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# Markovian Queueing System for Bulk Arrival and Retrial Attempts with Multiple Vacation Policy

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***Abstract*** — *In this paper, we have considered a single server Markovian queueing system with idle server state, busy server state, vacation state, and breakdown and repair state. The vacation policy is multiple vacation policy and the vacation period follows exponential. This Markovian queueing model works with retrial attempts for the service completion and impatience behaviour of the customers. We determine the total probability of these states and give the comparison between the first come first serve and bulk service. We have also explains the various performance measures.*

**Keywords —** *Markovian queueing system, multiple vacation policy, impatience behaviour, bulk service.*

## I. INTRODUCTION

Markovian queueing models have various ranges of applications such as the manufacturing system, the production system, the telecommunication network and computer system, transportation system and follows Poisson arrival rates and exponential service rate. In today's situation, everyone has a busy routine and a short amount of time. They want to do their work in a limited amount of time but it is not possible due to the fact that the customer's arrival is not pre-defined. Several queueing problems can be seen in a daily routine where customers arrive at the service station to take the service who are visiting to take the service may be demoralized by the queue length. They leave the service system without taking the service is under the balking case of behaviour. They wait a short time in the queueing system for receiving the service as soon as possible. If they do not receive the service then they become impatient and leave the service system under the reneging case of behaviour. We consider the single server with working vacations, breakdown and repair where the server may be a breakdown at any instant of time. The arrival rate of customers follows Poisson distribution and the service rate follows an exponential distribution. If arriving customers find that server is busy to provide service to another customer then they enter the orbit group. They try after the random instant of time for receiving service from the service station. The server will provide service to the customers from the retrial group according to the first come first serve rule. The server can take vacations if no customer is present to receive the service at the service station.

## II. LITERATURE REVIEW

F. Nuts (1967) have given the theory of general bulk service rule first in which customer arrive as per the Poisson process and served in batches according to the general distribution with general bulk service rule. Bar-Lev et al. (2007) worked on the M/G(m, M) /1 model in which items arrive at the group testing center according to the Poisson process to be tested and are served in batches with the batch size according to the general distribution, where m and M (>m) are the choice factors where each batch size can be among m and M. They thought about that the testing place has a limited population and present an expected profit objective function. They have developed the generating function for the steady-state probabilities of the embedded Markov chain and compute the optimal values of the decision variables (m, M) that maximize the expected profit. Kalyanaraman and Sundaramoorthy (2019) worked on the Markovian queueing system with a regular busy state, breakdown, repair state and working vacation state. The vacation state and repair breakdown state follow the negative exponential distribution of multiple vacations.

Arumuganathan (2008) investigated the operating qualities of a M[X]/G/1 queueing system with an unreliable server and single vacation. The server is failed, while it is providing service and the arrival rate of customers depends upon the all over conditions of the server. Failure time of the server is exponentially distributed and the repair times follow the general distribution. The model is identified with the embedded Markov chain technique and level crossing analysis. They have determined the expected number of customers in the system, expected length of a busy period and probability generating function of the consistent state system size at an arbitrary time. Parveen and Begum (2013) worked on three single servers, a bulk service queue with a general arrival pattern and multiple working vacations. They inferred the steady state Probability distribution at pre-arrival epoch and pre-departure epoch, mean queue length. Hui and Zhao (2005) examined the single server, retrial queueing system having infinite capacity with breakdowns. Arriving customers may have the choice to join the service system or join the retrial orbit to retry for taking service after some time if the server is busy. They have assumed the necessary and sufficient condition and follow the exponential distribution for the inter arrival times, service time, server uptimes, server downtimes, retrial orbit times. They have given the sufficient condition and the stationary probability of the waiting number of customers. Daw and Pender (2019) considered the batch arrival queueing system as per the Poisson distribution and service as per both exponential and general distributions. They have analyzed the transition mean, variance and moment generating function for the time varying arrival rates, limiting behaviour of process through a batch scaling of the queue. Laxmi and Kumar (2018) worked on the batch arrival infinite buffer, single server, vacations, breakdown and repair queueing system in which arrival of customers follows the Poisson distribution and service time follows an exponential distribution. They analyzed the probability generating function of the steady-state probabilities, closed-form expressions of the system size, some other performance measures. Kumar and Shinde (2018) determined the bulk arrival and bulk service queueing system and analyzed the average number of customers in the system, the average number of customers in the queue, the average waiting time of customers in the queue, average waiting time of customers in the system, response time. Srivastava et al (2020) discussed two types of service that follow Markovian queueing system. They assumed if number of customers more than 'a' then use bulk arrival otherwise FCFS. They found various measures by using supplementary variable technique.

