Some New Status Neighborhood Indices of Graphs

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Abstract: *The status of a vertex u in a connected graph G is the sum of the distances between u and all other vertices in a graph G. In this study, we define the atom bond connectivity (ABC) status neighborhood index, geometric-arithmetic (GA) status neighborhood index, arithmetic-geometric (AG) status neighborhood index, harmonic status neighborhood index, symmetric division status neighborhood index, inverse sum indeg status neighborhood index of a graph and compute exact formulas for some standard graphs, friendship graphs.*

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Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*35, 05*C*90.

I. Introduction

Many types of topological indices such as degree based graph indices, distance based graph indices and counting related graph indices are explored during past recent years. Among distance based graph indices Wiener index [1] is the oldest one and studied well. In this paper, we introduce and study *ABC* status neighborhood index, *GA* status neighborhood index, *AG* status neighborhood index of a graph.

Let *G* be a finite, simple, connected graph. Let $V(G)$ and $E(G)$ be its vertex and edge sets respectively. The degree $d_G(u)$ of a vertex *u* is the number of vertices adjacent to *u*. The distance $d(u, v)$ between any two vertices *u* and *v* is the length of shortest path connecting *u* and *v*. The status $\sigma(u)$ of a vertex *u* in a graph *G* is the sum of distances of all other vertices from *u* in *G*. Let $N(v)=N_G(v)=\{v:uv\in E(G)\}$. Let $\sigma_n(u)=\sum_{u\in N(v)}\sigma(u)$ be the status sum of neighbor vertices. For undefined term and notation, we refer [2].

The first and second status neighborhood indices of a graph are introduced by Kulli in [3], defined as

$$SN_1(G)=\sum_{uv\in E(G)}\left[\sigma_n(u)+\sigma_n(v)\right], \qquad SN_2(G)=\sum_{uv\in E(G)}\sigma_n(u)\sigma_n(v).$$

Some of the research works on the status and status neighborhood indices can be found in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

We now introduce the *ABC* status neighborhood index, *GA* status neighborhood index, *AG* status neighborhood index of a graph *G* as follows:

The atom bond connectivity (*ABC*) status neighborhood index of a graph *G* is defined as

$$ABCSN(G)=\sum_{uv\in E(G)}\sqrt{\frac{\sigma_n(u)+\sigma_n(v)-2}{\sigma_n(u)\sigma_n(v)}}.$$

The geometric-arithmetic (*GA*) status neighborhood index of a graph *G* is defined as

$$GASN(G)=\sum_{uv\in E(G)}\frac{2\sqrt{\sigma_n(u)\sigma_n(v)}}{\sigma_n(u)+\sigma_n(v)}.$$

The arithmetic-geometric (*AG*) status neighborhood index of a graph *G* is defined as

$$AGSN(G)=\sum_{uv\in E(G)}\frac{\sigma_n(u)+\sigma_n(v)}{2\sqrt{\sigma_n(u)\sigma_n(v)}}.$$

Recently many different graph indices were studied, for example, in [16, 17, 18, 19, 20, 21, 22, 23, 24].

The harmonic status neighborhood index of a graph G is defined as

$$HSN(G) = \sum_{uv \in E(G)} \frac{2}{\sigma_n(u) + \sigma_n(v)}.$$

Considering the harmonic status neighborhood index, we define the harmonic status neighborhood polynomial of a graph G as

$$HSN(G, x) = \sum_{uv \in E(G)} x^{\frac{2}{\sigma_n(u)+\sigma_n(v)}}.$$

The symmetric division status neighborhood index of a graph G is defined as

$$SDSN(G) = \sum_{uv \in E(G)} \left(\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)} \right).$$

Considering the symmetric division status neighborhood index, we define the symmetric division status neighborhood polynomial of a graph G as

$$SDSN(G, x) = \sum_{uv \in E(G)} x^{\left(\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)} \right)}.$$

The inverse sum indeg status neighborhood index of a graph G is defined as

$$ISSN(G) = \sum_{uv \in E(G)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(v) + \sigma_n(u)}.$$

Considering the inverse sum indeg status neighborhood index, we define the inverse sum indeg status neighborhood polynomial of a graph G as

$$ISSN(G, x) = \sum_{uv \in E(G)} x^{\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(v)+\sigma_n(u)}}.$$

The augmented status neighborhood index of a graph G is defined as

$$ASNI(G) = \sum_{uv \in E(G)} \left(\frac{\sigma_n(u) + \sigma_n(v) - 2}{\sigma_n(u)\sigma_n(v)} \right)^3.$$

Considering the augmented status neighborhood index, we define the augmented status neighborhood polynomial of a graph G as

$$ASNI(G, x) = \sum_{uv \in E(G)} x^{\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2} \right)^3}$$

In this paper, some newly defined status neighborhood indices of some standard graphs, friendship graphs are determined.

II. RESULTS FOR COMPLETE GRAPHS

In the following theorem, we compute the atom bond connectivity status neighborhood index, geometric-arithmetic status neighborhood index, arithmetic-geometric status neighborhood index of a complete graph K_n.

