Weakly Nonlinear Stability Analysis of Double Diffusive Convection in a Porous Medium with Magnetic Field and Concentration-Based Internal Heating

[1]Liberty Ebiwareme, [2]Chigozie Israel-Cookey
Department of Mathematics, Department of Physics, Rivers State University, Port Harcourt, Nigeria.

Abstract: *The Steady state finite amplitude of double diffusive convection with effects of concentration-based internal heating and magnetic field is investigated. The governing equation of the flow are reduced using the Darcy-Brinkman model in a porous medium. The result shows, both Darcy number and magnetic field increases the heat and mass transfer in the system, whereas Vadasz and other parameters decreases the mass transfer, but the heat transfer decreases as the magnetic parameter increases.*

Keywords: *Double diffusive convection, Weakly nonlinear analyses, Darcy-Brinkman, Concentration base, internal heat, porous medium*

I. INTRODUCTION

Double diffusive convection is simply defined as the phenomenon of convective heat transfer in a fluid saturated porous layer involving two components possessing different diffusivities which is driven by temperature and buoyancy [1]. This phenomenon became popular among scholars decades ago and has been extensively studied. Comprehensive treatment of double diffusive studies can be found in the following works [2], [3], [4], [5] and [6].

Motivated by the enormous applications of double diffusive convection found in science, engineering and others, researchers have for the past decades given this phenomenon attention. [7], [8], [9] and [10]. The myriads of applications of double diffusive convection includes oceanography, Solar Ponds modelling, magma chambers, salt fingers, ground water hydrology, petroleum reservoir, coal combustors, insulators, and thermal engineering. [11], [12], [13], [14], [15], [16], [17] have examined the linear and nonlinear stability analysis of double diffusive convection in a Maxwell fluid saturated porous layer with internal heat source. Similarly, the linear and nonlinear double-diffusive convection in a saturated anisotropic porous layer with Soret and internal heat source has been theoretically investigated by [18] using the stability analysis technique. The study reveals that, the stability of the system was enhanced by the presence of the thermal anisotropic parameter and concentration Rayleigh number, whereas Lewis number, internal Rayleigh number and mechanical anisotropic parameters triggers instability in the system. Additionally, the values of the Soret parameter significantly affected the marginal curves for stationary convection while negligible for the oscillatory mode.

[19] used the generalised Darcy model to investigate the linear and nonlinear double-diffusive convection in a fluid saturated anisotropic porous layer using the truncated Fourier for the finite amplitude modes and the linear stability analysis for the stationary and oscillatory convection. [20] considered the weakly nonlinear stability analysis double-diffusive convection with cross-diffusion in a fluid saturated porous medium. They found that, both fingers and diffusive instabilities occur simultaneously, finite amplitude occur when gradients of the initial properties stabilize, and the infinitesimal oscillation instability was enclosed the finite amplitude convection. [21] studied the linear and nonlinear double-diffusive convection in a fluid saturated porous layer with cross-diffusion effects using the Darcy model. It was found that Vadasz number exhibit dual effect on the threshold of oscillatory convection, while Lewis number and Solute Rayleigh number delay both the stationary and

oscillatory convection and Dufour together with Soret parameters stabilizes the system for stationary modes and destabilize in the oscillatory modes.

[22] carried out an analytical study of linear and nonlinear double-diffusive convection in a fluid saturated anisotropic porous layer with Soret effect. The result shows that, the system was stabilize for the stationary mode in the presence of Lewis number, while the trend was reversed in the oscillatory and finite amplitude modes. Also, the heat and mass transfer were enhanced by Lewis and anisotropic parameters, whereas the Soret parameter decreases the heat transfer and increased the mass transfer of the system. Pranesh et al analysed the linear and weakly nonlinear stability analyses of double-diffusive electro-convection in a micropolar pond. The study reveals that the heat and mass transfer increased as the electric Rayleigh number, electric field and concentration of suspended particles increased. The linear and weakly non-linear analyses of gravity modulation and electric field on the onset of Rayleigh-Bernard convection in a micropolar fluid has been investigated by [23]. [24] carried out a weak nonlinear oscillatory convection in a nonuniform heating porous medium with throughflow. It was found that the heat and mass transfer can be controlled and maintained by factors outside the system. [25] have examined the nonlinear double diffusive convection in a couple stress fluid with rotational modulation. The weak nonlinear thermal instabilities under vertical magnetic field, temperature modulation and heat source has been studied by [26]. [27] investigated the weakly nonlinear stability analysis of a thin magnetic fluid during spin coating. The nonlinear stability analysis of a Darcy flow with viscous dissipation has been considered by [28]. The onset of magnetoconvection in a rotating Darcy-Brinkman porous layer heated from below with temperature dependent heat source has been studied using linear stability analysis by [29]

However, none of the previous studies in our knowledge have carried out a weakly nonlinear stability analysis using the Darcy model, incorporating magnetic field and heat source in a fluid saturated porous medium. Therefore, in this present study, we consider the weakly nonlinear Stability analysis of double-diffusive convection in a porous medium with magnetic field and Concentration-Based Internal Heating. Our specific objective in this study is to use the minimal representation of Fourier series to perform weakly nonlinear stability analysis and ascertain the effect of the governing parameters on the heat and mass transfer via the Nusselt and Sherwood numbers

