International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I1P507

Volume 67 Issue 1, 53-59, January, 2021

Charecterizaton And Estimation of Area Biased Quasi Akash Distribution

R.B Ade[#1], D.P Teltumbade*[1], P.W Tasare[*2]
[#1]*Department of Statistics, Government Vidarbha Institute of Science and Humanities, Amravati, Maharashtra, India*

Abstract: *In this Paper, the area biased techniques of Quasi Akash distribution has been obtained. The distribution has two parameters. The various structural properties of the newly model have been studied. The parameter estimation of area biased Quasi Akash (ABQA) distribution by the method of maximum likelihood. And the likelihood ratio test of this distribution has been obtained.*

Keywords – *Weighted distribution, quasi Akash distribution, Reliability analysis, Order statistics, Maximum likelihood estimator, Likelihood ratio test*

I. Introduction

The concept of weight distribution is discussed by Fisher (1934). After being modified by C R Rao (1965) in a different way, in which with a weighty distribution many situations can be resolved. Weighted distribution is used in a variety of research fields related to reliability, environment, engineering and biomedicine. If the weight function looks at the size of one, the weight distribution reduces the size of the average distribution. If the weight function looks at the size of two, the weight distribution reduces the area of the biased distribution. Patil and Ord (1976) examined the use of weight-bearing statistics related to population and the environment can be found in Patil and Rao (1978). Van Deusen's (1986) relevant data related to the extent to chest height from a sample of the above point in color distribution. Lappi and Bailey (1987), used a randomized distribution of research in the analysis of the ascending data rate of sample size.Recently, Elangovan*et al* (2020) obtained a new area biased Aradhana distribution with application in Cancer data. This is showing a more flexibility than the classical distribution.Ade *et al*(2020) studied Area biased generalized uniform distribution and some of its important properties including coefficient of dispersion, coefficient of variation, hazard rate function, moment generating function, characteristic function, order statistics, Bonferroni and Lorenz curves, entropy measure.Shamker*et. al*.(2016)have been propose Quasi Akash distribution. Shanker has also obtained different structural properties including its survival function, failure rate function (hazard function), mean residual life function, moments and Stochastic ordering. The proposed quasi Akash distribution has better flexibility in handling real life data than the Akash distribution.

Consider the probability density function (pdf) of Quasi Akash distribution is given by

$$f(x;\alpha,\beta)=\frac{\theta^2(\alpha+\theta x^2)e^{-\theta x}}{(\alpha\theta+2)};\ \ x>0\ ,\alpha>0,\theta>0 \qquad (1.1)$$

And its first and second raw moment

$$E(X)=\frac{(\alpha\theta+6)}{\theta(\alpha\theta+2)}$$

$$E(X^2)=\frac{2(\alpha\theta+12)}{\theta^2(\alpha\theta+2)} \qquad (1.2)$$

II. AREA BIASED QUASI AKASH (ABQA) DISTRIBUTION

Suppose $X$is a non-negative random variable with probability density function $f(x)$. Let $w(x)$ be the non negative weight function, and then the probability density function of the weighted random variable $X_w$is given by:

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))};\ x>0$$

Where $w(x)$ be a non-negative weight function and$E\big(w(x)\big)=\int w(x)f(x)dx\ <\infty$.

For different weighted models, we have different choice of the weight function$w(x)$. When

$w(x) = x^c$, the resulting distribution is termed as weighted distribution. In this paper, we have to find the area biased version of weighteddistribution, we will take c = 2 in weights x^2, in order to get the area biased distribution and its pdf is given by:

$$f_1(x) = \frac{x^2 f(x)}{E(x^2)}\,; x > 0 \qquad (2.1)$$

Using the values of (1.1) and (1.2) in equation (2.1), we will get the pdf of area biased quasi Akashdistribution

$$f_1(x) = \frac{x^2\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)}; x > 0\,, \alpha > 0, \theta > 0 \qquad (2.2)$$