Theorem 1. Let K_n be a complete graph. Then

(1) $$ABCSN(K_n) = \frac{n\sqrt{n(n-2)}}{\sqrt{2}(n-1)}.$$

(2) $$GASN(K_n) = \frac{n(n-1)}{2}.$$

(3) $$AGSN(K_n) = \frac{n(n-1)}{2}.$$

Proof: Let K_n be a complete graph with *n* vertices and $\frac{n(n-1)}{2}$ edges. Then for any vertex *u* of K_n, $\sigma(u)= (n-1)$. By calculation, we have $\sigma_n(u)= (n-1)^2$ for any vertex *u* of K_n. Thus

(1) $$ABCSN(K_n) = \sum_{uv\in E(K_n)} \left(\frac{\sigma_n(u)+\sigma_n(v)-2}{\sigma_n(u)\sigma_n(v)}\right)^{\frac{1}{2}} = \left[\frac{(n-1)^2+(n-1)^2-2}{(n-1)^2(n-1)^2}\right]^{\frac{1}{2}} \frac{n(n-1)}{2}$$
$$= \frac{n\sqrt{n(n-2)}}{\sqrt{2}(n-1)}.$$

(2) $$GASN(K_n) = \sum_{uv\in E(K_n)} \frac{2\sqrt{\sigma_n(u)\sigma_n(v)}}{\sigma_n(u)+\sigma_n(v)} = \left[\frac{2\sqrt{(n-1)^2(n-1)^2}}{(n-1)^2+(n-1)^2}\right]\frac{n(n-1)}{2}$$
$$= \frac{n(n-1)}{2}.$$

(3) $$AGSN(K_n) = \sum_{uv\in E(K_n)} \frac{\sigma_n(u)+\sigma_n(v)}{2\sqrt{\sigma_n(u)\sigma_n(v)}} = \left[\frac{(n-1)^2+(n-1)^2}{2\sqrt{(n-1)^2(n-1)^2}}\right]\frac{n(n-1)}{2}$$
$$= \frac{n(n-1)}{2}.$$

In the following theorem, we compute the harmonic status neighborhood index and its polynomial of a complete graph K_n.

Theorem 2. Let K_n be a complete graph. Then

(1) $$HSN(K_n) = \frac{n}{2(n-1)}.$$

(2) $$HSN(K_n, x) = \frac{n(n-1)}{2} x^{\frac{1}{(n-1)^2}}.$$

Proof: Let K_n be a complete graph with *n* vertices and $\frac{n(n-1)}{2}$ edges. Then for any vertex *u* of K_n, $\sigma(u)= (n-1)$. By calculation, we have $\sigma_n(u)= (n-1)^2$ for any vertex *u* of K_n. Thus

(1) $$HSN(K_n) = \sum_{uv\in E(K_n)} \frac{2}{\sigma_n(u)+\sigma_n(v)} = \left[\frac{2}{(n-1)^2+(n-1)^2}\right]\frac{n(n-1)}{2}$$
$$= \frac{n}{2(n-1)}.$$

(2) $$HSN(K_n, x) = \sum_{uv\in E(K_n)} x^{\frac{2}{\sigma_n(u)+\sigma_n(v)}} = \frac{n(n-1)}{2} x^{\frac{2}{(n-1)^2+(n-1)^2}}$$
$$= \frac{n(n-1)}{2} x^{\frac{1}{(n-1)^2}}.$$

In the following theorem, we determine the symmetric division status neighborhood index and its polynomial of a complete graph K_n.

Theorem 3. Let K_n be a complete graph. Then

(1) $$SDSN(K_n) = n(n-1).$$

(2) $$SDSN(K_n, x) = \frac{n(n-1)}{2} x^2.$$

Proof: Let K_n be a complete graph with *n* vertices and $\frac{n(n-1)}{2}$ edges. Then by calculation, we have $\sigma_n(u)= (n-1)^2$ for any vertex *u* of K_n. Therefore

(1) $$SDSN(K_n) = \sum_{uv\in E(K_n)} \left(\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)} \right) = \left[\frac{(n-1)^2}{(n-1)^2} + \frac{(n-1)^2}{(n-1)^2} \right] \frac{n(n-1)}{2}$$
$$= n(n-1).$$

(2) $$SDSN(K_n, x) = \sum_{uv\in E(K_n)} x^{\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)}} = \frac{n(n-1)}{2} x^{\frac{(n-1)^2}{(n-1)^2} + \frac{(n-1)^2}{(n-1)^2}}$$
$$= \frac{n(n-1)}{2} x^2.$$

In the following theorem, we compute the inverse sum indeg status neighborhood index and its polynomial of a complete graph K_n.

Theorem 4. Let K_n be a complete graph. Then

(1) $$ISSN(K_n) = \frac{1}{4} n(n-1)^3.$$

(2) $$ISSN(K_n, x) = \frac{n(n-1)}{2} x^{\frac{1}{2}(n-1)^2}.$$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. Then by calculation, we obtain $\sigma_n(u) = (n-1)^2$ for any vertex u of K_n. Hence

(1) $$ISSN(K_n) = \sum_{uv\in E(K_n)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)} = \left[\frac{(n-1)^2 (n-1)^2}{(n-1)^2 + (n-1)^2} \right] \frac{n(n-1)}{2}$$
$$= \frac{1}{4} n(n-1)^3.$$

(2) $$ISSN(K_n, x) = \sum_{uv\in E(K_n)} x^{\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)}} = \frac{n(n-1)}{2} x^{\frac{(n-1)^2(n-1)^2}{(n-1)^2+(n-1)^2}}$$
$$= \frac{n(n-1)}{2} x^{\frac{1}{2}(n-1)^2}.$$

In the following theorem, we compute the augmented status neighborhood index and its polynomial of a complete graph K_n.

