International Journal of Mathematics Trends and Technology Volume 67 Issue 1, 60-65, January, 2021
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I1P509

Proper Lucky Labeling And Lucky Edge Labeling For The Extended Duplicate Graph Of Quadrilateral Snake

P. Indira[1], B. Selvam[2] , K. Thirusangu[3]

[1]Research Scholar, S.I.V.E.T College, University of Madras, Chennai, India
[1]Department of Mathematics, St.Thomas College of Arts and Science, Chennai, India
[2,3]Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai-600 073, India

Abstract *- In this paper, we investigate the extended duplicate graph of quadrilateral snake graph admits proper lucky labeling and lucky edge labeling.*

Keywords — *Quadrilateral snake graph, Duplicate graph, Extended duplicate graph, Lucky labeling, Proper lucky labeling, Lucky edge labeling.*

I. INTRODUCTION

In 1967, Rosa[3] have introduced the concept of graph labeling. Gallian[4] has given a dynamic survey of graph labeling. The concept of lucky labeling was introduced by A.Ahai et al.,[5]. Kins yenoke et.al.,[7] introduced the idea of proper lucky labeling. The notion of lucky edge labeling was introduced by Nellai Murugan[12]. E.Sampthkumar[1] introduced the concept of duplicate graph. Thirusangu et al.,[2] have introduced the notion of extended duplicate graph.

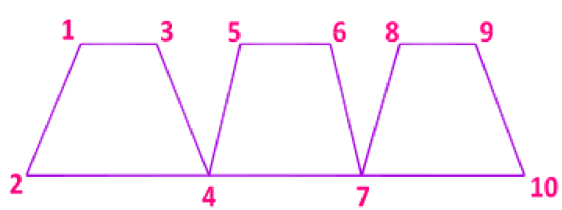
II. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges.

Definition: 2.1 Quadrilateral snake graph:

A quadrilateral snake QS_m is obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by joining u_i and u_{i+1} to two new vertex v_i and w_i respectively and then joining v_i and w_i , $1 \le i \le n-1$, where 'm' is the number of edges of the path. In general, a quadrilateral snake has 3m+1 vertices and 4m edges.

QUADRILATERAL SNAKE GRAPH (QS_3)



Definition : 2.2 Duplicate graph:

A Simple graph G with vertex set V and edge set E. The duplicate graph of G is DG = (V_1, E_1) where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective. The edge set E_1 of DG is defined as the edge $ab \in E$ iff both edges ab' and $a'b$ are in E_1.

Definition : 2.3 Extended duplicate graph of quadrilateral snake:

Let DG = (V_1,E_1) be a duplicate graph of the quadrilateral snake graph G(V,E). Extended duplicate graph of quadrilateral snake graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph and it is denoted by $EDG(QS_m)$. Clearly it has 6m+2 vertices and 8m+1 edges, where 'm' is the number of edges.

Definition : 2.4 Lucky labeling:
Let f: V(G) →N be a labeling of the vertices of a graph by positive intergers. Let S(v) denote the sum of labels of the neighbours of the vertex v in G. If v is an isolated vertex of G. We put S(v)=0. A labeling is lucky if S(u)≠S(v) whenever u and v are adjacent. The least integer k for which a graph G has a lucky labeling from the set {1,2,3...k} is the lucky number of G denoted by η(G).

Definition : 2.5 Proper lucky labeling:
A lucky labeling is proper lucky labeling if the labeling f is proper as well as lucky, that is if u and v are adjacent in G then f(u)≠f(v) and S(u)≠S(v). The proper lucky number of G is denoted by η_p(G) is the least positive integer K such that G has a proper lucky labeling with {1,2,3,...,k} as the set of labels.

Definition : 2.6 Lucky edge labeling:
Let G be a simple graph with vertex set V(G) and edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper colouring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges.

III. MAIN RESULTS

A. Proper Lucky- Labeling For Edg(Qsm) , M≥ 1

Here we present an algorithm and prove the existence of proper lucky labeling for the EDG of quadrilateral snake QS_m .

