattices and discuss some properties of them.

Given a hyperlattice L, by $P^*(L)$ we will denote the set of all nonempty subsets of L. If θ is an equivalence relation on L, then, for every $a \in L$, $[a]$ stands for the equivalence class of a with the represent θ. For any nonempty subset A of L, we denote $[A] = \{[a]_\theta | a \in A\}$. For any $A, B \in P^*(L)$, we denote $A\overline{\theta}B$ if the following conditions hold:

(1) for all $a \in A, \exists b \in B$ such that $\bar{a}\theta b$;

(2) for all $d \in B, \exists c \in A$ such that $c\theta d$.

Now, we can introduce the notion of hyper congruences on hyperlattices in the following manner.

Definition 1. Let $(L, \vee, \wedge)$ be a hyperlattice. An equivalence relation θ on L is called a hyper congruence on L if for all a, a′, b, b′ ∈ L the following implication holds: $a\theta a'$, and $b\theta b'$ imply $(a \vee b)\ \overline{\overline{\overline{\theta}}}(a \vee b)$ and $(a \wedge b)\ \overline{\overline{\overline{\theta}}}\ (a \wedge b)$.

Obviously, an equivalence relation θ on $(L, \vee, \wedge)$ is a hyper congruence if and only if for all $a, b, x \in L$, we have that $a\theta b$ implies $(a \vee x)\ \overline{\theta}\ (b \vee x)$ and $(a \wedge x)\overline{\theta}\ (b \wedge x)$.

Lemma 2. Let $(L, \vee, \wedge)$ be a hyperlattice, and let θ be a hyper congruence on L. For all $a, b, \in L$, then $[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$.$[a]_\theta \wedge [b]_\theta \subseteq [a \wedge b]_\theta$.

Proof. Suppose that $x \in [a]_\theta \vee [b]_\theta$, then there exist $x_1 \in [a]_\theta$ and $x_2 \in [b]_\theta$ such That $x \in x_1 \vee x_2 \in$. Since $a\theta x_1$, $b\theta x_2$, by Definition 1, we have $(a \vee b)\ \theta\ (x_1 \vee x_2)$. $x \in x_1 \vee x_2$ implies that there exists $y \in a \vee b$ such that $x\theta y$. Therefore, we have $x \in [a \vee b]_\theta$, which implies $[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$. Similarly, we can prove that $[a]_\theta \wedge [b]_\theta \subseteq [a \wedge b]_\theta$.

A hyper congruence relation θ on $(L, \vee, \wedge)$ is called $\vee$-complete if $[a]_\theta \vee [b]_\theta = [a \vee b]_\theta$ for all $a, b \in L$. Similarly, θ is called $\wedge$-complete if $[a]_\theta \vee [b]_\theta \subseteq [a \vee b]_\theta$ for all $a, b \in L$. We call θ complete if it is both $\vee$-complete and $\wedge$-Complete. Now, we briefly recall the rough set theory in Pawlak's sense.

Let θ be an equivalence relation on L, and let A be a nonempty subset of L.

Then, the sets $\overline{\theta}(A) = \{x \in L \,|[x]_\theta \cap A \neq \emptyset\}$ and $\underline{\theta}(A) = \{x \in L \,|[x]_\theta \subseteq A\}$ are called, respectively, the upper and lower approximations of A with respect to θ. $\theta(A) = (\underline{\theta}(A), \overline{\theta}(A))$ is called a rough set with respect to θ.

Proposition 3. Let θ be a hyper congruence on a hyperlattice $(L, \vee, \wedge)$. If A, B are two nonempty subsets of L, then

(i) $\overline{\theta}(A) \vee \overline{\theta}(B) \subseteq \overline{\theta}(A \vee B)$. In particular, if θ is a $\vee$ −complete, then $\overline{\theta}(A) \vee \overline{\theta}(B) = \overline{\theta}(A \vee B)$.