Theorem 5. Let K_n be a complete graph. Then

(1) $$ASNI(K_n) = \frac{(n-1)^{13}}{16n^2(n-2)^3}.$$

(2) $$ASNI(K_n, x) = \frac{n(n-1)}{2} x^{\frac{(n-1)^{12}}{8n^2(n-2)^3}}.$$

Proof: Let K_n be a complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. By calculation, we have $\sigma_n(u) = (n-1)^2$ for any vertex u of K_n. Thus

(1) $$ASNI(K_n) = \sum_{uv\in E(K_n)} \left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v) - 2} \right)^3 = \frac{n(n-1)}{2} \left(\frac{(n-1)^2 (n-1)^2}{(n-1)^2 + (n-1)^2 - 2} \right)^3$$
$$= \frac{(n-1)^{13}}{16n^2(n-2)^3}.$$

(2) $$ASNI(K_n, x) = \sum_{uv\in E(K_n)} x^{\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3} = \frac{n(n-1)}{2} x^{\left(\frac{(n-1)^2(n-1)^2}{(n-1)^2+(n-1)^2-2}\right)^3}$$

$$= \frac{n(n-1)}{2} x^{\frac{(n-1)^{12}}{8x^3(n-2)^3}}.$$

III. RESULTS FOR COMPLETE BIPARTITE GRAPHS

In the following theorem, we compute the atom bond connectivity status neighborhood index, geometric-arithmetic status neighborhood index, arithmetic-geometric index of a complete bipartite graph $K_{p,q}$.

Theorem 6. Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. Then

(1) $$ABCSN(K_{p,q}) = \left[\frac{pq\{2(p^2+q^2)-2(p+q)+2pq-2\}}{2(p^2+q^2)-6(p+q)+5pq+4}\right]^{\frac{1}{2}}.$$

(2) $$GASN(K_{p,q}) = \frac{(pq)^{\frac{3}{2}}\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^{\frac{1}{2}}}{(p^2+q^2)-(p+q)+pq}.$$

(3) $$AGSN(K_{p,q}) = \frac{(pq)^{\frac{1}{2}}\left[(p^2+q^2)-(p+q)+pq\right]}{2(p^2+q^2)-6(p+q)+5pq+4}.$$

Proof: If $K_{p,q}$ is a complete bipartite graph, then it has $p+q$ vertices and pq edges. The vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that $u \in V_1$ and $v \in V_2$ for every edge uv in $K_{p,q}$. Thus $d_B(u)=q$ and $d_B(v)=p$, where $B=K_{p,q}$. Then we have $\sigma(u)= q + 2p - 2$ and $\sigma(v)= p + 2q - 2$. By calculation, we obtain $\sigma_n(u)= p(q + 2p - 2)$ and $\sigma_n(v)= q(p + 2q - 2)$. Thus

(1) $$ABCSN(K_{p,q}) = \sum_{uv\in E(B)}\left(\frac{\sigma_n(u)+\sigma_n(v)-2}{\sigma_n(u)\sigma_n(v)}\right)^{\frac{1}{2}}$$

$$= pq\left[\frac{p(q+2p-2)+q(p+2q-2)-2}{p(q+2p-2)q(p+2q-2)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{pq\{2(p^2+q^2)-2(p+q)+2pq-2\}}{2(p^2+q^2)-6(p+q)+5pq+4}\right]^{\frac{1}{2}}.$$

(2) $$GASN(K_{p,q}) = \sum_{uv\in E(B)}\frac{2\sqrt{\sigma_n(u)\sigma_n(v)}}{\sigma_n(u)+\sigma_n(v)}$$

$$= \frac{2pq\left[p(q+2p-2)q(p+2q-2)\right]^{\frac{1}{2}}}{p(q+2p-2)+q(p+2q-2)}$$

$$= \frac{(pq)^{\frac{3}{2}}\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^{\frac{1}{2}}}{(p^2+q^2)-(p+q)+pq}.$$

(3) $$AGSN(K_{p,q}) = \sum_{uv\in E(B)}\frac{\sigma_n(u)+\sigma_n(v)}{2\sqrt{\sigma_n(u)\sigma_n(v)}}$$

$$= \frac{pq\{p(q+2p-2)+q(p+2q-2)\}}{2\left[p(q+2p-2)q(p+2q-2)\right]^{\frac{1}{2}}}$$

$$= \frac{\sqrt{pq}\left[(p^2+q^2)-(p+q)+pq\right]}{\left[2(p^2+q^2)-6(p+q)+5pq+4\right]^{\frac{1}{2}}}.$$

In the following theorem, we compute the harmonic status neighborhood index and its polynomial of a complete bipartite graph.

