Generalized Fractional Calculus Operators Involving Multivariable Aleph Function

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Abstract — *This paper involves the brief study of generalized fractional calculus and multivariable Aleph function. In this paper we presented three theorems consisting multivariable Aleph function and generalized fractional calculus operator. The Aleph function used in theorems is general by nature and can be reduced into many other functions.*

Keywords — *Fractional Calculus, Multivariable Aleph Function, Fractional Calculus operator.*

INTRODUCTION

Fractional calculus is a branch of mathematics which deals with various differential and integral operators. Fractional calculus is used in various areas such as science and engineering, Banking etc. Integral operators are studied extensively due to their impotence in applied problems of science and engineering. In this paper we will study the generalized fractional differentiation of multivariable Aleph function. Multivariable Aleph Function is an extension of multivariable I-function which is a generalization of multivariable H-function.

Definition 1: The properties and application of generalized fractional calculus operators are defined by Saigo [9] and the work was carried out by Samko et al. [13] further. For $a, b, c \in C$ and $x > 0$ the generalized fractional calculus operators are defined as follows:

$$(I_{0+}^{a.b.c} f)(x) = \frac{x^{-a-b}}{\Gamma a} \int_0^x (x-t)^{a-1} {}_2F_1\left(a+b, -c; a; 1-\frac{t}{x}\right) f(t)dt \qquad [R(a) > 0]$$

$$= \left(\frac{d}{dx}\right)^n (I_{0+}^{a+n, b-n, c-k} f)(x) \qquad (R(a) \leq 0; n = [R(-a)+1]); \tag{1}$$

used

$$= \left(-\frac{d}{dx}\right)^n (I_{-}^{a+n, b-n, c} f)(x) \qquad (R(a) \leq 0; n = [R(-a)+1]); \tag{2}$$

$$(D_{0+}^{a.b.c} f)(x) = (I_{0+}^{-a,-b,a+c} f)(x)$$

$$= \left(\frac{d}{dx}\right)^n (I_{0+}^{-a+n,-b-n,a+c-k} f)(x) \qquad (R(a) \leq 0; n = [R(-a)+1]); \tag{3}$$

$$(D_{-}^{a.b.c} f)(x) = (I_{-}^{-a,-b,a+c} f)(x)$$

$$= \left(-\frac{d}{dx} \right)^{n} (I_{-}^{-a+n,-b+n,a+c} f)(x) \qquad (R(a) \le 0; n = [R(-a)+1]); \tag{4}$$

where ${}_2F_1(.)$ is the Gaussian hyper geometric function defined by:

$${}_2F_1(p,q,r;u) = \sum_{k=0}^{\infty} \frac{(p)_k (q)_k}{(r)_k} . \frac{u^k}{k}$$

Lemma: For $a, b, c, \mu \in C, \mathrm{R}(a) > 0$ we have [11, 14]

$$\left(I_{0+}^{a,b,c} t^{\mu-1} \right)(x) = \frac{\Gamma\mu \, \Gamma(\mu + c - b)}{\Gamma(\mu - b) \, \Gamma(\mu + a + c)} x^{\mu-b-1}, \qquad R(\mu) > \max[0, R(b-c)] \tag{5}$$

and

$$\left(I_{-}^{a,b,c} t^{-\mu} \right)(x) = \frac{\Gamma(\mu + b) \, \Gamma(\mu + c)}{\Gamma\mu \, \Gamma(a + b + c + \mu)} x^{-\mu-b}, \qquad R(\mu) < \min[R(b), R(c)] \tag{6}$$

Definition 2: Multivariable Aleph Function

Multivariable aleph function is a generalization of multivariable I-function defined by C.K. Sharma and Ahmad [1] and multivariable I-function is generalization of multivariable G and H functions defined by Srivastava et al. [7].

The multivariable Aleph Function is defined by means of the multiple contour integral:

$$\aleph(z_1, z_2, \ldots\ldots\ldots z_r) = \aleph^{0,n;m_1,n_1,\ldots\ldots\ldots m_r,n_r}_{p_i,q_i,\tau_i;R;p_{i(1)},q_{i(1)},\tau_{i(1)};R^{(1)}\ldots\ldots\ldots p_{i(r)},q_{i(r)},\tau_{i(r)};R_{(r)}}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \ldots\ldots\ldots \int_{L_r} \psi(s_1, s_2 \ldots\ldots s_r) \prod_{k=1}^{r} \phi_k(s_k) z_k^{sk} ds_1 ds_2 \ldots\ldots\ldots ds_r \tag{7}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1, s_2, \ldots\ldots s_r) = \frac{\prod_{j=1}^{n} \Gamma\left(1 - a_j + \sum_{k=1}^{r} \alpha_j^{(k)} s_k\right)}{\sum_{i=1}^{r} \left[\tau_i \prod_{j=n+1}^{p_i} \Gamma\left(a_{ji} - \sum_{k=1}^{r} \alpha_{ji}^{(k)} s_k\right) \prod_{j=1}^{q_i} \Gamma\left(1 - b_{ji} + \sum_{k=1}^{r} \beta_{ji}^{(k)} s_k\right) \right]} \tag{8}$$