And the cumulative distribution function of area biased Quasi Akash (*ABQA)* distribution is obtained as

$$F_1(x) = \int_0^x f_1(x)dx$$

$$F_1(x) = \int_0^x \frac{x^2\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)}dx$$

After simplification, we will get the cumulative distribution function (cdf) of *ABQA* distribution is

$$F_1(x) = \frac{\theta^4}{2\,(\alpha\theta+12)}(\alpha\gamma(3,\theta x) + \theta(5,\theta x)) \qquad (2.3)$$

***A. Asymptotic Behavior*:**

We seek to investigate the behavior of the proposed model as in equation (2.) as $x \to 0$ and $x \to \infty$. This model involves considering $\lim_{x\to 0} f_1\,(x)$ and $\lim_{x\to\infty} f_1\,(x)$.

As $x \to 0$

$$\lim_{x\to 0} f_1\,(x) = \lim_{x\to 0}\left[\frac{x^2\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)}\right]$$
$$= 0$$

As $x \to \infty$

$$\lim_{x\to\infty} f_1\,(x) = \lim_{x\to\infty}\left[\frac{x^2\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)}\right]$$
$$= 0$$

These results confirm that the proposed distribution has a mode.

III. RELIABILITY ANALYSIS

In this session, we will introduce the reliability function or survival function, hazard rate function or failure rate and reverse hazard rate function, Mills ratio for the area biased Quasi Akashdistribution

A. Survival Function

The survival function of ABQAD is defined as

$$S(x) = 1 - F_1(x)$$

$$S(x) = 1 - \frac{\theta^4}{2\,(\alpha\theta+12)}(\alpha\gamma(3,\theta x) + \theta(5,\theta x))$$

B. Hazard Rate Function of ABQAD

The hazard function is also known as the hazard rate is defined as

$$h(x) = \frac{f_1(x)}{S(x)}$$

$$h(x) = \frac{x^2\theta^4(\alpha + \theta x^2)e^{-\theta x}}{2\,(\alpha\theta + 12) - \theta^4(\alpha\gamma(3, \theta x) + \theta(5, \theta x))}$$

C. Reverse Hazard Rate Function of ABQAD

The reverse hazard rate is given by

$$h_r(x) = \frac{f_1(x)}{F_1(x)}$$

$$h_r(x) = \frac{x^2(\alpha + \theta x^2)e^{-\theta x}}{(\alpha\gamma(3, \theta x) + \theta(5, \theta x))}$$

D. Mills Ratio

The Mills ratio of the ABQA distribution is given by,

$$Mills\ ratio = \frac{1}{h_r(x)}$$

$$= \frac{(\alpha\gamma(3, \theta x) + \theta(5, \theta x))}{x^2(\alpha + \theta x^2)e^{-\theta x}}$$

IV. MOMENTS AND ASSOCIATED MEASURES

Let Xdenotes the random variable of ABQA distribution with parameters αand β then the r^{th} order momentof ABQA distribution can be defined as

$$E(X^r) = \mu_r' = \int_0^\infty x^r\, f_1(x)dx$$

$$\mu_r' = \int_0^\infty x^{r+2}\, \frac{\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)}\, dx$$

After simplification,

$$\mu_r' = \frac{\theta^4}{2\,(\alpha\theta+12)}\left(\frac{\alpha\Gamma(r+3)}{\theta^{r+3}} + \frac{\Gamma(r+5)}{\theta^{r+4}}\right) \quad (4.1)$$