Theorem 7. Let $K_{p,q}$ be a complete bipartite graph. Then

(1) $$HSN(K_{p,q}) = \frac{pq}{(p^2+q^2)-(p+q)+pq}.$$

(2) $$HSN(K_{p,q},x) = pqx^{\frac{1}{(p^2+q^2)-(p+q)+pq}}.$$

Proof: If $K_{p,q}$ is a complete bipartite graph, then $\sigma_n(u)= p(q + 2p - 2)$ and $\sigma_n(v)= q(p + 2q - 2)$ for every edge uv in $K_{p,q}$, see Theorem 6. Let $B=K_{p,q}$.

(1) From definition, we have

$$HSN(K_{p,q}) = \sum_{uv\in E(B)} \frac{2}{\sigma_n(u)+\sigma_n(v)}$$

$$= pq\left[\frac{2}{p(q+2p-2)+q(p+2q-2)}\right]$$

$$= \frac{pq}{(p^2+q^2)-(p+q)+pq}.$$

(2) From definition, we have

$$HSN(K_{p,q},x) = \sum_{uv\in E(B)} x^{\frac{2}{\sigma_n(u)+\sigma_n(v)}}$$

$$= pqx^{\frac{2}{p(q+2p-2)+q(p+2q-2)}}$$

$$= pqx^{\frac{1}{(p^2+q^2)-(p+q)+pq}}.$$

In the following theorem, we compute the symmetric division status neighborhood index and its polynomial of a complete bipartite graph.

Theorem 8. Let $K_{p,q}$ be a complete bipartite graph. Then

(1) $$SDSN(K_{p,q}) = \frac{p^2(q+2p-2)^2+q^2(p+2q-2)^2}{(p+2q-2)(q+2p-2)}.$$

(2) $$SDSN(K_{p,q},x) = pqx^{\frac{p^2(q+2p-2)^2+q^2(p+2q-2)^2}{pq(p+2q-2)(q+2p-2)}}.$$

Proof: Let $B=K_{p,q}$ be a complete bipartite graph. Then $\sigma_n(u)= p(q + 2p - 2)$ and $\sigma_n(v)= q(p + 2q - 2)$ for every edge uv in $K_{p,q}$, see Theorem 6.

(1) $$SDSN(K_{p,q}) = \sum_{uv\in E(B)} \left(\frac{\sigma_n(u)}{\sigma_n(v)}+\frac{\sigma_n(v)}{\sigma_n(u)}\right)$$

$$= pq\left[\frac{p(q+2p-2)}{q(p+2q-2)}+\frac{q(p+2q-2)}{p(q+2p-2)}\right]$$

$$= \frac{p^2(q+2p-2)^2+q^2(p+2q-2)^2}{(p+2q-2)(q+2p-2)}.$$

(2) $$SDSN(K_{p,q},x)=\sum_{uv\in E(B)} x^{\frac{\sigma_n(u)}{\sigma_n(v)}+\frac{\sigma_n(v)}{\sigma_n(u)}}$$
$$= pqx^{\left[\frac{p(q+2p-2)}{q(p+2q-2)}+\frac{q(p+2q-2)}{p(q+2p-2)}\right]}$$
$$= pqx^{\frac{p^2(q+2p-2)^2+q^2(p+2q-2)^2}{pq(p+2q-2)(q+2p-2)}}.$$

In the following theorem, we determine the inverse sum indeg status neighborhood index and its polynomial of $K_{p,q}$.

Theorem 9. Let $K_{p,q}$ be a complete bipartite graph. Then

(1) $$ISSN(K_{p,q})=\frac{pq\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}{2(p^2+q^2)-2(p+q)+2pq}.$$

(2) $$ISSN(K_{p,q},x)=pqx^{\frac{2(p^2+q^2)-6(p+q)+5pq+4}{2(p^2+q^2)-2(p+q)+2pq}}.$$

Proof: Let $K_{p,q}=B$ be a complete bipartite graph. Then $\sigma_n(u)=p(q+2p-2)$ and $\sigma_n(v)=q(p+2q-2)$ for every edge uv in $K_{p,q}$, see Theorem 6.

(1) $$ISSN(K_{p,q})=\sum_{uv\in E(B)}\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)}$$
$$=\frac{pq\left[p(q+2p-2)q(p+2q-2)\right]}{q(p+2q-2)+p(q+2p-2)}$$
$$=\frac{pq\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}{2(p^2+q^2)-2(p+q)+2pq}.$$

(2) $$ISSN(K_{p,q},x)=\sum_{uv\in E(B)} x^{\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)}}$$
$$= pqx^{\frac{p(q+2p-2)q(p+2q-2)}{q(p+2q-2)+p(q+2p-2)}}$$
$$= pqx^{\frac{2(p^2+q^2)-6(p+q)+5pq+4}{2(p^2+q^2)-2(p+q)+2pq}}.$$

In the following theorem, we complete the augmented status neighborhood index and its polynomial of $K_{p,q}$.

Theorem 10. Let $K_{p,q}$ be a complete bipartite graph. Then

(1) $$ASNI(K_{p,q})=p^4q^4\left(\frac{\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}{2(p^2+q^2)-2(p+q)+2pq-2}\right)^3.$$

(2) $$ASNI(K_{p,q},x)=pqx^{\left(\frac{pq\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}{2(p^2+q^2)-2(p+q)+2pq-2}\right)^3}.$$

Proof: Let $K_{p,q}=B$ be a complete bipartite graph. Then $\sigma_n(u)=p(q+2p-2)$ and $\sigma_n(v)=q(p+2q-2)$ for every edge uv in $K_{p,q}$.