II. MATHEMATICAL FORMULATION

The governing equations of continuity, momentum, energy and Boussinesq approximation of the flow are given as follows

Continuity equation

$$\vec{\nabla}^{*}.\vec{V} = 0 \qquad (1)$$

Momentum equation

$$\frac{\rho_0}{\phi}\frac{\partial \vec{V}^{*}}{\partial t^{*}} = -\vec{\nabla}P^{*} + \rho_0 g[\beta_T(T^{*} - T_0) + \beta_c(c^{*} - c_0)]\vec{k} - \frac{\mu}{\kappa}\vec{V}^{*} + \mu_e {\vec{\nabla}^{*}}^{2}\vec{V}^{*} + F_L \qquad (2)$$

Energy equation

$$A\frac{\partial T^{*}}{\partial t^{*}} + (\vec{V}^{*}.\vec{\nabla})T^{*} = \alpha_T {\Delta^{*}}^{2} T^{*} + Q(c^{*} - c_0) \qquad (3)$$

Solute concentration

$$\phi\frac{\partial c^{*}}{\partial t^{*}} + (\vec{V}^{*}.\vec{\nabla})c^{*} = \kappa_c {\Delta^{*}}^{2} c^{*} \qquad (4)$$

Boussinesq approximation

$$\rho(T^{*}, c^{*}) = \rho_0[1 - \alpha_T(T^{*} - T_0) + \alpha_c(c^{*} - c_0)] \qquad (5)$$

Ohm' s Law

$$\vec{J}^{*} = \alpha_c[\vec{E}^{*} + \vec{V}^{*} \times \vec{B}^{*}], \vec{\nabla}^{*}.\vec{J}^{*} = 0 \qquad (6)$$

Subject to the boundary conditions

$$\vec{V} = 0 \text{ on } z^* = 0, h \tag{7a}$$

$$\vec{T}^* = T_0 + \Delta T,\ c^* = c_0 + \Delta c \text{ on } z^* = 0 \tag{7b}$$

$$\vec{T}^* = T_0,\ c^* = c_0 \text{ on } z^* = h \tag{7c}$$

For electrically insulating boundaries, the electrostatic potential is constant, hence $\vec{E} = 0$. Thus, the current density in Eq. (6) reduced to

$$\vec{J}^* = \alpha_c(\vec{V}^* \times \vec{B}^*) \tag{8}$$

Then the Lorentz force become

$$\vec{F}_L = \vec{J}^* \times \vec{B}^* \tag{9}$$

Substitution of Eq. (8), gives

$$\vec{F}_L = \alpha_c B_0^2 (u^*, v^*, 0) \tag{10}$$

Using the following scales $h, \frac{h^2 A}{\alpha_T}, \frac{\alpha_T}{h}, \frac{\kappa}{(\mu\alpha_T)}$ representing length, time, velocity and pressure. Similarly, $T = \frac{(T^* - T_0)}{\Delta T}, c = \frac{(c^* - c_0)}{\Delta c}$ and $\varepsilon = \frac{\phi}{A}$ denoting the temperature, concentration and porosity constant respectively.

The dimensionless equations governing the system become.

$$\vec{\nabla}.\vec{V} = 0 \tag{11}$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\vec{V} + Ha^2(u, v, 0) = -\nabla P + Ra\vec{k} - Rsc\vec{k} \tag{12}$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T = \nabla^2 T + Ric \tag{13}$$

$$Le\varepsilon\frac{\partial c}{\partial t} + Le(\vec{V}.\vec{\nabla})c = \nabla^2 c \tag{14}$$

The boundary conditions are

$$w = 0, T = 1, c = 1 \qquad \text{on} \quad z = 0 \tag{15}$$

$$w = 0, T = 0, c = 0 \qquad \text{on} \quad z = 1 \tag{16}$$

Basic State

Assuming the basic state of the system to be quiescent and described by

$$V_b = 0, T = T_b(z), c = c_b(z), p = p_b(z) \tag{17}$$

Substituting Eq. (17) into Eqs. $(11) - (14)$ using the boundary condition (15) and (16) yield the equations governing the basic state as follows

$$\frac{d^2 T_b}{dz^2} + Ric_b = 0 \tag{18}$$

$$\frac{d^2 c_b}{dz^2} = 0 \tag{19}$$

$$\frac{dP_b}{dz} = RaT_b - Rsc_b \tag{20}$$

Subject to the boundary conditions

$$T_b = c_b = 1 \qquad \text{on} \quad z = 0 \tag{21}$$

$$T_b = c_b = 0 \qquad \text{on} \quad z = 1 \tag{22}$$

The integration of Eqs. (18)$-$ (20) under the boundary conditions Eqs. (21) and (22), yield the basic temperature, $T_b(z)$, concentration , $c_b(z)$, and pressure, $P_b(z)$ respectively as follows

$$\left.\begin{array}{c} T_b(z) = \frac{1}{6}[6(1-z) + (2z - 3z^2 + z^3)] \\ c_b(z) = 1 - z \\ P_b(z) = \int (Ta\, T_b(z) - Rsc_b(z))dz \end{array}\right\} \tag{23}$$