Algorithm-1

Procedure - [Proper lucky labeling for EDG(QS_m), m ≥ 1]

```
V(G) ← {v1,v2, v3,… ,v3m,v3m+1, v’1,v’2,v’3,…,v’3m ,v’3m+1}
E(G) ← {e1, e2, e3, …,e4m,e4m+1,e’1,e’2,e’3,… ,e’4m }
if  m≥1
      for i = 1 to 4
           vi  ← 1
           v’i ←2
     end for
end if
if  m≥2
      v7 ←3
     for i = 1 to 2
         vi+4 ← 1
     end for
     for i = 0 to (m-2)
             for j = 0 to 1
                  v’5+3i+2j ← 2
                  v’6+3i    ← 3
             end for
     end for
end if
if  m > 2
      for  i = 0  to  (m-3)
               for  j = 0  to  1
                   v8+3i   ← 1
                   v9+3i+j ← 3
               end  for
      end  for
end if
end procedure
```

Theorem 1 : The Proper lucky number for the extended duplicate graph of quadrilateral snake graph

$$EDG(QSm) = \begin{cases} 2, \ if \ \ m = 1 \\ 3, \ if \ \ m \geq 2 \end{cases}$$

Proof: Let QSm be the quadrilateral snake graph and EDG(QSm) be the extended duplicate graph of quadrilateral snake Define the set of vertices and edges are

$V(G) = \{v_1, v_2, v_3, \ldots, v_{3m}, v_{3m+1}, v'_1, v'_2, v'_3, \ldots, v'_{3m}, v'_{3m+1}\}$

$E(G) = \{e_1, e_2, e_3, \ldots, e_{4m}, e_{4m+1}, e'_1, e'_2, e'_3, \ldots, e'_{4m}\}$

Let us define the mapping f: V(G) → N such that the labeling is a proper lucky labeling if $f(u) \neq f(v)$ & $S(u) \neq S(v)$, whenever u and v are adjacent in G and S(v) denote the sum of labels of the neighbours of the vertex v in G.

Case 1 : If m≥1, by using algorithm 1, assign the label '1' to v_i and '2' to v'_i for $1 \leq i \leq 4$.

Case 2: If m≥2, labeling the vertices v_1, v_2, v_3, v_4 and v'_1, v'_2, v'_3, v'_4 as in case 1 and the vertex v_5 and v_6 receive the label '1' and v_7 receive the label '3'.

For i = 0 to (m-2) and $0 \leq j \leq 1$, the vertices $v'_{5+3i+2j}$ receive the label '2' ; the vertices v'_{6+3i} receive the label '3'. For i = 0 to (m-3) & $0 \leq j \leq 1$, the vertices v_{8+3i} receive the label '1'; the vertices v_{9+3i+j} receive the label '3' .

From the above cases, we observe that $f(u) \neq f(v)$.

Thus, 6m+2 vertices are labeled by {1,2,3}.

Claim: To prove that EDG(QSm) is the proper lucky labeling .

i.e., to prove $S(u) \neq S(v)$.

The sum neighbourhood of the vertices are as follows:

if m≥1,

$S(v_1) = 4 = S(v_3)$, $S(v_2) \to 6$, $S(v_4) \to 8$,

$S(v'_1) = 2 = S(v'_3)$, $S(v'_2) \to 3$, $S(v'_4) \to 6$ and

if m≥2,

$S(v'_5) \to 2$, $S(v'_6) \to 3$, $S(v'_7) \to 6$ and

for i = 0 to (m-2), $S(v_{5+3i}) \to 5$, $S(v_{6+3i}) \to 4$, $S(v_{7+3i}) \to 9$

for i = 0 to (m-3), $S(v'_{8+3i}) \to 6$, $S(v'_{9+3i}) \to 4$, $S(v'_{10+3i}) \to 10$.

Clearly, we get that $S(u) \neq S(v)$.

Therefore, the extended duplicate graph of quadrilateral snake graph is proper lucky labeling and the proper lucky number is

$$\eta(G) = EDG(QSm) = \begin{cases} 2, \ if \ \ m = 1 \\ 3, \ if \ \ m \geq 2 \end{cases}$$

Example 1 : Proper lucky labeling diagram in EDG(QSm) is shown in figure(1) & figure(2)