(1) $$ASNI(K_{p,q})=\sum_{uv\in E(B)}\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3$$

$$= pq\left(\frac{p(q+2p-2)q(p+2q-2)}{p(q+2p-2)+q(p+2q-2)-2}\right)^3$$

$$= p^4q^4\left(\frac{2(p^2+q^2)-6(p+q)+5pq+4}{2(p^2+q^2)-2(p+q)+2pq-2}\right)^3.$$

(2) $$ASNI(K_{p,q},x) = \sum_{uv\in E(B)} x^{\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3}$$

$$= pqx^{\left(\frac{p(q+2p-2)q(p+2q-2)}{q(p+2q-2)+p(q+2p-2)-2}\right)^3} = pqx^{\left(\frac{pq\left[2(p^2+q^2)-6(p+q)+5pq+4\right]}{2(p^2+q^2)-2(p+q)+2pq-2}\right)^3}.$$

IV. RESULTS FOR WHEEL GRAPHS

A wheel graph, denoted by W_n, is the join of K_1 and C_n. A graph W_4 is presented in Figure 1.

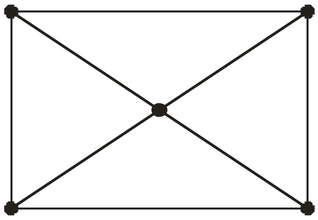

Figure 1. Wheel graph W_4

A wheel graph has n+1 vertices and 2n edges. In this graph, we find two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_{W_n}(u) = d_{W_n}(v) = 3\}$ $\quad |E_1| = n.$

$E_2 = \{uv \in E(W_n) \mid d_{W_n}(u) = 3, d_{W_n}(v) = n\}$ $\quad |E_2| = n.$

Therefore by calculation, there are two types of status edges as follows:

$E_1 = \{uv \in E(W_n) \mid \sigma(u) = \sigma(v) = 2n-3\},$ $\quad |E_1| = n.$

$E_2 = \{uv \in E(W_n) \mid \sigma(u) = n, \sigma(v) = 2n-3\},$ $\quad |E_2| = n.$

By calculation, we find that there are two types of status neighborhood edges as in Table 1.

$\sigma_n(u), \sigma_n(v) \setminus uv \in E(W_n)$	$(5n-6, 5n-6)$	$(5n-6, n(2n-3))$
Number of edges	n	n

Table 1. Status neighborhood edge partition of W_n

Theorem 11. Let W_n be a wheel graph with n + 1 vertices and 2n edges. Then

(1) $$ABCSN(W_n) = \frac{n(10n-14)^{\frac{1}{2}}}{5n-6} + n\left(\frac{2n^2+2n-8}{10n^3-27n^2+18n}\right)^{\frac{1}{2}}.$$

(2) $$GASN(W_n) = n + \frac{n\sqrt{10n^3-27n^2+18n}}{n^2+n-3}.$$

(3) $$AGSN(W_n) = n + \frac{n(n^2+n-3)}{\sqrt{10n^3-27n^2+18n}}.$$

Proof: From definition and by using Table 1, we deduce

(1) $$ABCSN(W_n) = \sum_{uv\in E(W_n)} \left(\frac{\sigma_n(u)+\sigma_n(v)-2}{\sigma_n(u)\sigma_n(v)}\right)^{\frac{1}{2}}$$

$$= n\left(\frac{(5n-6)+(5n-6)-2}{(5n-6)(5n-6)}\right)^{\frac{1}{2}} + n\left(\frac{(5n-6)+(2n^2-3n)-2}{(5n-6)(2n^2-3n)}\right)^{\frac{1}{2}}$$

$$= \frac{n(10n-14)^{\frac{1}{2}}}{5n-6} + n\left(\frac{2n^2+2n-8}{10n^3-27n^2+18n}\right)^{\frac{1}{2}}.$$

(2) $$GASN(W_n) = \sum_{uv\in E(W_n)} \frac{2\sqrt{\sigma_n(u)\sigma_n(v)}}{\sigma_n(u)+\sigma_n(v)}$$

$$= n\frac{2\sqrt{(5n-6)(5n-6)}}{5n-6+5n-6} + n\frac{2\sqrt{(5n-6)(2n^2-3n)}}{5n-6+2n^2-3n}$$

$$= n + \frac{n\sqrt{10n^3-27n^2+18n}}{n^2+n-3}.$$

(3) $$AGSN(W_n) = \sum_{uv\in E(W_n)} \frac{\sigma_n(u)+\sigma_n(v)}{2\sqrt{\sigma_n(u)\sigma_n(v)}}$$

$$= \frac{n(5n-6+5n-6)}{2\sqrt{(5n-6)(5n-6)}} + \frac{n(5n-6+2n^2-3n)}{2\sqrt{(5n-6)(2n^2-3n)}}$$

$$= n + \frac{n(n^2+n-3)}{\sqrt{10n^3-27n^2+18n}}.$$

In the following theorem, we compute the harmonic status neighborhood index and its polynomial of a wheel graph W_n.