Linearization and perturbation Solution

Superimposing small disturbances to study the stability of the base state of the form

$$\vec{V} = \vec{V}_b + \vec{v}, p = p_b(z) + p, T = T_b(z) + \theta, c = c_b(z) + \varphi \tag{24}$$

where $\vec{v}, p, \theta, \varphi$ represents the perturbed quantities

Substituting Eq. (24) into Eqs. (11) to (14) using Eqs. (15)$-$ (16), we obtained the linearized equations as

$$\vec{\nabla}.\vec{v} = 0 \tag{25}$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\vec{v} + Ha^2(u, v, 0) = -\nabla p + Ra\theta\vec{k} - Rs\varphi\vec{k} \tag{26}$$

$$\frac{\partial\theta}{\partial t} + (\vec{v}.\vec{\nabla})\theta + f(z)w = \vec{\nabla}^2\theta + Ri\varphi \tag{27}$$

$$Le\varepsilon\frac{\partial\varphi}{\partial t} + Le(\vec{v}.\nabla)\theta - Lew = \vec{\nabla}^2\varphi \tag{28}$$

$$f(z) = \frac{\partial T_b}{\partial z} = -1 + \frac{Ri}{6}(2 - 6z + 3z^2) \tag{29}$$

Boundary conditions are:

$$w = \theta = \varphi = 0 \tag{30}$$

III. METHOD OF SOLUTION

Weakly Nonlinear Analysis

For the nonlinear analysis, we restrict our study to only two- dimensional rolls, where all inherent physical quantities do not dependent on y. Taking the stream function of the form

$$u = \frac{\partial\psi}{\partial z}, w = -\frac{\partial\psi}{\partial x} \tag{31}$$

Substituting Eq. (31) into Eqs. (26) $-$ (28). By eliminating the pressure term, the linearized equations reduced to

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\nabla^2\psi + Ha^2\frac{\partial^2\psi}{\partial z^2} = -Ra\frac{\partial\theta}{\partial x} + Rs\frac{\partial\psi}{\partial x} \tag{32}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = Ri\varphi + f(z)\frac{\partial\psi}{\partial x} + \left(\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right) \tag{33}$$

$$\left(Le\varepsilon\frac{\partial}{\partial t} - \nabla^2\right)\varphi = Le\left(\frac{\partial\psi}{\partial x}\frac{\partial\varphi}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial\varphi}{\partial x}\right) - Le\frac{\partial\psi}{\partial x} \tag{34}$$

Now, the boundary conditions defined in terms of the stream function reduce to

$$\psi = \theta = \varphi = 0 \text{ on } z = 0, 1 \tag{35}$$

To obtain information on the rates of heat and mass transfers, we assume that the basic circulation is undistorted except the temperature and specie concentration fields which are distorted close to the threshold of convection (Mamou & Vasseur, 1999). Hence, we describe the finite amplitude free convection using minimal Fourier representation in the form

$$\psi(x,z,t) = \psi_1(t)\, sin(\,ax)\, sin(\,\pi z) \qquad (36)$$

$$\theta(x,z,t) = \theta_1(t)\, cos(\,ax)\, sin(\,\pi z) + \theta_2(t)\, sin(\,2\pi z) \qquad (37)$$

$$\varphi(x,z,t) = \varphi_1(t)\, cos(\,ax)\, sin(\,\pi z) + \varphi_2(t)\, sin(\,2\pi z) \qquad (38)$$

where $\psi_1(t), \theta_1(t), \theta_2(t), \varphi_1(t), \varphi_2(t)$ are real amplitudes to be determined by the dynamics of the system.

Substituting Eqs. $(36) - (38)$ into the coupled nonlinear partial differential equation in Eqs. $(32) - (34)$ and equating coefficients up to $2\pi z$ inclusive and ignoring higher frequency terms, we obtain

$$\frac{d\psi_1}{dt} = -\frac{Va}{\delta}[\delta\psi_1 + \pi^2 Ha^2\psi_1 + aRa\theta_1 - aRs\varphi_1] \qquad (39)$$

$$\frac{d\theta_1}{dt} = -\delta\theta_1 + f(z)a\psi_1 - 2a\pi\theta_2\psi_1 + R_1\varphi_1 \qquad (40)$$

$$\frac{d\theta_2}{dt} = \frac{a\pi}{2}\theta_1\psi_1 - 4\pi^2\theta_2 + Ri\varphi_2 \qquad (41)$$

$$Le\varepsilon\frac{d\varphi_1}{dt} = -\delta\varphi_1 - Lea\psi_1 - 2Le\pi a\varphi_2\psi_1 \qquad (42)$$

$$Le\varepsilon\frac{d\varphi_2}{dt} = -4\pi^2\varphi_2 + Le\frac{a\pi}{2}\varphi_1\psi_1 \qquad (43)$$

where $\delta = \pi^2 + a^2$

IV. STEADY FINITE AMPLITUDE SOLUTION

For the case of steady-state solutions, we set the time derivatives on the left-hand side of Eqs. $(39) - (43)$ equal to zero. Then the result become