The model description of this paper is given in section 2. Queue size distribution is explained under section 3. Formulation of equations is given in section 4. The solution of the queueing model is presented in section 5. In next section 6, we describe the performance measures of this model and conclusion is given in section 7.

## III. MODEL DESCRIPTION

We have considered the following assumptions according to the transition diagram:

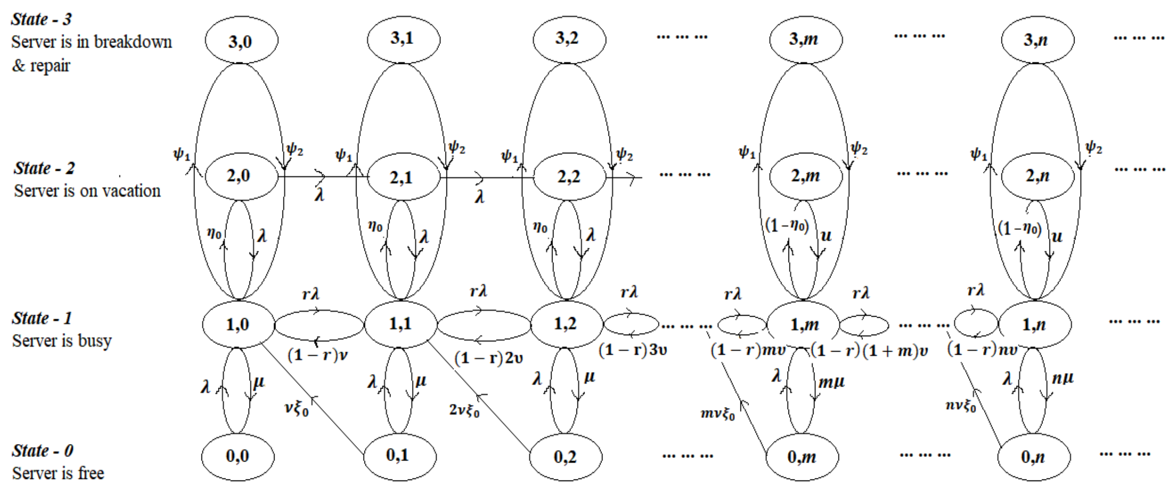


Figure – 1

For this unreliable server Markovian queueing model:

1. Server serves the customers one by one by using the first come first serve rule.

2. The arrival of customers follows the Poisson distribution with mean $1/\lambda$ and service of customers having a single unreliable server follows the expontial distribution with mean $1/\mu$.

3. If any customers comes to the service station and server is busy to providing service to other customers then customers may join the retrial orbit with probability $r$ or leave the queueing system with probability $(1-r)$.

4. Customer tries for the request after some time from the retrial orbit and follows the exponential distribution with mean $1-\nu$. If the server is not free then they stay in the retrial orbit and again tries for the request.

5. If customer tries after some time for receiving service with probability $\xi_0$ and if server is busy then customer may be impatient and leave the service station without receiving service with probability $1 - \xi_0$. This is known as the reneging behaviour of the customer.

6. The server may be failed at any time follows exponential distribution with failure rate $\psi_1$ and starts giving service to service to customers with repair rate $\psi_2$.

7. Server may go for the vacation at any instant of time f the number of customers is not sufficient to giving service in bulk manner.

Let random variable $N(t)$ represent the total customers in the system at any time t, and $s(t) = 0, 1, 2, 3$ defines the states, server is idle, server is busy, server is on vacation and server is in breakdown & repair state at time t.

We define $$S(t) = \begin{cases} 3, \text{Server is in breakdoun } \textit{and repair} \\ 2, \text{Server is on vacation} \\ 1, \text{Server is busy} \\ 0, \text{Server is free} \end{cases}$$