and

$$\phi_k(s_k) = \frac{\prod_{j=1}^{m_k} \Gamma\left(d_j^{(k)} - \delta_j^{(k)} s_k\right) \prod_{j=1}^{n_k} \Gamma\left(1 - c_j^{(k)} + \gamma_j^{(k)} s_k\right)}{\sum_{i=1}^{r} \left[\tau_{i(k)} \prod_{l=m_k+1}^{q_{i(k)}} \Gamma\left(1 - d_{ji(k)}^{(k)} + \delta_{ji(k)}^{(k)} s_k\right) \prod_{l=n_k+1}^{p_i(k)} \Gamma\left(c_{ji(k)}^{(k)} - \gamma_{ji(k)}^{(k)} s_k\right) \right]} \tag{9}$$

The condition for absolute convergence of multiple Mellin Barnes type contour can be obtained by extension of the corresponding conditions for multivariable H-function given as

$$\left| \arg z_k \right| < \frac{1}{2} A_i^{(k)} \pi$$

where

$$A_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i(k)} \sum_{j=n_k+1}^{p_{i(k)}} \gamma_{ji(k)}^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i(k)} \sum_{j=m_{k+1}}^{q_{i(k)}} \delta_{ji(k)}^{(k)} > 0 \qquad (10)$$

with $k = 1,\ldots r, i = 1,\ldots R$ and $i^{(k)} = 1,\ldots R^{(k)}$

The complex numbers z_i are not zero. In this paper we consider the existence and absolute convergence conditions of the multivariable Aleph function as following

$$\aleph(z_1, z_2, \ldots, z_r) = 0(|z_1|^{\alpha_1}, \ldots, |z_r|^{\alpha_r}), \max(|z_1|, \ldots, |z_r|) \to 0$$

$$\aleph(z_1, z_2, \ldots, z_r) = 0(|z_1|^{\beta_1}, \ldots, |z_r|^{\beta_r}), \max(|z_1|, \ldots, |z_r|) \to \infty$$

where $k = 1, 2\ldots r; \quad \alpha_k = \min[\mathrm{Re}(d_j^{(k)} / \delta_j^{(k)}], \quad j = 1, \ldots, m_k$

and $\beta_k = \max[\mathrm{Re}((c_j^{(k)} - 1) / \gamma_j^{(k)}], \quad j = 1, \ldots, n_k$

For convenience, we will use the following notations in this paper-

$$V = m_1, n_1; \ldots; m_r, n_r$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \ldots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)}$$

$$A = \left\{\left(a_j; \alpha_j^{(1)}, \ldots, \alpha_j^{(r)}\right)_{1,n}\right\}, \left\{\tau_i\left(a_{ji}; \alpha_{ji}^{(1)}, \ldots, \alpha_{ji}^{(r)}\right)_{n+1,p_i}\right\}, \left\{\left(c_j^{(1)}; \gamma_j^{(1)}\right)_{1,n_1}\right\},$$
$$\left.\tau_{i^{(1)}}\left(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\right\}, \ldots, \left\{\left(c_j^{(r)}; \gamma_j^{(r)}\right)_{1,n_r}\right\}, \left.\tau_{i^{(r)}}\left(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\right\} \qquad (11)$$

$$B = \left\{\tau_i\left(b_{ji}; \beta_{ji}^{(1)}, \ldots, \beta_{ji}^{(r)}\right)_{m+1,q_i}\right\}, , \left\{\left(d_j^{(1)}; \delta_j^{(1)}\right)_{1,m_1}\right\},$$
$$\left.\tau_{i^{(1)}}\left(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\right\}, \ldots, \left\{\left(d_j^{(r)}; \delta_j^{(r)}\right)_{1,m_r}\right\}, \left.\tau_{i^{(r)}}\left(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\right\} \qquad (12)$$

Main Results:

Theorem: 1 In this section we drive the generalized fractional integral of multivariable aleph functions. Let $a, b, c, \mu \in C, \Re(a) > 0, a \in R, l \in R_+ = (0, \infty)$ .then there holds the following result:

$$I_{0+}^{a,b,c}\left[t^{\frac{\upsilon_1}{l}-1} \aleph_{p_i,q_i,\tau_i,R:w}^{0,n;\upsilon}\begin{bmatrix} x_1 t^{\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n t^{\rho_n} \end{bmatrix}\right]$$

$$= z^{\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2+\ldots\ldots+\rho_r s_r-b-1\right)} \aleph_{p_i+2,q_i+2\tau_i;R;w}^{0,n+2,V}\left[\begin{matrix} z_1^{\rho_1} \\ z_2^{\rho_2} \\ \vdots \\ z_n^{\rho_n} \end{matrix}\left|\begin{matrix} \left(1-\frac{\upsilon_1}{l}, \rho_1, \ldots, \rho_n\right)\left(1-\frac{\upsilon_1}{l}-c+b, \rho_1, \ldots, \rho_n\right), A \\ \left(1-\frac{\upsilon_1}{l}+b, \rho_1, \ldots, \rho_n\right)\left(1-\frac{\upsilon_1}{l}-c+b, \rho_1, \ldots, \rho_n\right), B \end{matrix}\right.\right] \qquad (13)$$

Proof: To prove the theorem, we represent the Aleph function in its contour form using equation no (7) and on changing the order of equation (which is permissible under the conditions stated above) we get

$$I_0^{a,b,c}\left[t^{\frac{\nu_1}{l}-1} \aleph^{0,n;\upsilon}_{p_i,q_i,\tau_i;R:w}\begin{pmatrix} x_1 t^{\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n t^{\rho_n} \end{pmatrix}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\cdots\cdots\int_{L_r}\psi(s_1,s_2\ldots\ldots s_r)\prod_{m=1}^{r}\phi_m(s_m)\; x_1^{s_1}x_2^{s_2}\ldots\ldots\ldots x_r^{s_r}$$

$$\times\, I_0^{a,b,c}[t^{\left(\frac{\nu_1}{l}+\rho_1 s_1+\rho_2 s_2+\ldots\ldots\ldots+\rho_r s_r\right)-1}]ds_1\ldots\ldots ds_r$$

Now using equation no (5), we get

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\cdots\cdots\int_{L_r}\psi(s_1,s_2\ldots\ldots s_r)\prod_{m=1}^{r}\phi_m(s_m)\; x_1^{s_1}x_2^{s_2}\ldots\ldots\ldots x_r^{s_r}$$

$$x^{\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\ldots\ldots+\rho_r s_r-b-1\right)}\frac{\Gamma\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\ldots\ldots+\rho_r s_r\right)\Gamma\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\ldots\ldots+\rho_r s_r+c-b\right)}{\Gamma\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\ldots\ldots+\rho_r s_r-b\right)\Gamma\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\ldots\ldots+\rho_r s_r+a+c\right)}ds_1\ldots\ldots ds_r$$

combining this expression in form of Aleph function we get the RHS of Theorem 1.

Theorem: 2 Let $a,b,c,\mu \in C, \Re(a)>0, a\in □, l\in □_+=(0,\infty)$ .Then there holds the following result:

$$I_-^{a,b,c}\left[t^{-a-\frac{\upsilon_1}{l}} \aleph^{0,n;\upsilon}_{p_i,q_i,\tau_i;R:w}\begin{pmatrix} x_1 t^{-\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n t^{-\rho_n} \end{pmatrix}\right]$$

$$= z^{-\left(a+\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2+\ldots\ldots+\rho_r s_r-b\right)}\aleph^{0,n+2,V}_{p_i+2,q_i+2,\tau_i;R;w}\left[\begin{matrix} z_1^{\rho_1} \\ z_2^{\rho_2} \\ \vdots \\ z_n^{\rho_n}\end{matrix}\left|\begin{matrix}\left(1-\frac{\upsilon_1}{l}-a-b,\rho_1,\ldots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-c-a,\rho_1,\ldots,\rho_n\right) & A \\ \left(1-\frac{\upsilon_1}{l}-a+b,\rho_1,\ldots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-2a-c-b,\rho_1,\ldots,\rho_n\right) & B\end{matrix}\right.\right] \quad (13)$$

Proof: To prove this we follow the same process as in theorem1 and using the result of Lemma (6) we easily obtain the RHS of theorem 2.