$$\lambda\psi_1 + aRa\theta_1 - aRs\varphi_1 = 0 \qquad (44)$$

$$f(z)a\psi_1 - \delta\theta_1 - 2\pi a\psi_1\theta_2 + Ri\varphi_1 = 0 \qquad (45)$$

$$\frac{a\pi}{2}\psi_1\theta_1 - 4\pi^2\theta_2 + Ri\varphi_2 = 0 \qquad (46)$$

$$-2Le\pi a\varphi_2\psi_1 - \delta\varphi_1 - Lea\psi_1 = 0 \qquad (47)$$

$$Le\frac{a\pi}{2}\varphi_1\psi_1 - 4\pi^2\varphi_2 = 0 \qquad (48)$$

where $\lambda = \delta + \pi^2 Ha^2, \quad \delta = \pi^2 + a^2$

On solving for the amplitudes $\theta_1, \theta_2, \varphi_1, \varphi_2$ in terms of ψ_1, we obtain

$$\theta_1 = \frac{a^3Le^2(4F\pi^2+Ri)\psi_1^3-16a\pi^2(LeRi-F(z)\delta)\psi_1}{\pi^2(4\delta+a^2\psi_1^2)(4\delta+a^2Le^2\psi_1^2)} \tag{49}$$

$$\theta_2 = \frac{a^2[\pi^2 f(z)((4\delta+a^2Le^2\psi_1^2)-LeRi(4\pi^2+Le\delta]\psi_1^2}{2\pi^3(4\delta+a^2\psi_1^2)(4\delta+a^2Le^2\psi_1^2)} \tag{50}$$

$$\varphi_1 = -\frac{4aLe\psi_1}{4\delta+a^2Le^2\psi_1^2} \tag{51}$$

$$\varphi_2 = -\frac{a^2Le^2\psi_1^2}{8\pi\delta+2a^2Le^2\pi\psi_1^2} \tag{52}$$

Substituting Eqs. (49) and (51) for θ_1 and φ_2 into Eq. (44) yields

$$\psi_1(a_2\psi_1^4 + a_1\psi_1^2 + a_0) = 0 \tag{53}$$

where $a_2 = Le\lambda\pi^2a^4$

$a_1 = a^4Le^2(8\pi^2Le^2 + Ri)Ra + 4a^4\pi^2[LeRs + \delta\lambda(1 + Le^2)]$

$a_0 = 16\pi^2\delta^2\lambda + 16a^2\pi^2[f(z)\delta - LeRi]Ra + 16a^2\pi^2Le\delta Rs$

The required root of Eq.(53) is given by

$$\psi_1^2 = \frac{-a_1+\sqrt{a_1^2-4a_2a_0}}{2a_2} \tag{54}$$

Equating the discriminant in Eq.(54) equal to zero.

i.e, $a_1^2 - 4a_2a_0 = 0$, yields a quadratic equation which gives the expression for the finite-amplitude Rayleigh number, $Ra^{F,}$. This characterizes the onset of finite amplitude steady motion given by

$$d_2(Ra^F)^2 + d_1(Ra^F) + d_0 = 0 \tag{55}$$

where

$d_2 = a^8Le^4(4F(z)\pi^2 + Ri)^2$

$d_1 = 8a^6\pi^2Le^2Ri[Le(a^2Rs + 8\pi^2\lambda) + \delta\lambda(1 + Le^2] + 32F(z)a^6\pi^4Le^2[a^2LeRs + \delta\lambda(Le^2 - 1]$

$d_0 = 16a^4\pi^4[a^2LeRs + \delta\lambda(1 - Le^2]^2$

V. HEAT AND MASS TRANSPORT

The determination of the Nusselt and Sherwood numbers, which gives quantitatively the heat and mass transport in the system play a pivotal role for the study of convection in porous medium. This is so because at the threshold of convection, the increase in thermal Rayleigh number, Ra is more readily detected by its effect on the heat and mass transfer in the system. However, heat and mass transport are by only due to conduction alone in the basic state. Therefore, in this section, we will attempt to estimate the heat and mass transfer occurring at the lower boundary ($z = 0$) in terms of the Nusselt number, Nu and the Sherwood number, Sh respectively.

The Nusselt number, Nu and Sherwood number, Sh are defined below following (Gupta and Singh 2014) as

$$Nu = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial \theta}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial T_b}{\partial z} dx}\right]_{z=0} \tag{56}$$

$$Sh = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial \varphi}{\partial z} dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial c_b}{\partial z} dx}\right]_{z=0} \tag{57}$$