## IV. STATIONARY QUEUE SIZE DISTRIBUTION

For state 0, n $\geq 0$ when the server is free state:

$$\lambda P_{0,0} = \mu P_{1,0} \quad \ldots(1)$$

$$(\lambda + m\nu\xi_0)P_{0,m} = m\mu P_{1,m} \quad \ldots(2)$$

$$(\lambda + n\nu\xi_0)P_{0,n} = n\mu P_{1,n} \quad \ldots(3)$$

For state 1, n $\geq 0$ when the server is in busy state:

$$(\psi_1 + \eta_0 + r\lambda + \mu)P_{1,0} = (1 - r)\nu P_{1,1} + \psi_2 P_{3,0} + \lambda P_{2,0} + \nu\xi_0 P_{0,1} + \lambda P_{0,0} \quad \ldots(4)$$

$$(1 - \eta_0 + r\lambda + m\mu + \psi_1)P_{1,m}$$

$$= \psi_2 P_{3,m} + \mu P_{2,m} + \lambda P_{0,m} + r\lambda P_{1,m-1} + (1 - r)(m + 1)\nu P_{1,m+1} + (m + 1)\nu\xi_0 P_{0,m+1} \quad \ldots(5)$$

$$(1 - \eta_0 + \psi_1 + n\mu + (1 - r)n\nu + r\lambda)P_{1,n}$$

$$= \mu P_{2,n} + \psi_2 P_{3,n} + \lambda P_{0,n} + r\lambda P_{1,n-1} + r\lambda(n + 1)\nu P_{1,n+1} + (n + 1)\nu\xi_0 P_{0,n+1} \quad \ldots(6)$$

For state 2, when the server is on vacation:

$$2\lambda P_{2,0} = \eta_0 P_{0,1} \quad \ldots(7)$$

$$(\mu + \lambda)P_{2,m} = (1 - \eta_0)P_{1,m} + \lambda P_{1,m-1} \quad \ldots(8)$$

$$\mu P_{2,n} = (1 - \eta_0)P_{1,n} + \lambda P_{1,n-1} \quad \ldots(9)$$

For state 3, when the server is on breakdown and repair state:

$$\psi_2 P_{3,0} = \psi_1 P_{1,0} \quad \ldots(10)$$

$$\psi_2 P_{3,m} = \psi_1 P_{1,m} \quad \ldots(11)$$

$$\psi_2 P_{3,n} = \psi_1 P_{1,n} \quad \ldots(12)$$

From equation (1), (2) and (3), we have the following results

$$\left.\begin{aligned} P_{1,0} &= \left(\frac{\lambda}{\mu}\right) P_{0,0} \\ P_{1,m} &= \left(\frac{\lambda + m\nu\xi_0}{m\mu}\right) P_{0,m} \\ P_{1,n} &= \left(\frac{\lambda + n\nu\xi_0}{n\mu}\right) P_{0,n} \end{aligned}\right\} \quad \ldots(13)$$

Now by using (4) for n = 0 and (10), we have

$$(\Psi_1 + \eta_0 + r\lambda + \mu)P_{1,0} = (1 - r)\nu P_{1,1} + \psi_1 P_{1,0} + \lambda P_{2,0} + \nu\xi_0 P_{0,1} + \lambda P_{0,0}$$

$$(\eta_0 + r\lambda + \mu)P_{1,0} = (1 - r)\nu P_{1,1} + \lambda P_{2,0} + \nu\xi_0 P_{0,1} + \lambda P_{0,0}$$

By using (1), we get

$$\left(\frac{\eta_0\lambda + r\lambda^2}{\mu}\right) P_{0,0} = (1 - r)\nu P_{1,1} + \lambda P_{2,0} + \nu\xi_0 P_{0,1}$$

For m = 1 by using (13)

$$P_{1,1} = \left(\frac{\lambda + \nu\xi_0}{\mu}\right) P_{0,1}$$

$$P_{1,1} = A_1 P_{0,1} \quad \ldots(14)$$

$$\left(\frac{\eta_0\lambda + r\lambda^2}{\mu}\right) P_{0,0} = (1 - r)\nu A_1 P_{0,1} + \lambda P_{2,0} + \nu\xi_0 P_{0,1}$$

By using (7)

$$\left(\frac{\eta_0\lambda + r\lambda^2}{\mu}\right) P_{0,0} = \left[(1 - r + A_1 + \xi_0)\nu + \frac{\eta_0}{2}\right] P_{0,1}$$

$$P_{0,1} = \frac{2(\eta_0\lambda + r\lambda^2)}{\mu\left[(1 - r + A_1 + \xi_0)\nu + \frac{\eta_0}{2}\right]} P_{0,0}$$

$$P_{0,1} = \frac{B_2}{C_2} P_{0,0} \quad \ldots(15)$$

Now by using n = 1 in (6)

$$[1 - \eta_0 + \psi_1 + \mu + (1 - r)\nu + r\lambda]P_{1,1} = \mu P_{2,1} + \psi_2 P_{3,1} + \lambda P_{0,1} + r\lambda P_{1,0} + 2r\lambda\nu P_{1,2} + 2\nu\xi_0 P_{0,2}$$