Theorem 12. The W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $$HSN(W_n) = \frac{n}{5n-6} + \frac{n}{n^2+n-3}.$$

(2) $$HSN(W_n, x) = nx^{\frac{1}{5n-6}} + nx^{\frac{1}{n^2+n-3}}.$$

Proof: (1) From definition and using Table 1, we deduce

$$HSN(W_n) = \sum_{uv\in E(W_n)} \frac{2}{\sigma_n(u)+\sigma_n(v)}$$

$$= n\frac{2}{5n-6+5n-6} + n\frac{2}{5n-6+2n^2-3n}$$

$$= \frac{n}{5n-6} + \frac{n}{n^2+n-3}.$$

(2) Using definition and Table 1, we derive

$$HSN(W_n, x) = \sum_{uv\in E(W_n)} x^{\frac{2}{\sigma_n(u)+\sigma_n(v)}}$$

$$= nx^{\frac{2}{5n-6+5n-6}} + nx^{\frac{2}{5n-6+2n^2-3n}}$$

$$= nx^{\frac{1}{5n-6}} + nx^{\frac{1}{n^2+n-3}}.$$

In the following theorem, we determine the symmetric division status neighborhood index and its polynomial of a wheel graph W_n.

Theorem 13. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $$SDSN(W_n) = 2n + \frac{4n^4 - 12n^3 + 34n^2 - 60n + 36}{10n^2 - 27n + 18}.$$

(2) $$SDSN(W_n, x) = nx^2 + nx^{\frac{4n^4-12n^3+34n^2-60n+36}{10n^3-27n^2+18n}}.$$

Proof: (1) From definition and using Table 1, we obtain

$$SDSN(W_n) = \sum_{uv\in E(W_n)} \left(\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)} \right)$$

$$= n\left(\frac{5n-6}{5n-6} + \frac{5n-6}{5n-6} \right) + n\left(\frac{5n-6}{2n^2-3n} + \frac{2n^2-3n}{5n-6} \right)$$

$$= 2n + \frac{4n^4 - 12n^3 + 34n^2 - 60n + 36}{10n^2 - 27n + 18}.$$

(2) From definition and using Table 1, we have

$$SDSN(W_n, x) = \sum_{uv\in E(W_n)} x^{\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)}}$$

$$= nx^{\frac{5n-6}{5n-6} + \frac{5n-6}{5n-6}} + nx^{\frac{5n-6}{2n^2-3n} + \frac{2n^2-3n}{5n-6}}$$

$$= nx^2 + nx^{\frac{4n^4-12n^3+34n^2-60n+36}{10n^3-27n^2+18n}}.$$

In the following theorem, we determine the inverse sum indeg status neighborhood index and its polynomial of a wheel graph W_n.

Theorem 14. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $$ISSN(W_n) = \frac{1}{2}n(5n-6) + \frac{n(10n^3 - 27n^2 + 18n)}{2n^2 + 2n - 6}.$$

(2) $$ISSN(W_n, x) = nx^{\frac{1}{2}(5n-6)} + nx^{\frac{10n^3-27n^2+18n}{2n^2+2n-6}}.$$

Proof: (1) From definition and using Table 1, we deduce

$$ISSN(W_n) = \sum_{uv\in E(W_n)} \left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)} \right)$$

$$= n\frac{(5n-6)(5n-6)}{5n-6+5n-6} + n\frac{(5n-6)(2n^2-3n)}{5n-6+2n^2-3n}$$

$$= \frac{1}{2}n(5n-6) + \frac{n(10n^3 - 27n^2 + 18n)}{2n^2 + 2n - 6}.$$

(2) From definition and using Table 1, we have

$$ISSN(W_n, x) = \sum_{uv\in E(W_n)} x^{\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)}}$$

$$= nx^{\frac{(5n-6)(5n-6)}{5n-6+5n-6}} + nx^{\frac{(5n-6)(2n^2-3n)}{2n^2+2n-6}}$$

$$= nx^{\frac{1}{2}(5n-6)} + nx^{\frac{10n^3-27n^2+18n}{2n^2+2n-6}}.$$

In the following theorem, we compute augmented status neighborhood index and its polynomial of a wheel graph W_n.

Theorem 15. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $$ASNI(W_n) = n\left[\frac{(5n-6)^2}{10n-14}\right]^3 + n\left[\frac{10n^3-27n^2+18n}{2n^2+2n-8}\right]^3.$$

(2) $$SSNI(W_n,x) = nx^{\left[\frac{(5n-6)^2}{10n-14}\right]^3} + nx^{\left[\frac{10n^3-27n^2+18n}{2n^2+2n-8}\right]^3}.$$

Proof: (1) Using definition and Table 1, we obtain

$$ASNI(W_n) = \sum_{uv\in E(W_n)}\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3$$

$$= n\left(\frac{(5n-6)(5n-6)}{5n-6+5n-6-2}\right)^3 + n\left(\frac{(5n-6)(2n^2-3n)}{5n-6+2n^2-3n-2}\right)^3$$

$$= n\left[\frac{(5n-6)^2}{10n-14}\right]^3 + n\left[\frac{10n^3-27n^2+18n}{2n^2+2n-8}\right]^3.$$

(2) From definition and using Table 1, we have

$$ASNI(W_n,x) = \sum_{uv\in E(W_n)} x^{\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3}$$

$$= nx^{\left(\frac{(5n-6)(5n-6)}{5n-6+5n-6-2}\right)^3} + nx^{\left(\frac{(5n-6)(2n^2-3n)}{5n-6+2n^2-2n-2}\right)^3}$$

$$= nx^{\left[\frac{(5n-6)^2}{10n-14}\right]^3} + nx^{\left[\frac{10n^3-27n^2+18n}{2n^2+2n-8}\right]^3}.$$

V. FRIENDSHIP GRAPHS

A friendship graph F_n is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. A graph F_4 shown in Figure 2.