Substituting the values of $T_b(z), c_b(z), \theta$ and φ from Eqs. $(36), (37)$ and (38) in Eqs. $(56) - (57)$, we obtain

$$Nu = 1 + \frac{\left(\frac{2\pi}{a}\right)^2 \theta_2}{\frac{2\pi}{a}\left(\frac{Ri}{3} - 1\right)} = 1 - \frac{6\pi\theta_2}{3 - Ri} \tag{58}$$

$$Sh = 1 - 2\pi\varphi_2 \tag{59}$$

Substituting the values of θ_2 and φ_2 into Eqs. (58) and (59), give the expressions for Nu and Sh in terms of ψ_1 as

$$Nu = 1 + \frac{3a^2}{(3 - Ri)\pi^2}\left[\frac{LeRi(4\pi^2 + Le\delta) - \pi^2 f(z)(4\delta + a^2 Le\psi_1^2)}{(4\delta + a^2\psi_1^2)((4\delta + a^2 Le\psi_1^2)}\right] \tag{60}$$

$$Sh = 1 + \left[\frac{a^2 Le^2 \psi_1^2}{4\delta + 2a^2 Le^2 \psi_1^2}\right] \tag{61}$$

VI. ANALYSIS OF RESULT

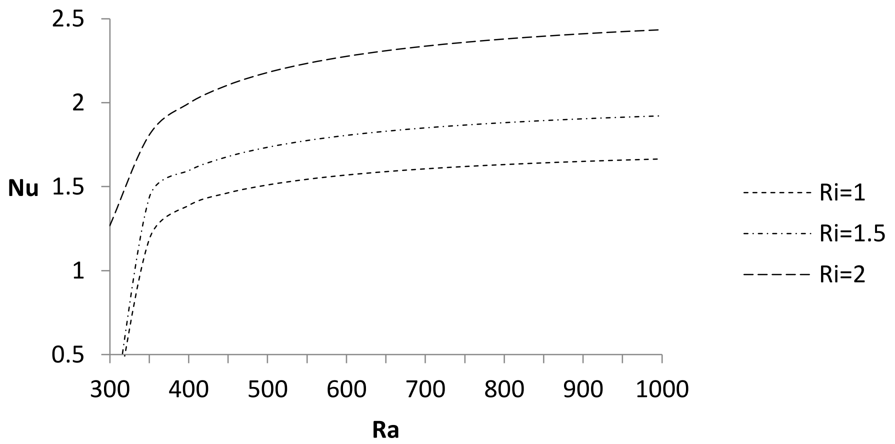

Figure 1. Influence of internal heat parameter, $\boldsymbol{Ri}$ on Nusselt number for fixed values of $\boldsymbol{Rs} = \mathbf{100}, \boldsymbol{a} = \mathbf{1}, \boldsymbol{Ha} = \mathbf{1}, \boldsymbol{Le} = \mathbf{2.0}$ for stationary convection

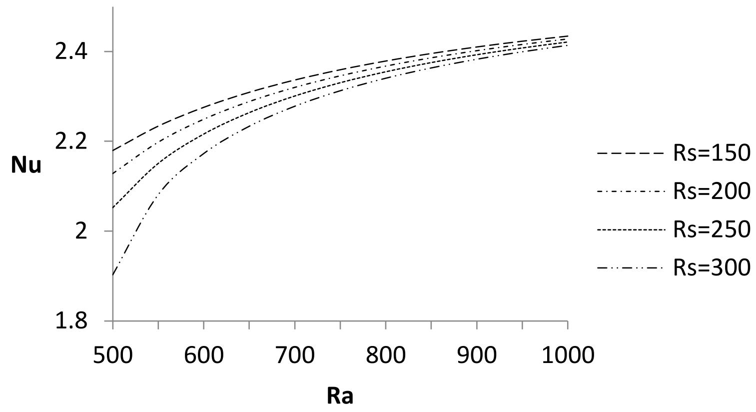

Figure 2. Influence of solutal Rayleigh number, $\boldsymbol{Rs}$ on Nusselt number for fixed values of $\boldsymbol{Ri} = \mathbf{2}, \boldsymbol{a} = \mathbf{1}, \boldsymbol{Ha} = \mathbf{1}, \boldsymbol{Le} = \mathbf{2.0}$ for stationary convection

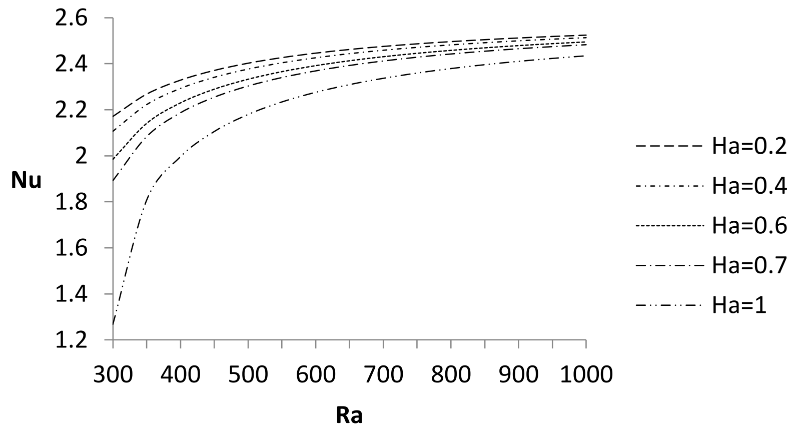

Figure 3. Influence of magnetic field parameter, $\boldsymbol{Ha}$ on Nusselt number for fixed values of $\boldsymbol{Rs} = \mathbf{100}, \boldsymbol{a} = \mathbf{1}, \boldsymbol{Ri} = \mathbf{2}, \boldsymbol{Le} = \mathbf{2.0}$ for stationary convection

