

# Towards the problem of homobaricity in bubble dynamics

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**Abstract :** Spherical gas bubbles oscillating radially under the action of an acoustic field in a liquid are considered. The history of homobaricity approach assuming uniformity of pressure inside the bubble but non-uniformity of temperature and density fields is described. Mistakes of some publications in this field are analyzed.

**Keywords:** bubbles, homobaricity, oscillation

## I. INTRODUCTION

Bubbly liquids have found widespread application in conventional and nuclear power engineering, cryogenic technology and chemical engineering.

At present a big number of publications are available in which different aspects of the problem of radial oscillations of gas bubbles in the liquid are studied. It has been revealed [1] that in the case of small oscillations of a bubble within a wide range of the equilibrium values of its radius the heat transfer dominates over other dissipation mechanisms, i.e. viscosity and compressibility of the liquid. The assumptions of the uniformity of all parameters in the bubbles adopted in some of these studies, considerably simplify the problem, but hold under certain restrictions only.

The present work analyzed some of the publications in this field. The detailed review of the publications for gas and vapor bubbles was done in [2, 3].

## II. BASIC ASSUMPTIONS

The assumption of uniform pressure (homobaricity) in the bubble, despite non-uniformity of the temperature and density fields was used for the first time in [4]. A more detailed justification of this assumption and application of it to solving a wide range of problems can be found in [5-9]. Later, other authors also used this idea giving the reference to the original work [4] (see for example [10]).

## III. DISCUSSION OF RESULTS

In paper [11] the idea and even term homobaricity were used without giving any reference to the original publication [4].

Firstly, it is very annoying and incomprehensible that the author of this article is completely unfamiliar with such an abundance of works on the subject of the article, performed in academic schools of academicians L.I. Sedov, R.I. Nigmatulin, V.E. Nakoryakov, V.A. Akulichev [7, 8, 12, 13] (or intentionally ignoring them). In these works, not only free and forced small oscillations of gas bubbles were considered, but also linear and non-linear oscillations of a single gas, steam, vapor-gas bubbles, and vibrations of vapor bubbles in binary systems were also studied in detail considering many other factors, such as fluid viscosity, capillary forces on the interface, acoustic radiation. In addition, Russian researchers have published a huge number of works on the theory of linear and nonlinear waves in bubbly systems with a detailed description of all the thermophysical processes inside the bubbles themselves, as well as related to their interaction with the carrier fluid. Many results of these studies are presented in a two-volume monograph by R.I. Nigmatulin "Dynamics of multiphase media" [7].

As for the article itself, it was not done carefully but artfully. Let us analyze some details of this article.

The hypothesis of homobaricity is accepted in bubbles. It replaces the impulse equation and the description of radial gas motions in bubbles when solving the equation. Since the inhomogeneity of the temperature  $T_g$  in the bubble is taken into account, under the condition of homobaricity, according to the equation of state  $p = \rho_g \cdot R_g \cdot T_g$ , it is therefore necessary to take into account the density inhomogeneity. This circumstance, in turn, determines the use of the continuity equation inside the bubble. Instead, the author proposes to write the law of conservation of mass as

$$\frac{4}{3} \frac{\pi R^3}{V} = const \quad (1)$$

Here  $R$  is the bubble radius,  $p$  - pressure,  $T$  - temperature,  $\rho$  is density. Subscript  $g$  refers to the parameters of gas.  $R_g$  is the gas constant. So, in [11] there is contradiction to the equation of state. If pressure field inside the bubble is uniform, but temperature field is not uniform, then density field must be nonuniform.

Moreover, the parameter  $V$  is not determined in the article [11], only by its meaning it is possible to guess that  $V$  is the average specific volume for the entire mass of gas in the bubble.

For a clearer presentation of further errors of the author of the article [11], we denote by  $V$  the local value of the specific volume associated with the density:  $V = 1 / \rho_g$ . The average value of the specific volume, we denote by  $\bar{V}$  which is associated with the density of the gas in the bubble by the following expression:

$$\bar{V} = 1 / \bar{\rho}_g, \quad \bar{\rho}_g = \left( \int_0^R 4\pi \rho_g r^2 dr \right) / \left( \frac{4}{3} \pi R^3 \right) \quad (2)$$

Then expression (1) should be written as

$$\frac{4}{3} \frac{\pi R^3}{\bar{V}} = const \quad (3)$$

The equation of state for gas  $p = \rho_g \cdot R_g \cdot T_g$ , after linearization, near equilibrium state (0) has the form:

$$\frac{p'}{p_0} = \frac{\rho'_g}{\rho_{g0}} + \frac{T'}{T_0} \quad (4)$$

The density perturbation  $\rho'_g$  with the specific volume perturbation  $V'$  are related as

$$\frac{\rho'_g}{\rho_{g0}} = - \frac{V'}{V_0} \quad (5)$$

With this in mind, equation (4) can be written as

$$\frac{p'}{p_0} + \frac{V'}{V_0} = \frac{T'}{T_0} \quad (6)$$

Equation (3), taking into account the fact that  $\bar{V}_0 = V_0$ , after linearization, has the form

$$\frac{\bar{V}'}{V_0} = 3 \frac{R'}{R_0} \quad (7)$$

Here, we specially note that  $\bar{V}'$  is the perturbation of the average specific volume throughout the bubble, and the parameter  $V'$  is the perturbation of the local density in the bubble (which in this problem depends on time and on the coordinate  $r$ ).

Further, assuming that the pressure perturbations of the incompressible fluid at infinity changes according to a sinusoidal law (which corresponds to forced oscillations), a solution is found for the distribution of temperature perturbations (which has long been known [14,15]). Subsequently, this temperature distribution is averaged over the entire volume of the bubble. And then, this averaged expression for temperature is further presented in formula (6) of the present article or formula(9) from [11] which is written for local values of temperature (where  $V'$  and  $T'$  depend on  $r$ ) and it is stated in [11] that the expression for pulsations of perturbations of the average specific volume is obtained (formula (33) in [11]). Consequently, expression (33) in the article [11] is obtained completely incorrectly. Further, this marked expression for the specific volume is substituted into the formula (10) of the article [11] (in our article it is equation (7)). And according to the author [11], an expression is obtained that relates the perturbation of the bubble radius and specific volume (formula (34) in the article [11]). By virtue of the above, it is absolutely not true.

Thus, in all calculations, it is accepted that  $\bar{V}'$  and  $V'$  is the same parameter  $V'$ . In this regard, all the formulas for the effective values of the polytropic coefficients, for the problems of forced bubble oscillations, and the corresponding graphic illustrations are incorrect (apparently, the difference between the author's graphs and those obtained in A. Prosperetti's work [10] is connected with the aforementioned error of the author [11]). In this regard, all other graphic illustrations are very doubtful.

In fact, all of these dependencies presented in the article [11] and their graphical representations can be obtained using well-known solutions that were studied in detail in the works mentioned at the beginning of our paper.

We believe that the author [11] took up the solution of a known problem and completed it in a semi-artisanal way.

#### IV. CONCLUSION

The question of homobaricity (pressure uniformity) in bubbles is analyzed. It is shown that some authors are not using homobaricity properly and solve the related problems in a semi-artisanal way.

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