Theorem: 3 Let $a,b,c,\mu \in C, \Re(a)>0, a\in □, l\in □_+=(0,\infty)$ .Then there holds the following result:

$$I_0^{a,b,c}\left[t^{\frac{v_1}{l}-1}\aleph^{0,n;\upsilon}_{p_i,q_i,\tau_i,R:w}\begin{pmatrix} x_1 t^{\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n t^{\rho_n}\end{pmatrix}\right]$$

$$= z^{\left(b+\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2+\ldots\ldots+\rho_r s_r-1\right)}\aleph^{0,n+2,V}_{p_i+2,q_i+2,\tau_i;R;w}\left[\begin{matrix} z_1^{\rho_1} \\ z_2^{\rho_2} \\ \cdot \\ \cdot \\ z_n^{\rho_n}\end{matrix}\left|\begin{matrix}\left(1-\frac{\upsilon_1}{l},\rho_1,\ldots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-c-a+b,\rho_1,\ldots,\rho_n\right) & A \\ \left(1-\frac{\upsilon_1}{l}-b,\rho_1,\ldots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-c,\rho_1,\ldots,\rho_n\right) & B\end{matrix}\right.\right] \tag{14}$$

Proof: To prove this theorem we write the contour form of Aleph function given in equation (7) and changing the order of integration and further using result (5) we easily arrive at RHS.

In this section we drive the generalized fractional integral and multivariable aleph functions.

Theorem:4 Let $a,b,c,\mu\in C,\Re(a)>0, a\in \square, l\in \square_+=(0,\infty)$ .Then there holds the following result:

$$D_-^{a,b,c}\left[t^{a-\frac{\upsilon_1}{l}}\aleph^{0,n;\upsilon}_{p_i,q_i,\tau_i,R:w}\begin{pmatrix} x_1 t^{-\rho_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n t^{-\rho_n}\end{pmatrix}\right]$$

$$= z^{\left(a-\frac{\upsilon_1}{l}+b+\rho_1 s_1+\rho_2 s_2+\ldots\ldots+\rho_r s_r\right)}\aleph^{0,n+2,V}_{p_i+2,q_i+2,\tau_i;R;w}\left[\begin{matrix} z_1^{\rho_1} \\ z_2^{\rho_2} \\ \cdot \\ \cdot \\ z_n^{\rho_n}\end{matrix}\left|\begin{matrix}\left(1-\frac{\upsilon_1}{l}+a+b,\rho_1,\ldots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}+a,\rho_1,\ldots,\rho_n\right) & A \\ \left(1+a-\frac{\upsilon_1}{l},\rho_1,\ldots,\rho_n\right) & \left(1+a-\frac{\upsilon_1}{l},\rho_1,\ldots,\rho_n\right) & B\end{matrix}\right.\right] \tag{15}$$

Proof: To prove this theorem we write the contour form of Aleph function given in equation (7) and changing the order of integration and further using result (6) we easily arrive at RHS.

Particular cases:

Case-I: If $p_i = q_i = n = 0$ then the Aleph function of n –variables reduces to product of N aleph functions of one variable then by using theorem 1, we find the following result:

$$I_0\left[t^{\upsilon_1/l-1}\prod_{\alpha=1}^{N}\aleph_{p_{i(\alpha)},q_{i(\alpha)},\tau_{i(\alpha)};R(\alpha)}^{m_\alpha,n_\alpha}(x_\alpha t^{\rho_\alpha})\right]$$

$$= z^{\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2+\dots\dots+\rho_r s_r-b-1\right)}\aleph_{2,2,\tau_i;R;w}^{0,2,V}\left[\begin{matrix} z_1^{\rho_1}\\ z_2^{\rho_2}\\ \cdot\\ \cdot\\ z_N^{\rho_N}\end{matrix}\left|\begin{matrix}\left(1-\frac{\upsilon_1}{l},\rho_1,\dots,\rho_N\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\dots,\rho_N\right)\\ \left(1-\frac{\upsilon_1}{l}+b,\rho_1,\dots,\rho_N\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\dots,\rho_N\right)\end{matrix}\right.\right] \tag{16}$$