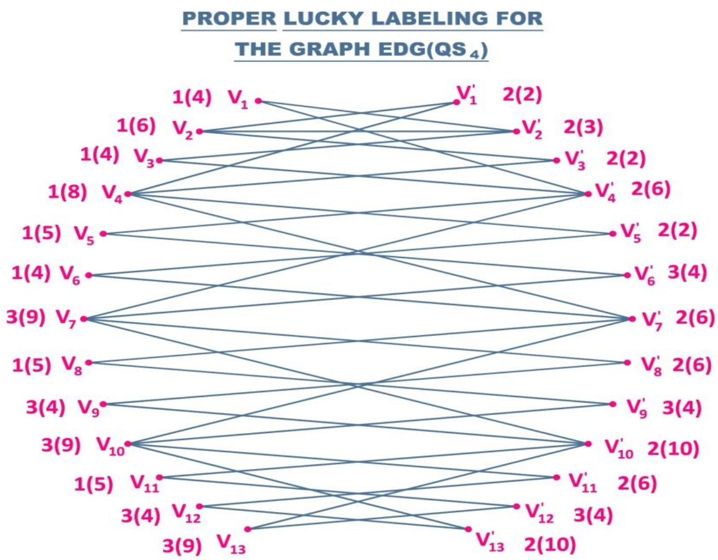


Fig 1: EDG(QS_4)

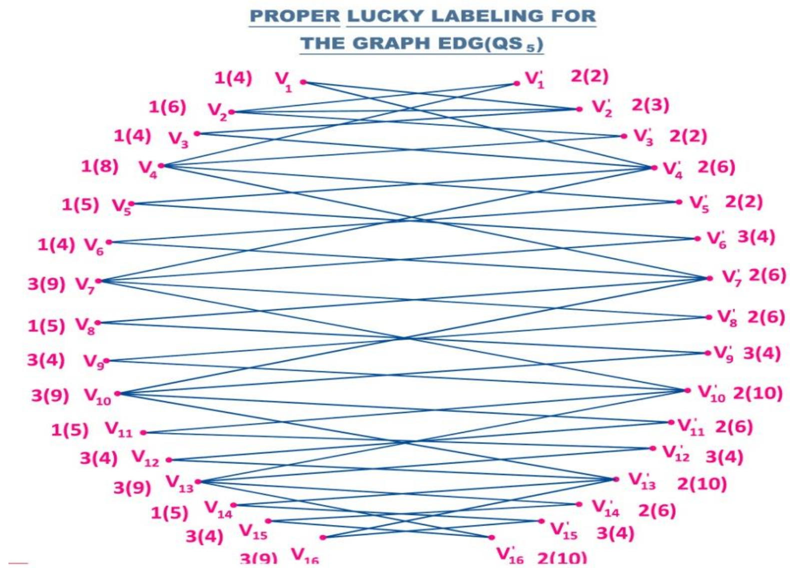


Fig 2: EDG(QS_5)

B. Lucky Edge Labeling For Edg(Qsm)

Algorithm-2

Procedure – [Lucky labeling for EDG(QS_m), $m \geq 1$]

$V(G) \leftarrow \{v_1, v_2, v_3, \ldots, v_{3m}, v_{3m+1}, v'_1, v'_2, v'_3, \ldots, v'_{3m}, v'_{3m+1}\}$

$E(G) \leftarrow \{e_1, e_2, e_3, \ldots, e_{4m}, e_{4m+1}, e'_1, e'_2, e'_3, \ldots, e'_{4m}\}$

if $m \geq 1$

$v_1 \leftarrow 1, v_2 \leftarrow 2, v_3 \leftarrow 3, v_4 \leftarrow 1, v'_1 \leftarrow 1, v'_2 \leftarrow 2, v'_3 \leftarrow 3, v'_4 \leftarrow 1$

if $m \geq 2$

for $i = 0$ to $\left\lfloor \frac{(m-2)}{3} \right\rfloor$

for $j = 0$ to 1

$v_{5+9i+j} \rightarrow 2;\ v'_{5+9i+j} \rightarrow 2;\ v_{7+9i} \rightarrow 4;\ v'_{7+9i} \rightarrow 4.$

end for

end for

for $i = 0$ to $\left\lfloor \frac{(m-3)}{3} \right\rfloor$ **do**

$v_{8+9i} \rightarrow 3;\ v'_{8+9i} \rightarrow 3;\ v_{9+9i} \rightarrow 1;\ v'_{9+9i} \rightarrow 1;\ v_{10+9i} \rightarrow 5;\ v'_{10+9i} \rightarrow 5.$

end for

for $i = 0$ to $\left\lfloor \frac{(m-4)}{3} \right\rfloor$ **do**

$v_{11+9i} \rightarrow 2;\ v'_{11+9i} \rightarrow 2;\ v_{12+9i} \rightarrow 1;\ v'_{12+9i} \rightarrow 1;\ v_{13+9i} \rightarrow 4;\ v'_{13+9i} \rightarrow 4.$

end for

end if

end procedure

Theorem 2 : The extended duplicate graph of quadrilateral snake graph is lucky edge labeling and its number $\eta(G)$ is 9.