(ii) $\overline{\theta}(A) \wedge \overline{\theta}(B) \subseteq \overline{\theta}(A \wedge B)$. In particular, if θ is a $\wedge$ −complete, then $\overline{\theta}(A) \wedge \overline{\theta}(B) = \overline{\theta}(A \wedge B)$.

Proof:

Suppose that $x \in \overline{\overline{\theta}}(A) \vee \overline{\theta}(B)$. There exist $x_1 \in \overline{\theta}(A)$ and $x_2 \in \overline{\theta}(B)$ such that $x \in x_1 \vee x_2$.

It follows that there exists $a, b \in L$ such that $a \in [x_1]_\theta \cap A$ and $b \in [x_2]_\theta \cap B$. Since θ is a hyper congruence on L, we have $a \vee b \subseteq [x_1]_\theta \vee [x_2]_\theta \subseteq [x_1 \vee x_2]_\theta$ by lemma 2.

On the other hand, since $a \vee b \subseteq A \vee B$, we obtain $a \vee b \subseteq [x_1 \vee x_2]_\theta \cap (A \vee B)$, which implies $x \in x_1 \vee x_2 \subseteq \overline{\overline{\theta}}(A \vee B)$. Therefore $\overline{\theta}(A) \vee \overline{\theta}(B) \subseteq \overline{\theta}(A \vee B)$.

If θ is $\vee$ −complete, let $x \in \overline{\overline{\theta}}(A \vee B)$, then $[x]_\theta \cap (A \vee B) \neq \emptyset$. Therefore, there exists $y \in [x]_\theta \cap (A \vee B)$, and so for some $a \in A$ and $b \in B$, we have $y \in a \vee b$. Since θ is a $\vee$ −complete, we can obtain $x \in [y]_\theta \subseteq [a \vee b]_\theta = [a]_\theta \vee [b]_\theta$.

Thus, there exists $x_1 \in [a]_\theta$ and $x_2 \in [b]_\theta$ such that $x \in x_1 \vee x_2$.

It follows that $a \in [x_1]_\theta \cap A$ and $b \in [x_2]_\theta \cap B$. Hence, $x_1 \in \overline{\overline{\theta}}(A)$ and $x_2 \in \overline{\overline{\theta}}(B)$, and we have $x \in x_1 \vee x_2 \subseteq \overline{\theta}(A) \vee \overline{\theta}(B)$. Therefore, $\overline{\theta}(A) \vee \overline{\theta}(B) = \overline{\theta}(A \vee B)$.

(2) is similar to that of (1).

Proposition 4: Let θ be a hyper congruence on a hyperlattice $(L, \vee, \wedge)$ and A, B are two nonempty subsets of L, then

(i) If A and B are two $\vee$-hyperideals of L, then $\overline{\theta}(A) \vee \overline{\theta}(B) = \overline{\theta}(A \vee B)$.

(ii) If A and B are two $\wedge$-hyperideals of L, then $\overline{\theta}(A) \wedge \overline{\theta}(B) = \overline{\theta}(A \wedge B)$.

(1) Let $x \in \overline{\theta}(A \vee B)$, then there exist $a \in A$ and $b \in B$ such that $[x]_\theta \cap (a \vee b) \neq \emptyset$, which implies that there exists $t \in a \vee b$ such that $x \,\theta\, t$. Since A is a $\vee$-hyperideal of L, we have $a \vee b \subseteq A$. It follows that $t \in A$. Hence, we obtain that $[x]_\theta \cap A = [t]_\theta \cap A \neq \emptyset$, which implies $x \in \overline{\theta}(A)$. In a similar way, we have $x \in \overline{\theta}(B)$. Thus, $x \in x \vee x \subseteq \overline{\theta}(A) \vee \overline{\theta}(B)$.

Combining proposition 3, we have $\overline{\theta}(A) \vee \overline{\theta}(B) = \overline{\theta}(A \vee B)$.

(2) The proof is similar to that of (1).