Putting $r = 1,2,3,4$ in equation (4.1), we will get the mean of ABQA distribution which is given by

$$E(X) = \frac{3(\alpha\theta+20)}{\theta(\alpha\theta+12)}$$

And putting $r = 2$ we obtain the second moment is

$$E(X^2) = \frac{12(\alpha\theta + 5)}{\theta^2(\alpha\theta + 12)}$$

Therefore,

$$Variance = \sigma^2 = \frac{(3\alpha^2\theta^2 - 156\alpha\theta - 2880)}{\theta^2(\alpha\theta + 12)^2}$$

$$Standard\ Deviation = \sigma = \frac{\sqrt{(3\alpha^2\theta^2 - 156\alpha\theta - 2880)}}{\theta(\alpha\theta + 12)}$$

$Coefficient\ of\ Variation = \frac{\sigma}{\mu}$

$$= \frac{\sqrt{(3\alpha^2\theta^2 - 156\alpha\theta - 2880)}}{3(\alpha\theta + 20)}$$

$Coefficient\ of\ Dispersion(\gamma) = \frac{\sigma^2}{\mu}$

$$= \frac{(3\alpha^2\theta^2 - 156\alpha\theta - 2880)}{3\theta(\alpha\theta + 12)(\alpha\theta + 20)}$$

A. *Moment generation function and characteristic function*

Let XbeABQA distribution, then the MGF of $X$is obtained as

$$M_X(t) = E(e^{tx}) = \int_0^\beta e^{tx} f_1(x)\, dx$$

$$= \int_0^\beta (1 + (tx) + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots + \cdots) f_1(x) dx$$

$$= \int_0^\beta \sum_{r=0}^{\infty} \frac{(tx)^r}{r!} f_1(x)\, dx$$

$$= \sum_{r=0}^{\infty} \frac{(t)^r}{r!} E(x^r)$$

Using equation (4.1) we have

$$M_X(t) = \sum_{r=0}^{\infty} \frac{(t)^r}{r!} \frac{\theta^4}{2\,(\alpha\theta + 12)} \left(\frac{\alpha\Gamma(r+3)}{\theta^{r+3}} + \frac{\Gamma(r+5)}{\theta^{r+4}} \right)$$

Similarly, the characteristic function of ABQA distribution can be obtained as

$$\varphi_X(t) = M_X(it)$$

$$\Rightarrow M_X(it) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{\theta^4}{2\,(\alpha\theta + 12)} \left(\frac{\alpha\Gamma(r+3)}{\theta^{r+3}} + \frac{\Gamma(r+5)}{\theta^{r+4}} \right)$$

B. Harmonic Mean

Let X be anABQAD, then the harmonic mean is obtained as

$$\frac{1}{H} = \int_0^\infty \frac{1}{x} f_1(x)\, dx$$

$$= \int_0^\beta \frac{1}{x} \frac{x^2\theta^4(\alpha + \theta x^2)e^{-\theta x}}{2\,(\alpha\theta + 12)} dx$$

After simplification

$$\frac{1}{H} = \frac{\theta(\alpha\theta + 6)}{2(\alpha\theta + 12)}$$

Thus,

$$H = \frac{2(\alpha\theta + 12)}{\theta(\alpha\theta + 6)}$$

V. ORDER STATISTICS OF ABQA DISTRIBUTION

The probability density function of the j^{th} order statistics $X_{(j)}$ for $1 \le j \le n$ is

$$f_{X_{(j)}}(x) = \frac{n!}{(j-1)!(n-j)!} [F(x)]^{j-1}[1 - F(x)]^{n-j} f_1(x) \tag{5.1}$$

Substitute the value of pdf and cdf of area biased Quasi Akashdistribution in equation (5.1), we get

$$f_{X_{(j)}}(x) = \frac{n!}{(j-1)!\,(n-j)!} \left[\frac{\theta^4}{2\,(\alpha\theta + 12)} \big(\alpha\gamma(3, \theta x) + \theta(5, \theta x)\big)\right]^{j-1}$$

$$\times \left[1 - \left(\frac{x}{\beta}\right)^{\alpha+3}\right]^{n-j} \frac{x^2\theta^4(\alpha + \theta x^2)e^{-\theta x}}{2\,(\alpha\theta + 12)} \tag{5.2}$$