Proof: Let QSm be the quadrilateral snake graph and EDG(QSm) be the extended duplicate graph of quadrilateral snake. Define the set of vertices and edges are

$V(G) = \{v_1, v_2, v_3, \ldots, v_{3m}, v_{3m+1}, v'_1, v'_2, v'_3, \ldots, v'_{3m}, v'_{3m+1}\}$

$E(G) = \{e_1, e_2, e_3, \ldots, e_{4m}, e_{4m+1}, e'_1, e'_2, e'_3, \ldots, e'_{4m}\}$

Define a mapping $f: V(G) \rightarrow N$ such that the labeling is a lucky edge labeling if $E(e_1) \neq E(e_2)$, whenever e_1 & e_2 are adjacent edges.

By using the algorithm 2, we have

Case 1: If $m \geq 1$,

$v_1 \rightarrow 1, v'_1 \rightarrow 1, v_2 \rightarrow 2, v'_2 \rightarrow 2, v_3 \rightarrow 3, v'_3 \rightarrow 3, v_4 \rightarrow 1, v'_4 \rightarrow 1$

Case 2: If $m>1$,

To Label the vertices for v_1, v_2, v_3, v_4 and v'_1, v'_2, v'_3, v'_4 as in case(1) and

For $i = 0$ to $\left\lfloor \frac{(m-2)}{3} \right\rfloor$ & $j = 0$ to 1,

$f(v_{5+9i+j}) \rightarrow 2 \leftarrow f(v'_{5+9i+j})$; $f(v_{7+9i}) \rightarrow 4 \leftarrow f(v'_{7+9i})$.

For $i = 0$ to $\left\lfloor \frac{(m-3)}{3} \right\rfloor$,

$f(v_{8+9i}) \rightarrow 3 \leftarrow f(v'_{8+9i})$; $f(v_{9+9i}) \rightarrow 1 \leftarrow f(v'_{9+9i})$; $f(v_{10+9i}) \rightarrow 5 \leftarrow f(v'_{10+9i})$.

For $i = 0$ to $\left\lfloor \frac{(m-4)}{3} \right\rfloor$,

$f(v_{11+9i}) \rightarrow 2 \leftarrow f(v'_{11+9i})$; $f(v_{12+9i}) \rightarrow 1 \leftarrow f(v'_{12+9i})$; $f(v_{13+9i}) \rightarrow 4 \leftarrow f(v'_{13+9i})$.

Thus, $6m+2$ vertices are labeled.

Claim: To prove that EDG(QSm) is a lucky edge labeling. i.e., to prove $E(e_i) \neq E(e_j)$ for every e_i & e_j are adjacent edges.

If $m \geq 1$,

$E(e_1) \rightarrow 3 \leftarrow E(e'_1)$; $E(e_2) \rightarrow 2 \leftarrow E(e'_2)$

$E(e_3) \rightarrow 5 \leftarrow E(e'_3)$; $E(e_4) \rightarrow 4 \leftarrow E(e'_4)$ and also $E(e_{4m+1}) \rightarrow 4$.

If $m \geq 2$,

$E(e_5) \rightarrow 3 \leftarrow E(e'_5)$; $E(e_6) \rightarrow 5 \leftarrow E(e'_6)$;

$E(e_7) \rightarrow 4 \leftarrow E(e'_7)$; $E(e_8) \rightarrow 6 \leftarrow E(e'_8)$.

If $m \geq 3$,

for $i = 0$ to $\left\lfloor \frac{(m-3)}{3} \right\rfloor$,

$E(e_{9+12i}) \rightarrow 7 \leftarrow E(e'_{9+12i})$; $E(e_{10+12i}) \rightarrow 9 \leftarrow E(e'_{10+12i})$;

$E(e_{11+12i}) \rightarrow 5 \leftarrow E(e'_{11+12i})$; $E(e_{12+12i}) \rightarrow 6 \leftarrow E(e'_{12+12i})$.

If $m \geq 4$,

for $i = 0$ to $\left\lfloor \frac{(m-4)}{3} \right\rfloor$,

$E(e_{13+12i}) \rightarrow 7 \leftarrow E(e'_{13+12i})$; $E(e_{14+12i}) \rightarrow 9 \leftarrow E(e'_{14+12i})$;

$E(e_{15+12i}) \rightarrow 3 \leftarrow E(e'_{15+12i})$; $E(e_{16+12i}) \rightarrow 5 \leftarrow E(e'_{16+12i})$.