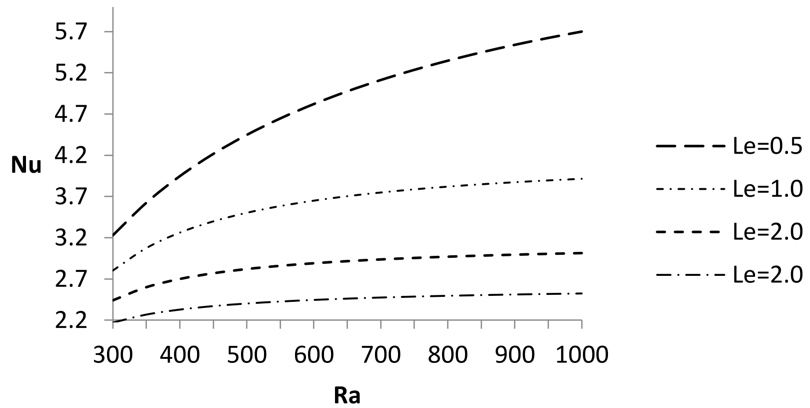

Figure 4. Influence of Lewis number, $\boldsymbol{Le}$ on Nusselt number for fixed values of $\boldsymbol{Rs} = \mathbf{100}, \boldsymbol{a} = \mathbf{1}$,

$\boldsymbol{Ri} = \mathbf{2}, \boldsymbol{Ha} = \mathbf{0.2}$ for stationary convection

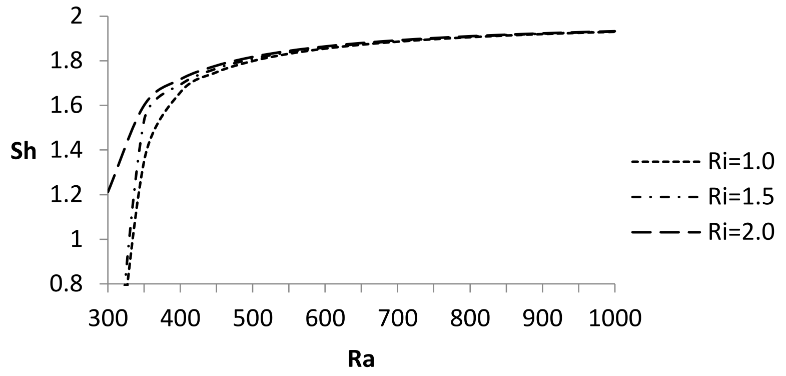

Figure 5. Influence of internal heat parameter, $\boldsymbol{Ri}$ on Sherwood number, for fixed $\boldsymbol{Rs = 100, a = 1, Ha = 1, Le = 0.2}$ for oscillatory convection

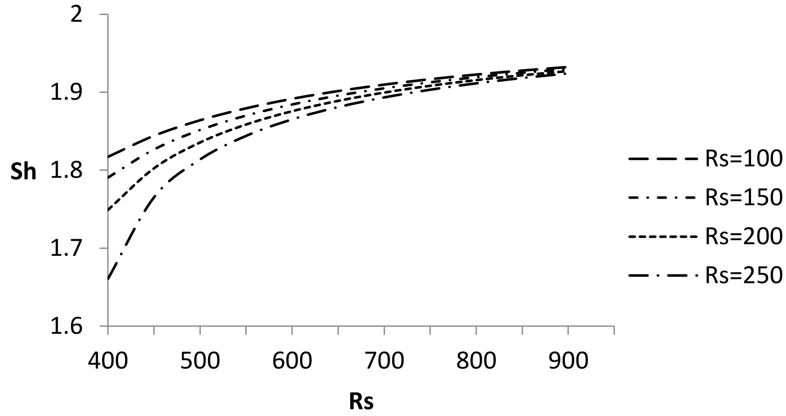

Figure 6. Influence of solutal Rayleigh number, $\boldsymbol{Rs}$ on Sherwood number, for fixed values of $\boldsymbol{Ri = 2, a = 1, Ha = 1, Le = 2}$ for oscillatory convection

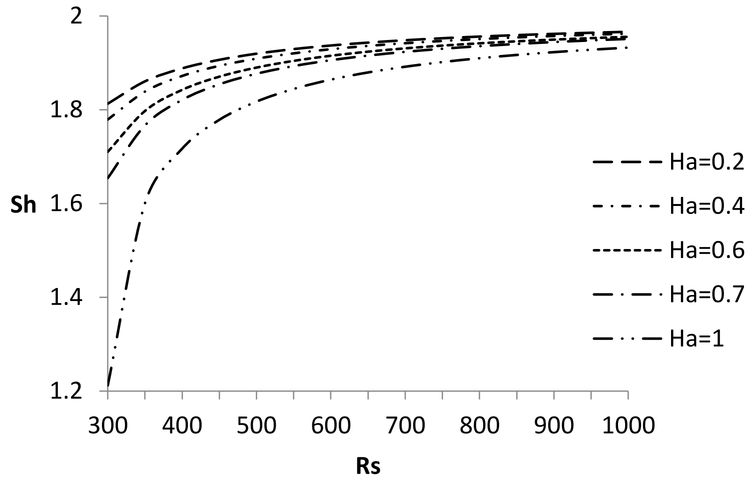

Figure 7. Influence of magnetic field parameter number, $\boldsymbol{Ha}$ on Sherwood number, for fixed $\boldsymbol{Rs = 100, a = 1, Ri = 2, Le = 2.0}$ for oscillatory convection