Put $j = 1$ in equation (5.2), we will get the probability density function of first order statistics of ABQAD.

$$f_{X_{(1)}}(x) = n\left[1 - \frac{\theta^4}{2\,(\alpha\theta + 12)} \big(\alpha\gamma(3, \theta x) + \theta(5, \theta x)\big)\right]^{n-1} \frac{x^2\theta^4(\alpha + \theta x^2)e^{-\theta x}}{2\,(\alpha\theta + 12)}$$

Put $j = n$ in equation (6.2), we will get the probability density function of n^{th} order statistics of ABQAD.

$$f_{X_{(n)}}(x) = n\left[\frac{\theta^4}{2\,(\alpha\theta + 12)} \big(\alpha\gamma(3, \theta x) + \theta(5, \theta x)\big)\right]^{\mathrm{n}-1} \frac{x^2\theta^4(\alpha + \theta x^2)e^{-\theta x}}{2\,(\alpha\theta + 12)}$$

VI. BONFERRONI AND LORENZ CURVES

The Bonferroniand Lorenz curves is obtained by

$$B(p) = \frac{1}{p\mu} \int_0^q x\, f_1(x) dx$$

& $L(p) = pB(p) = \frac{1}{\mu}\int_0^q x\, f_1(x) dx$

$$Where\ \mu = E(x) = \frac{3(\alpha\theta+20)}{\theta(\alpha\theta+12)} \quad and \quad q = F^{-1}(p)$$

$$\therefore\ B(p) = \frac{\theta(\alpha\theta+12)}{p3(\alpha\theta+20)} \int_0^q \frac{x^3\theta^4(\alpha+\theta x^2)e^{-\theta x}}{2\,(\alpha\theta+12)} dx$$

$$B(p) = \frac{\theta^5}{p6(\alpha\theta+20)} \int_0^q x^3(\alpha+\theta x^2)e^{-\theta x}\, dx$$

After simplification

$$B(p) = \frac{\theta^5}{p6(\alpha\theta+20)} \left(\alpha\gamma(4,\theta q) + \theta(6,\theta q)\right)$$

And

$$L(p) = pB(p) = \frac{\theta^5}{6(\alpha\theta+20)} \left(\alpha\gamma(4,\theta q) + \theta(6,\theta q)\right)$$

VII. MAXIMUM LIKELIHOOD ESTIMATION (MLEs)

In this section, we will discuss the maximum likelihood estimators (MLE) we try to obtain values of the parameters of area biased Quasi Akash(ABQA)distribution. Considerbe the random sample of size *n* from the ABQA distribution, then the likelihood function is given by

$$L(x;\alpha,\beta) = \left(\frac{\theta^4}{2\,(\alpha\theta+12)}\right)^n e^{-\theta\sum_{i=1}^n x} \prod_{i=1}^n x_i^2\theta^4(\alpha+\theta x^2)$$

The log likelihood function is given by

$$\log l = 4nlog\theta - nlog(2(\alpha\theta+12) + \sum_{i=1}^n log x_i^2 + \sum_{i=1}^n log(\alpha+\theta x_i^2) - \theta\sum_{i=1}^n x_i \qquad (7.1)$$

The MLE of αand βcan be obtained by differentiating equation (8.1) with respect to αand β, we get

$$\frac{\partial log\ l}{\partial\alpha} = \frac{-n\theta}{(\alpha\theta+12)} + \sum_{i=1}^n \frac{1}{\alpha+\theta x^2} = 0 \qquad (7.2)$$

$$\frac{\partial log\ l}{\partial\theta} = \frac{4n}{\theta} - \frac{\alpha n}{\alpha\theta+12} + \sum_{i=1}^n \frac{(x_i^2)}{(\alpha+\theta x_i^2)} - \sum_{i=1}^n x_i = 0 \qquad (7.3)$$

The simultaneous solution of the above likelihood equations (7.2) and (7.3) are nonlinear equation. so analytical solution are not possible. Then we can solve above nonlinear equations with the help of R Software.