If $m \geq 5$,

for $i = 0$ to $\left\lfloor \frac{(m-5)}{3} \right\rfloor$,

$E(e_{17+12i}) \rightarrow 6 \leftarrow E(e'_{17+12i})$; $E(e_{18+12i}) \rightarrow 8 \leftarrow E(e'_{18+12i})$;

$E(e_{19+12i}) \rightarrow 4 \leftarrow E(e'_{19+12i})$; $E(e_{20+12i}) \rightarrow 6 \leftarrow E(e'_{20+12i})$.

Clearly, we observe that $E(e_1) \neq E(e_2)$.

Therefore, the extended duplicate graph of quadrilateral snake graph is a lucky edge labeling and its number $\eta(G) = 9$.

Example 2: Lucky labeling diagram in EDG(QSm) is shown in figure(3) & figure(4).

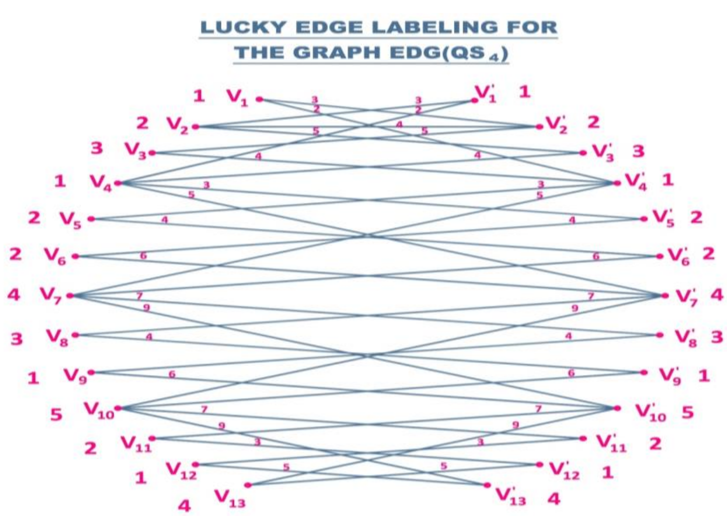


Fig.3: EDG(QS_4)

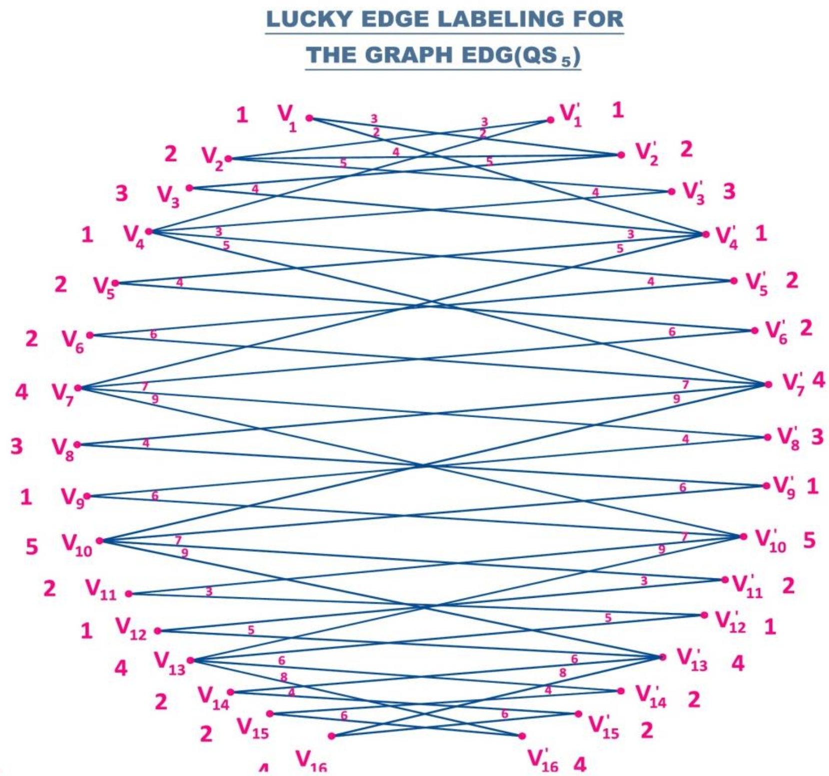


Fig 4: EDG(QS_5)

VI. CONCLUSIONS

In this research paper, we have presented algorithms and proved the extended duplicate graph of quadrilateral snake graph QS_m, $m \geq 1$ admits proper lucky labeling and lucky edge labeling.

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