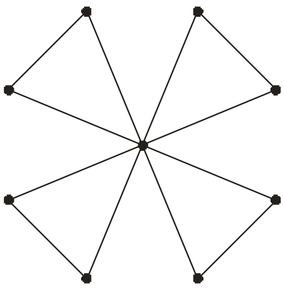

Figure 2. Friendship graph F_4

A graph F_n has $2n$+1 vertices and $3n$ edges. In this graph F_n, we find two types of edges as follows:

$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}$, $|E_1| = n$.

$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}$, $|E_2| = 2n$.

Thus by calculation, in F_n there are two types of status edges as follows:

$E_1 = \{uv \in E(F_n) \mid \sigma(u) = \sigma(v) = 4n-2\}$, $|E_1| = n$.

$E_2 = \{uv \in E(F_n) \mid \sigma(u) = 2n, \sigma(v) = 4n-2\}$, $|E_2| = 2n$.

Therefore by calculation, we obtain that there are two types of status neighborhood edges as given in Table 2.

$\sigma_n(u), \sigma_n(v) \setminus uv \in E(F_n)$	$(6n-2, 6n-2)$	$(6n-2, 2n(4n-2))$
Number of edges	n	$2n$

Table 2. Status neighborhood edge partition of F_n

Theorem 16. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(1) $$ABCSN(F_n) = \frac{n\sqrt{12n-6}}{6n-2} + n\left[\frac{4n^2+n-2}{6n^3-5n^2+n}\right]^{\frac{1}{2}}.$$

(2) $$GASN(F_n) = n + \frac{4n\sqrt{12n^3-10n^2+2n}}{4n^2+n-1}.$$

(3) $$AGSN(F_n) = n + \frac{n(4n^2+n-1)}{\sqrt{12n^3-10n^2+2n}}.$$

Proof: From definition and by using Table 2, we obtain

(1) $$ABCSN(F_n) = \sum_{uv\in E(F_n)} \left[\frac{\sigma_n(u)+\sigma_n(v)-2}{\sigma_n(u)\sigma_n(v)}\right]^{\frac{1}{2}}$$

$$= n\left[\frac{6n-2+6n-2-2}{(6n-2)(6n-2)}\right]^{\frac{1}{2}} + 2n\left(\frac{6n-2+8n^2-4n-2}{(6n-2)(8n^2-4n)}\right)^{\frac{1}{2}}$$

$$= \frac{n\sqrt{12n-6}}{6n-2} + n\left[\frac{4n^2+n-2}{6n^3-5n^2+n}\right]^{\frac{1}{2}}.$$

(2) $$GASN(F_n) = \sum_{uv\in E(F_n)} \frac{2\sqrt{\sigma_n(u)\sigma_n(v)}}{\sigma_n(u)+\sigma_n(v)}$$

$$= n\frac{2\sqrt{(6n-2)(6n-2)}}{6n-2+6n-2} + 2n\frac{2\sqrt{(6n-2)(8n^2-4n)}}{6n-2+8n^2-4n}$$

$$= n + \frac{4n\sqrt{12n^3-10n^2+2n}}{4n^2+n-1}$$

(3) $$AGSN(F_n) = \sum_{uv\in E(F_n)} \frac{\sigma_n(u)+\sigma_n(v)}{2\sqrt{\sigma_n(u)\sigma_n(v)}}$$

$$= \frac{n(6n-2+6n-2)}{2\sqrt{(6n-2)(6n-2)}} + \frac{2n(6n-2+8n^2-4n)}{2\sqrt{(6n-2)(8n^2-4n)}}$$

$$= n + \frac{n(4n^2+n-1)}{\sqrt{12n^3-10n^2+2n}}.$$

In the following theorem, we compute the harmonic status neighborhood index and its polynomial of a friendship graph F_n.

Theorem 17. The F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(1) $$HSN(F_n) = \frac{n}{6n-2} + \frac{2n}{4n^2+n-1}.$$

(2) $$HSN(F_n, x) = nx^{\frac{1}{6n-2}} + 2nx^{\frac{1}{4n^2+n-1}}.$$

Proof: (1) From definition and using Table 2, we deduce

$$HSN(F_n) = \sum_{uv \in E(F_n)} \frac{2}{\sigma_n(u) + \sigma_n(v)}$$
$$= n\frac{2}{6n-2+6n-2} + 2n\frac{2}{6n-2+8n^2-4n}$$
$$= \frac{n}{6n-2} + \frac{2n}{4n^2+n-1}.$$

(2) From definition and by using Table 2, we derive

$$HSN(F_n, x) = \sum_{uv \in E(F_n)} x^{\frac{2}{\sigma_n(u)+\sigma_n(v)}}$$
$$= nx^{\frac{2}{6n-2+6n-2}} + 2nx^{\frac{2}{6n-2+8n^2-4n}}$$
$$= nx^{\frac{1}{6n-2}} + 2nx^{\frac{1}{4n^2+n-1}}.$$

In the following theorem, we compute the symmetric division status neighborhood index and its polynomial of a friendship graph F_n.