VIII. LIKELIHOOD RATIO TEST

Let$x_1, x_2, x_3, \ldots\ldots, x_n$ be a random sample from the ABQA distribution. We use the hypothesis

$H_0: f(x) = f(x,\alpha,\beta)$Against$H_1: f(x) = f_1(x,\alpha,\beta)$

In order test the random sample of size n comes from Akash distribution and area biased Quasi Akash (ABQA) distribution then following test statistics is used

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^n \frac{f_1(x,\alpha,\beta)}{f(x,\alpha,\beta)}$$

$$\Delta = \left(\frac{\theta^2(\alpha\theta+2)}{2(\alpha\theta+12)}\right)^n \prod_{i=1}^n x_i^2$$

We reject the null hypothesis, if

$$\Delta = \left(\frac{\theta^2(\alpha\theta+2)}{2(\alpha\theta+12)}\right)^n \prod_{i=1}^{n} x_i^{\,2} > k$$

$$\Delta = \prod_{i=1}^{n} x_i^{\,2} > k\left(\frac{2(\alpha\theta+12)}{\theta^2(\alpha\theta+2)}\right)^n$$

or

$\Delta^* = \prod_{i=1}^{n} x_i^{\,2} > k^*$Where$k^* = k\left(\frac{2(\alpha\theta+12)}{\theta^2(\alpha\theta+2)}\right)^n$

For large sample size n, $2log\Delta$ distributed as chi-square distribution with 1 degree of freedom(df) and also p-value is obtained from the chi-square distribution. Thus we reject the null hypothesis, when the probability value is given by

$$p(\Delta^* > \theta^*)$$

Where$\theta^* = \prod_{i=1}^{n} x_i$is less than specified level of significanceand $\prod_{i=1}^{n} x_i^{\,2}$is observed value of the statistics Δ^*.

IX. CONCLUSION

In this paper, we have studied a new distribution called as the area biased Quasi Akash distribution. By using certain special functions, its statistical properties, moments, failure rate, survival function has been obtained. The parameters have been estimated by using maximum likelihood method and also order statistics have been obtained.

REFERENCES

[1] Abebe, Berhane & Shanker Rama. (2018). A Discrete Quasi Akash Distribution with Applications. TurkiyeKlinikleri journal of Biostatistics. 10. 187-199.
[2] Ade R.B, Aradhye G.R ,Pawar D.D. (2020). Some Statistical Properties of Area biased Generalized Uniform Distribution. IJSR 9.
[3] Elangovan, Re &Mohanasundari, R. (2020). A New Area-Biased Distribution with Applications in Cancer Data. Science, Technology and Development. VIII. 1-14.
[4] Fisher, R. A., (1934), the effects of methods of ascertainment upon the estimation of frequencies. Annals of Eugenics, 6, 13-25.
[5] Lappi J, Bailey RL., (1987), Estimation of diameter increment function or other tree relations using angle-count samples, Forest science 33: 725-739.
[6] Patil, G. P. & Rao, C. R. (1978). Weighted distributions and size biased sampling with applications to wildlife populations and human families. Biometrics 34, 179-189..
[7] Patil, G. P. & Ord, J. K. (1976). On size biased sampling and related form invariantWeighted distributions. Sankhya, Series B 38, 48-61.
[8] Rao, C. R., (1965), "On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete", G.P. Patiled; Pergamum Press and Statistical publishing Society, Calcutta. 320-332.
[9] Shanker. R, (2016). A Quasi Akash Distribution and its application, Assam Statistical Review, 30(1), 135-160
[10] Van Deusen P.C., (1986), fitting assumed distributions to horizontal point sample diameters. *For Sci* 32(1): 146-148.