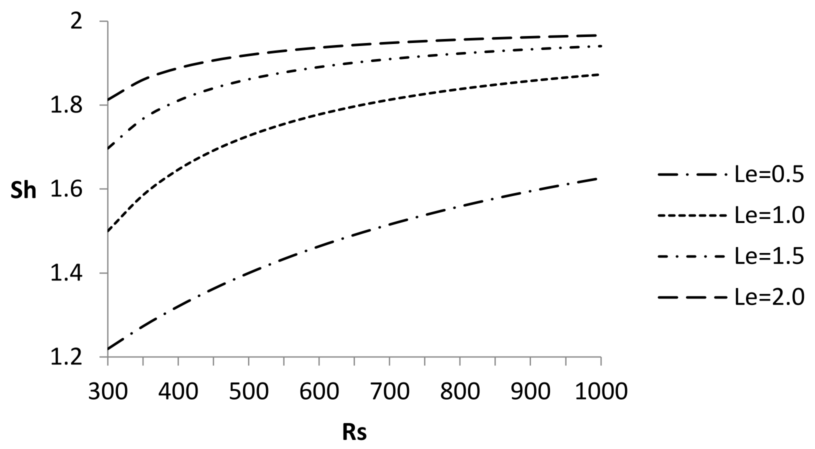

Figure 8. Effect of Lewis number, $\boldsymbol{Le}$ on Sherwood number, $\boldsymbol{Sh}$ for fixed values of $\boldsymbol{Rs = 100, a = 1, Ha = 0.2, Ri = 2.0}$ for oscillatory convection.

VII. DISCUSSION OF RESULTS

Figure 1 show the effect of variations in the internal heat parameter, Ri on the thermal Rayleigh number, Ra for stationary convection and fixed values of $Rs = 100, Ha = 1, Le = 2.0, a = 1$. It is evident that, increase in the internal heat parameter, Ri increases the value of Nu**.** This is an indication that, internal heat parameter has the effect of increasing the heat transfer on the system.

Figure 2 depict the influence of solutal Rayleigh number, Rs for stationary convection on the thermal Rayleigh number, Ra for fixed values of $Ha = 1, Le = 2.0, Ri = 2, a = 1$. The result show that, increase in the solutal Rayleigh number increases the value of the Nusselt number. This implies that, solutal Rayleigh number cause an increase in the heat transfer of the system.

Figure 3 show the effect of the variation of magnetic field, Ha on the thermal Rayleigh number, Ra for stationary convection with fixed $Rs = 100, Ri = 2, Le = 2.0, a = 1$. It is found that increase in magnetic field decreases the value of Nu. This show that, magnetic field parameter decreases the heat transfer in the system.

Figure 4 show the influence of Lewis number, Le on thermal Rayleigh number, Ra for with fixed values of $Rs = 100, Ha = 0.2, Ri = 2, a = 1$ for stationary convection. The result shows that, increase in the Lewis number, Le increases the value of the Nusselt number, Nu. This indicates, Lewis number increases the rate of heat transfer on the system.

Figure 5 show the influence of the internal heat source parameter, Ri on thermal Rayleigh number, Ra for fixed values of $Rs = 100, Ha = 1, Le = 2.0, a = 1$. The result show that, increase in the internal heat parameter increases the mass transfer in the system.

Figure 6 show the influence of the solutal Rayleigh number, Rs on thermal Rayleigh number, Ra for fixed values of $Ri = 2, Ha = 1, Le = 2.0, a = 1$ for oscillatory convection. The result indicates, increase in the solutal Rayleigh number decreases the mass transfer in the system.

Figure 7 depicts the influence of magnetic field, Ha on the thermal Rayleigh number, Ra with fixed values of $Rs = 100, Ri = 2, Le = 2.0, a = 1$ for oscillatory convection. The result shows that, increase in magnetic field decreases the mass transfer in the system. This implies that, magnetic field stabilize the system.

Figure 8 depict the effect of Lewis number, Le on thermal Rayleigh number, Ra with fixed values of $Rs = 100, Ha = 0.2, Ri = 2, a = 1$ for oscillatory convection. The result show that, increase in the value of the Lewis number, Le increases the Sherwood number, Sh. Hence, Lewis number increases the mass transfer in the system.

REFERENCES

[1] Gupta, V.K., and Singh, A.K. (2014). Double-diffusive reaction-convection in a viscous fluid layer. Journal of Industrial Mathematics, 6(4), Article ID: IJIM-00488, 12 Pages

[2] Joseph, D.D. (1976). Stability of Fluid Motions. Vol. 1 and 2. Springer-Verlag, Berlin

[3] Nield, D.A., and Bejan, A. (2017). Convection in Porous Media, Fifth edition, Springer-Verlag, New York, US.