Theorem 18. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(1) $$SDSN(F_n) = 2n + \frac{16n^4 - 16n^3 + 13n^2 - 6n + 1}{6n^2 - 5n + 1}.$$

(2) $$SDSN(F_n, x) = nx^2 + 2nx^{\frac{16n^4-16n^3+13n^2-6n+1}{12n^3-10n^2+2n}}.$$

Proof: (1) From definition and using Table 2, we obtain

$$SDSN(F_n) = \sum_{uv \in E(F_n)} \left(\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)} \right)$$
$$= n\left(\frac{6n-2}{6n-2} + \frac{6n-2}{6n-2} \right) + 2n\left(\frac{6n-2}{8n^2-4n} + \frac{8n^2-4n}{6n-2} \right)$$
$$= 2n + \frac{16n^4 - 16n^3 + 13n^2 - 6n + 1}{6n^2 - 5n + 1}.$$

(2) From definition and using Table 2, we have

$$SDSN(F_n, x) = \sum_{uv \in E(F_n)} x^{\frac{\sigma_n(u)}{\sigma_n(v)} + \frac{\sigma_n(v)}{\sigma_n(u)}}$$
$$= nx^{\frac{6n-2}{6n-2} + \frac{6n-2}{6n-2}} + 2nx^{\frac{6n-2}{8n^2-4n} + \frac{8n^2-4n}{6n-2}}$$
$$= nx^2 + 2nx^{\frac{16n^4-16n^3+13n^2-6n+1}{12n^3-10n^2+2n}}.$$

In the following theorem, we compute the inverse sum indeg status neighborhood index and its polynomial of a friendship graph F_n.

Theorem 19. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(1) $$ISSN(F_n) = n(3n-1) + \frac{8n^2(6n^2 - 5n + 1)}{4n^2 + n - 1}.$$

(2) $$ISSN(F_n, x) = nx^{3n-1} + 2nx^{\frac{4n(6n^2-5n+1)}{4n^2+n-1}}.$$

Proof: (1) Using definition and Table 2, we obtain

$$ISSN(F_n) = \sum_{uv \in E(F_n)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)}$$
$$= n\frac{(6n-2)(6n-2)}{6n-2+6n-2} + 2n\frac{(6n-2)(8n^2-4n)}{6n-2+8n^2-4n}$$

$$= \frac{1}{2} n(6n-2) + \frac{8n^2(6n^2-5n+1)}{4n^2+n-1}.$$

(2) From definition and using Table 2, we deduce

$$ISSN(F_n, x) = \sum_{uv \in E(F_n)} x^{\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)}}$$

$$= nx^{\frac{(6n-2)(6n-2)}{6n-2+6n-2}} + 2nx^{\frac{(6n-2)(8n^2-4n)}{6n-2+8n^2-4n}}$$

$$= nx^{3n-1} + 2nx^{\frac{4n(6n^2-5n+1)}{4n^2+n-1}}.$$

In the following theorem, we compute augmented status neighborhood index and its polynomial of a friendship graph F_n.

Theorem 20. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(1) $$ASNI(F_n) = n\left[\frac{(6n-2)^2}{12n-6}\right]^3 + 2n\left[\frac{4n(6n^2-5n+1)}{4n^2+n-1}\right]^3.$$

(2) $$ASNI(F_n, x) = nx^{\left[\frac{(6n-2)^2}{12n-6}\right]^3} + 2nx^{\left[\frac{4n(n^2-5n+1)}{4n^2+n-2}\right]^3}.$$

Proof: (1) From definition and using Table 2, we derive

$$ASNI(F_n) = \sum_{uv \in E(F_n)} \left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3$$

$$= n\left(\frac{(6n-2)(6n-2)}{6n-2+6n-2-2}\right)^3 + 2n\left(\frac{(6n-2)(8n^2-4n)}{6n-2+8n^2-4n-2}\right)^3$$

$$= n\left[\frac{(6n-2)^2}{12n-6}\right]^3 + 2n\left[\frac{4n(6n^2-5n+1)}{4n^2+n-2}\right]^3.$$

(2) Using definition and Table 2, we deduce

$$ASNI(F_n, x) = \sum_{uv \in E(F_n)} x^{\left(\frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u)+\sigma_n(v)-2}\right)^3}$$

$$= nx^{\left(\frac{(6n-2)(6n-2)}{6n-2+6n-2-2}\right)^3} + 2nx^{\left(\frac{(6n-2)(8n^2-4n)}{6n-2+8n^2-4n-2}\right)^3}$$

$$= nx^{\left[\frac{(6n-2)^2}{12n-6}\right]^3} + 2nx^{\left[\frac{4n(6n^2-5n+1)}{4n^2+n-2}\right]^3}.$$

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