Now by using (12) and put n = 1

$$[1 - \eta_0 + \psi_1 + (1 - r)\nu + r\lambda + \mu]P_{1,1} = \mu P_{2,1} + \psi_1 P_{1,1} + \lambda P_{0,1} + r\lambda P_{1,0} + 2r\lambda\nu P_{1,2} + 2\nu\xi_0 P_{0,2}$$

$$[1 - \eta_0 + (1 - r)\nu + \mu + r\lambda]P_{1,1} = \mu P_{2,1} + \lambda P_{0,1} + r\lambda P_{1,0} + 2r\lambda\nu P_{1,2} + 2\nu\xi_0 P_{0,2}$$

By using (9) for n = 1 and (14), we have

$$\mu P_{2,1} = (1-\eta_0)P_{1,1} + \lambda P_{1,0}$$

$$[(1-r)\nu + \mu + r\lambda]A_1 P_{0,1} = \lambda P_{1,0} + \lambda P_{0,1} + r\lambda P_{1,0} + 2r\lambda\xi_0 P_{1,2} + 2\nu\xi_0 P_{0,2}$$

$$[(1-r)\nu A_1 + \mu A_1 + r\lambda A_1 - \lambda]P_{0,1} = \lambda P_{1,0} + r\lambda P_{1,0} + 2r\lambda\xi_0 P_{1,2} + 2\nu\xi_0 P_{0,2} \quad \ldots(16)$$

Now in (13) put n = 2, we get

$$P_{1,2} = \left(\frac{\lambda+2\nu\xi_0}{2\mu}\right)P_{0,2} = \frac{B_1}{C_1}P_{0,2}$$

Further we solve (16) by using (1) and (15), we get

$$P_{0,2} = \frac{B_3}{C_3}P_{0,0} \quad \ldots(17)$$

Where:

$$B_3 = \frac{r\lambda A_1 B_2 + \nu(1-r)A_1 B_2 + \mu A_1 B_2 - \lambda B_2}{C_2} - \frac{\lambda^2(1+r)}{\mu}$$

$$C_3 = {}^{2r\lambda\xi_0 B_1}\!\big/_{C_1} + 2\xi_0\nu$$

Put n = 2 in (6), (9) and (12), we get

$$(1-\eta_0 + \psi_1 + 2\mu + 2(1-r)\nu + r\lambda)P_{1,2} = \mu P_{2,2} + \psi_2 P_{3,2} + \lambda P_{0,2} + r\lambda P_{1,1} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3} \quad \ldots(18)$$

$$\mu P_{2,2} = (1-\eta_0)P_{1,2} + \lambda P_{1,1} \quad \ldots(19)$$

$$\psi_2 P_{3,2} = \psi_1 P_{1,2} \quad \ldots(20)$$

Using (19) and (20) in (18), we get

$$(2\mu + 2(1-r)\nu + r\lambda)P_{1,2} = \lambda(1+r)P_{1,1} + \lambda P_{0,2} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3} \quad \ldots(21)$$

$$(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1}P_{0,2} = \lambda(1+r)P_{1,1} + \lambda P_{0,2} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3}$$

$$\left[(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right]P_{0,2} = \lambda(1+r)P_{1,1} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3}$$

$$\left[(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right]\frac{B_3}{C_3}P_{0,0} = \lambda(1+r)A_1 P_{0,1} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3}$$

$$\left[(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right]\frac{B_3}{C_3}P_{0,0} = \lambda(1+r)A_1\frac{B_2}{C_2}P_{0,0} + 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3}$$

$$\left[\left\{(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right\}\frac{B_3}{C_3} - \lambda(1+r)A_1\frac{B_2}{C_2}\right]P_{0,0} = 3r\lambda\nu P_{1,3} + 3\nu\xi_0 P_{0,3}$$

Put n = 3 in (13), we get

$$P_{1,3} = \left(\frac{\lambda + n\nu\xi_0}{n\mu}\right)P_{0,3} = \frac{B_4}{C_4}P_{0,3}$$

$$\left[\left\{(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right\}\frac{B_3}{C_3} - \lambda(1+r)A_1\frac{B_2}{C_2}\right]P_{0,0} = \left(3r\lambda\nu\frac{B_4}{C_4} + 3\nu\xi_0\right)P_{0,3}$$

$$P_{0,3} = \frac{\left[\left\{(2\mu + 2(1-r)\nu + r\lambda)\frac{B_1}{C_1} - \lambda\right\}\frac{B_3}{C_3} - \lambda(1+r)A_1\frac{B_2}{C_2}\right]}{\left(\frac{3r\lambda\nu B_4}{C_4} + 3\nu\xi_0\right)} P_{0,0}$$

$$P_{0,3} = \frac{B_5}{C_5} P_{0,0}$$

In general, we have

$$P_{0,n} = P_{0,1} + P_{0,2} + P_{0,3} + \cdots \qquad \ldots(22)$$

By using (13), we get

$$P_{1,n} = \left(\frac{\lambda + n\nu\xi_0}{n\mu}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] \qquad \ldots(23)$$