[4] Ingham, D. B., and Pop, I. (2005). Transport Phenomena in Porous Media, Vol. III, Pergamon, Oxford

[5] Chandrasekhar, S. (1961). Hydrodynamic and Hydromagnetic Stability. Clarendon Press, Oxford, UK.

[6] Vafai, K. (2005). Handbook of Porous Media, second edition. Taylor and Francis Group, LLC, US

[7] Malashetty, M.S., and Gaikward, S.N., and Swamy, M. (2006).

[8] Wang, C.Y. (2002). Transport of Porous Media, 46(2002), 37.

[9] Hayat, T., Ali, N., and Asghar, S. (2007). Physics Letters A 363(2007) 97

[10] Bahloul, A., Boutana, N., and Vasseur, P. (2003). Journal of Fluid Mechanics, 491(2003) 325

[11] Narayan, M., Sibanda, P., and Motsa, S.S., and Sidhdeswar, P.G. (2012). On the Double-diffusive convection and cross-diffusion effects in a horizontal wavy surface in a porous medium. Boundary vale Problems, 2012, 1/88.

[12] Stern, M.E. (1960). The Salt fountain and Thermohaline convection. Tellus, 172-175.

[13] Akbarzadeh, A., and Manins, A. (1988). Convective Layers generated by side Walls in Solar Ponds. Sol. Energy, 41(6), 521-529.

[14] Stern, M.E. (1969). Collective instability of Salt fingers. Journal of Fluid Mechanics, 35, 209-218.

[15] Fernando, H.J.S., and Brandt, A. (1994). A recent Advances in Double-diffusive convection. Application of Mechanical Review, 47, c1-c7

[16] Huppert, H.E., and Sparks, R.S.J. (1984). Double-diffusive convection due to crystallization in Magma. Annual Review of Earth Planet Science, 12, 11-37.

[17] Zhao, M., Zhang, Q., and Shaowei, W. (2014). Linear and Nonlinear Stability Analysis of Double-diffusive convection in a Maxwell fluid saturated porous layer with Internal heat Source. Journal of Applied Mathematics, Article ID, 489279, 12 Pages.

[18] Altawallbeh, A. A., Bhadauria, B.S., and Hashim, I. (2013). Linear and Nonlinear Double-diffusive convection in a saturated porous layer with Soret and Internal heat source, 59(2013), 103-111.

[19] Malashetty, M.S., and Swamy, M. (2005). Linear and Non-linear double diffusive convection in a fluid saturated anisotropic porous layer. Proceedings of Internal Conference on Advances in Applied Mathematics, 2005, 253-264.

[20] Rudraiah, N., and Siddheshwar, P.G. (1998). A Weak nonlinear stability analysis of double diffusive convection with cross-diffusion in a fluid saturated porous medium. Heat and Mass Transfer, 33(4), 287-293.

[21] Malashetty, M.S., and Biradar, B.S. (2012). Linear and nonlinear double-diffusive convection in a fluid-saturated porous medium with cross-diffusion effects. Transport of Porous Medium, 9(2012), 649-675.

[22] Gaikwad, S.N., Malashetty, M.S., and Prasad, K.R. (2009). An analytical study of linear and nonlinear double diffusive convection in a fluid saturated anisotropic porous medium with Soret effect. Applied Mathematical Modelling, 33(9), 3617-3635

[23] Pranesh, S., and Sameena, T. (2015). Linear and weakly nonlinear stability analysis of double diffusive electro-convection in a micropolar pond. IOSR Journal of Mathematics, 11(6), 44-70.

[24] Pranesh, S. (2014). Linear and weakly nonlinear stability analysis of gravity modulation and electric field on the onset of Rayleigh-Bernard convection in a micropolar pond. Journal of Advances in Mathematics, 9(3), 2057-2082.

[25] Palle, K. (2015). Weak nonlinear oscillatory convection in a non-uniform heating porous medium with throughflow. International Journal of Engineering and Mathematical Modelling, 2(3), 63-78.

[26] Premila, K., and Sudhaamsh, G. (2017). Nonlinear double diffusive convection in a couple stress fluid with rotational modulation. International Journal of Engineering Research and Application, 7(7), 25-29.

[27] Bhadauria, B.S., and Palle, K. (2014). Weak nonlinear thermal instabilities under vertical magnetic field, temperature modulation and heat source. International Journal of Engineering Research and Application, 4(1), 200-208.

[28] Cha'o-kuang, C., and Dong-Yu, L. (2010). Weakly nonlinear stability analysis of a thin magnetic fluid during spin coating. Mathematical Problems in Engineering, 4(17), 987891-987898.

[29] Celli, M., Alves, D.B., and Barletta, A. (2014). Nonlinear stability analysis of a Darcy flow with viscous dissipation. International Journal of Engineering Research and Application, 2(4), 927-936.

[30] Edmond, O. O., Israel-Cookey, C. and Emeka, A. (2020). Onset of Magnetoconvection in a rotating Darcy-Brinkman porous layer heated from below with temperature dependent heat source. International Journal of Mathematics Trends and Technology-Volume 66 Issue 5.