Case-II: If $\tau_i = \tau_{i(1)} = \dots = \tau_{i(s)} = 1$ then the multivariable Aleph function converted into the multivariable H-function defined by Srivastava et al [7] and theorem 1 reduces into following result

$$I_0\left[t^{\frac{v_1}{l}-1}\aleph_{p_i,q_i,\tau_i,R:w}^{0,n;\upsilon}\left(\begin{matrix} x_1 t^{\rho_1}\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ x_n t^{\rho_n}\end{matrix}\right)\right]$$

$$= z^{\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2+\dots\dots+\rho_r s_r-b-1\right)}H_{p_i+2,q_i+2,\tau_i;R;w}^{0,n+2,V}\left[\begin{matrix} z_1^{\rho_1}\\ z_2^{\rho_2}\\ \cdot\\ \cdot\\ z_n^{\rho_n}\end{matrix}\left|\begin{matrix}\left(1-\frac{\upsilon_1}{l},\rho_1,\dots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\dots,\rho_n\right) & A'\\ \left(1-\frac{\upsilon_1}{l}+b,\rho_1,\dots,\rho_n\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\dots,\rho_n\right) & B'\end{matrix}\right.\right] \tag{17}$$

Where

$$A' = \left\{\left(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{1,n}\right\},\left\{\left(a_{ji};\alpha_{ji}^{(1)},\dots,\alpha_{ji}^{(r)}\right)_{n+1,p_i}\right\},\left\{\left(c_j^{(1)};\gamma_j^{(1)}\right)_{1,n_1}\right\},$$
$$\left(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)}\right)_{n_1+1,p_{i^{(1)}}}\Big\},\dots,\left\{\left(c_j^{(r)};\gamma_j^{(r)}\right)_{1,n_r}\right\},\left(c_{ji^{(r)}}^{(r)};\gamma_{ji^{(r)}}^{(r)}\right)_{n_r+1,p_{i^{(r)}}}\Big\}$$

$$B' = \left\{\left(b_{ji};\beta_{ji}^{(1)},\dots,\beta_{ji}^{(r)}\right)_{m+1,q_i}\right\},,\left\{\left(d_j^{(1)};\delta_j^{(1)}\right)_{1,m_1}\right\},$$
$$\left(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)}\right)_{m_1+1,q_{i^{(1)}}}\Big\},\dots,\left\{\left(d_j^{(r)};\delta_j^{(r)}\right)_{1,m_r}\right\},\left(d_{ji^{(r)}}^{(r)};\delta_{ji^{(r)}}^{(r)}\right)_{m_r+1,q_{i^{(r)}}}\Big\}$$

Case-III: If we put n = 2 in theorem 1**,** we obtain aleph function of two variable defined by K. Sharma [8]. And the result becomes

$$I_0\left[t^{\frac{v_1}{l}-1}\aleph_{p_i,q_i,\tau_i,R:w}^{0,n;\upsilon}\left(\begin{matrix} x_1 t^{\rho_1}\\ \cdot\\ x_2 t^{\rho_2}\end{matrix}\right)\right]$$

$$= z^{\left(\frac{\upsilon_1}{l}+\rho_1 s_1+\rho_2 s_2-b-1\right)} \aleph^{0,n+2,V}_{p_i,q_i,\tau_i;R;w} \left[\begin{array}{c|cc} z_1^{\rho_1} & \left(1-\frac{\upsilon_1}{l},\rho_1,\rho_2\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\rho_2\right) \\ \cdot & & \\ z_2^{\rho_2} & \left(1-\frac{\upsilon_1}{l}+b,\rho_1,\rho_2\right) & \left(1-\frac{\upsilon_1}{l}-c+b,\rho_1,\rho_2\right) \end{array} \right] \quad (18)$$

Case IV: By putting n=1 in theorem 1, we obtain aleph function of one variable defined by Sudland [10].

$$I_0\left[t^{\frac{v_1}{l}-1} \aleph^{0,n;\upsilon}_{p_i,q_i,\tau_i,R:w}\left(xt^{\rho}\right)\right]$$

$$= z^{\left(\frac{\upsilon_1}{l}+\rho s-b-1\right)} \aleph^{M,n+2,V}_{p_i+2,q_i+2,\tau_i;R;w} \left[z^{\rho} \left| \begin{array}{l} \left(1-\frac{\upsilon_1}{l},\rho\right)\ \left(1-\frac{\upsilon_1}{l}-c+b,\rho\right),\ (a_j,A_j)_{1,n},\left[c_i(a_{ji},A_{ji})\right]_{n+1,p_i;r} \\ \left(1-\frac{\upsilon_1}{l}+b,\rho\right)\ \left(1-\frac{\upsilon_1}{l}-c+b,\rho\right),\ (b_j,B_j)_{1,n}\left[c_i(b_{ji},B_{ji})\right]_{m+1,q_i;r} \end{array} \right. \right]$$

(19)

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