By using (9)

$$P_{2,n} = \left(1-\eta_0\right)\left(\frac{\lambda + n\nu\xi_0}{n\mu^2}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] + \frac{\lambda P_{1,n-1}}{\mu} \qquad \ldots(24)$$

Similarly by using (12), we have

$$P_{3,n} = \left(\frac{\psi_1}{\psi_2}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] \qquad \ldots(25)$$

Now by using (22), (23), (24) and (25), we have the give distribution

$$P_{r,n} = \begin{cases} P_{0,1} + P_{0,2} + P_{0,3} + \cdots & \text{, for } r = 0 \\ \left(\frac{\lambda + n\nu\xi_0}{n\mu}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] & \text{, for } r = 1 \\ \left(1-\eta_0\right)\left(\frac{\lambda + n\nu\xi_0}{n\mu^2}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] + \frac{\lambda P_{1,n-1}}{\mu} & \text{, for } r = 2 \\ \left(\frac{\psi_1}{\psi_2}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] & \text{, for } r = 3 \end{cases}$$

For finding the value of $P_{0,0}$, the normalization expression is given by

$$\sum_{r=0}^{3}\sum_{n=0}^{\infty} P_{i,r} = 1$$

$$\left(P_{0,1} + P_{0,2} + P_{0,3}\right)\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\psi_1}{\psi_2}\right) + \frac{\lambda}{\mu}P_{1,n-1} = 1$$

$$P_{0,0}\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\psi_1}{\psi_2}\right) + \frac{\lambda}{\mu}P_{1,n-1} = 1$$

$$P_{0,0} = \frac{1 - \frac{\lambda}{\mu}P_{1,n-1}}{\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\psi_1}{\psi_2}\right)}$$

## V. PERFORMANCE MEASURES

**(i) The Probability that server is idle:**

$$P_0 = \sum_{n=0}^{\infty} P_{0,n}$$

$$= \frac{1 - \frac{\lambda}{\mu} P_{1,n-1}}{\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\Psi_1}{\Psi_2}\right)\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)} \left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right]$$

**(ii) The Probability that server is busy:**

$$P_1 = \sum_{n=0}^{\infty} P_{1,n}$$

$$= \frac{1 - \frac{\lambda}{\mu} P_{1,n-1}}{\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\Psi_1}{\Psi_2}\right)\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)} \left(\frac{\lambda + n\nu\xi_0}{n\mu}\right) \times \left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right]$$

**(iii) The Probability that server on vacation:**

$$P_2 = \sum_{n=0}^{\infty} P_{2,n}$$

$$= \frac{1 - \frac{\lambda}{\mu} P_{1,n-1}}{\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\Psi_1}{\Psi_2}\right)\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)} \times \left[(1 - \eta_0)\left(\frac{\lambda + n\nu\xi_0}{n\mu^2}\right)\left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right] + \frac{\lambda P_{1,n-1}}{\mu}\right]$$

**(iv) The Probability that server is on breakdown and repair state:**

$$P_3 = \sum_{n=0}^{\infty} P_{3,n}$$

$$= \frac{1 - \frac{\lambda}{\mu} P_{1,n-1}}{\left(1 + \frac{\lambda + n\nu\xi_0}{n\mu^2} + \frac{\Psi_1}{\Psi_2}\right)\left(\frac{B_2}{C_2} + \frac{B_3}{C_3} + \frac{B_5}{C_5}\right)} \left(\frac{\Psi_1}{\Psi_2}\right) \times \left[P_{0,1} + P_{0,2} + P_{0,3} + \cdots\right]$$

## VI. CONCLUSION

We have worked on the Markovian queueing model with retrial attempts of customers, balking and reneging behaviour of the customers. We have considered the four states of the system such as idle state, busy state, vacation state breakdown and repair state. We have analyzed the probabilities of the different states and the performances measures of these states. This model is helpful to reduce the balking and reneging behaviour of the customers such as computer network, telecommunication system, hospitals, and supermarket.

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