Proposition 5: Let θ be a hypercongurence relation on a hyperlattice $(L, \vee, \wedge)$. If A and B are $\vee$ -hyperideals ($\wedge$ -hyperideals) of L, then $\overline{\theta}(A \cap B) = \overline{\theta}(A) \cap \overline{\theta}(B)$.

Proof: Let $x \in \overline{\theta}(A) \cap \overline{\theta}(B)$, we have $[x]_\theta \cap A \neq \emptyset$ and $[x]_\theta \cap B \neq \emptyset$. Then, there exist $x_1 \in A$ and $x_2 \in B$ such that $x_1 \theta x$ and $x_2 \theta x$. It follows from θ which is a hyper congruence relation that $x_1 \vee x_2 \overline{\theta} x \vee x$, which implies that there exists $t \in x_1 \vee x_2$ such that $t \,\theta x$. Since A and B are $\vee$-hyperideals of L, we have $x_1 \vee x_2 \subseteq A \cap B$. So, $t \in A \cap B$. It follows that $[x]_\theta \cap (A \cap B) = [t]_\theta \cap (A \cap B) \neq \emptyset$, which implies $x \in \overline{\theta}(A \cap B)$. Hence, $\overline{\theta}(A) \cap \overline{\theta}(B) \subseteq \overline{\theta}(A \cap B)$. On the other hand, it is clear that $\overline{\theta}(A \cap B) \subseteq \overline{\theta}(A) \cap \overline{\theta}(B)$. Therefore, $\overline{\theta}(A \cap B) = \overline{\theta}(A) \cap \overline{\theta}(B)$. In a similar way, if A and B are $\wedge$ -hyperideals of L, we can also obtain $\overline{\theta}(A \cap B) = \overline{\theta}(A) \cap \overline{\theta}(B)$.

Next, we will introduce and investigate a new algebraic structure called rough hyperideals in meet hyper lattices. Let us begin with introducing the following definitions.

Definition 6: Let θ be a hypercongruence on a hyperlattice $(L, \vee, \wedge)$, and let A be a non empty subset of L. A is called a lower (an upper)rough sub hyperlattice of L if $\underline{\theta}(A)(\overline{\theta}(A))$ is a sub hyperlattice of L. A is called a rough sub hyperlattice of L if A is both a lower rough sub hyperlattice and an upper rough sub hyperlattice of L.

Similarly, A is called a lower (an upper) rough $\vee$ -hyperideal of L if $\underline{(A)}(\overline{\theta}(A))$ is a $\vee$-hyperideal of L.

And we call A as rough ∨-hyperideal of L if A is both a lower rough ∨-hyperideal and an upper rough ∨-hyperideal of L. In a similar way, a rough ∧ -hyperideal of L can be defined.

Example 7 : Let L = {a, b, c, d} be the hyperlattice . Let θ be a hyper congruence relation on the hyperlattice L with the following equivalent classes: $[a]_\theta = \{a, b\}$, $[c]_\theta = \{c, d\}$. Considering A = {a, b, c}, we can obtain that $\underline{(A) = \{a, b\}}$ $\overline{\theta}(A) = L$.

Notice that {a, b} and L are ∨-hyperideals, so A is a rough ∨-hyperideal of L. If A = {b, c, d}, we have that $\underline{(A)} = \{c, d\}$ and $\overline{\theta}(A) = L$. we obtain that {c, d} and L are ∧-hyperideals, so A is a rough ∧-hyperideal of L.

Example 8: In example 7, A = {a, b, c} is a rough ∧-hyperideal of (L, ∨, ∧), but A is not a ∨-hyperideal of L.

Conclusion:

Hence, we have successfully introduced the Rough hyperideals in meet hyperlattice. And we investigated some of their properties.

Reference:

[1] https://www.researchgate.net/publication/340109245_Fuzzy_Soft_Hyperideals_In_meet_Hyperlattices
[2] https://www.researchgate.net/search?context=publicSearchHeader&q=