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Analysis of Inventory Control Model with Modified Weibully Distributed Deterioration Rate, Partially Backlogged Shortages and Time Dependent Holding Cost

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***Abstract** - A deterministic inventory control model with deterioration is developed in this paper. Time to deteriorate follows three parameters modified Weibull distribution and exponential distribution with a two parameter Weibull demand rate. Holding cost is taken as a linear function of time keeping in mind the criteria of modern era. Shortages are allowed to occur which are partially backlogged and partially suffer a lost sale cost. The aim of the paper is to understand the retailer's replenishment decisions under more practical circumstances such as time-dependent demand rate and lost sale situation in an economic order quantity model for deteriorating items. The mathematical model is explored by a numerical example to validate the applicability of the proposed model. We minimize the average total inventory cost subject to decision variables (inventory cycle time and inventory level) and perform the sensitivity analysis of the optimal solution on significant parameters to understand the stability and practicability of our results.*

Keywords — *Linear demand rate, partially backlogged shortages, three parameter modified Weibull distribution*

I. INTRODUCTION

The problem of deterioration and its effect is very common with all inventory management models. It is observed in two categories of items- one that gets spoiled, decayed, damaged or expire with time such as eatables, flowers and medicines and the other that is a loss of parts or value through time that is caused by new technology or alternatives such as fashion goods and electronic items.

Most researchers assumed that the demand is lost or backlogged during the shortage period. But in reality, many customers are willing to wait for the replenishment. The cost associated with this waiting time is called lost sale cost and is generally time-dependent. Dye, C.Y., Hsieh, T.P., and Ouyang, L.Y., [4] considered the lost sale cost in the inventory model they developed. Yadav and Vats [12] established an inventory model under partial backlogging and constant holding cost.

In practical scenario, the inventory holding cost is not constant but is time-dependent in many cases. Giri, Goswami and Chaudhary [6] & Mishra, Singh and Kumar [9] analyzed inventory models with holding cost as a function of time. Parmar and Gothi [10] developed an EPQ model for deteriorating items under three-parameter Weibull distribution and time-dependent holding cost with shortages. G.P. Samanta, Jhuma Bhowmick [7] proposed a three continuous order-level inventory models with shortages and the demand rate as a ramp type function of time. Sujan Chandra [2] provided an inventory model where demand rate was taken as a ramp type function of time and a time-dependent holding cost.

Alin Rosca and Natalia Rosca [1] developed an EOQ model with a modified Weibull distribution deterioration rate and shortages with partial backlogging but with exponential demand rate. A lot of research has been done under both the categories. Whitin [11] first studied the deterioration of fashion items. Ghare and Schrader [5] first developed inventory model with a constant deterioration rate and no shortages and later expanded the studies for exponentially decaying inventories. Many researchers have then developed inventory models considering different decaying scenarios and various demand rates and shortage situations. While Covert and Philip [2] used Weibull distribution to describe time to deterioration, Chung and Ting [8] proposed a model with time-varying demand and partial backlogging.

In this paper, we have considered an inventory model that has a linearly varying holding cost and time-dependent demand rate when the time to deteriorate follows a modified three-parameter Weibull distribution and Exponential distribution. Shortages are allowed to occur, which are partially backlogged. The objective is to minimize the total cost of an inventory system based on certain costs, including the lost sale cost during the duration of shortage. The mathematical model is derived under these conditions of an inventory system. A numerical example is followed by the cost equations and cost minimization to illustrate the model. Sensitivity analysis is performed in the later segment of the paper to understand the effect of change in the decision parameter values. The last section consists of a graphical representation of the sensitivity analysis and the conclusion is drawn based on it.

II. NOTATIONS

The following notations have been considered for our mathematical model:

1. Q_1 : The on-hand positive inventory level of the item at time t = 0.
2. Q_2 : The on-hand positive inventory level of the item at time t = μ.
3. R(t) : Demand rate varying over time.
4. θ(t) : Deterioration rate per unit per unit time.
5. A : Ordering cost per order that is known and is constant.
6. C_d : Deterioration cost per unit per unit time.
7. C_h : Inventory holding cost per unit per unit time.
8. C_s : Shortage cost per unit.
9. C_p : Purchasing cost per unit.
10. C_l : Lost sale cost per unit per unit time
11. PC : Purchase cost per unit.
12. I_B : Backlog order quantity during time $[t_1, T]$
13. T : The fixed length of each cycle.
14. TC : The average total cost for the time period [0, T]
15. θ : Deterioration rate $(\theta > 0)$
16. δ : Backlogging parameter $(0 < \delta < 1)$

III. ASSUMPTIONS

We have taken the following assumptions to develop the inventory model

1. The inventory system is considered over an infinite time horizon.
2. Replenishment is instantaneous.
3. Holding cost is a linear function of time and it is $C_h = h + rt$ (h, r > 0)
4. Lead time is assumed to be zero.
5. Shortages are allowed and they are partially backlogged.
6. The inventory system involves only one item and one stocking point.
7. The two parameter Weibull demand rate at any time is given by
   $R(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha << 1$ and $\beta > 0$.
8. Deterioration rate follows a three parameter modified Weibull deterioration rate (unit/unit time) in time period [0, μ] and exponential deterioration rate in the time interval [μ, T]; the deterioration rate is given by
   $$\theta(t) = \begin{cases} \psi + \xi\eta t^{\eta-1} \\ \theta \end{cases}$$
   where $0 < \psi < 1, 0 < \xi < 1, \eta > 0$ and $\theta > 0$

IV. MATHEMATICAL MODEL AND ANALYSIS

Based on the above stated assumptions, the inventory cycle as shown in the figure below starts at initial stage (time t = 0) with on-hand inventory level Q_1. During the time period $[0, t_1]$, the inventory starts depleting partially due to demand and partially due to combined effects of three-parameter modified Weibull distribution and exponential distribution deterioration rates in time period $[0, \mu]$ and $[\mu, t_1]$ respectively. The depletion continues until time $t = t_1$ when inventory falls down to zero and then the inventory system suffers shortages which is accumulated from time period $[t_1, T]$. A backlog of I_B units is created from time $[t_1, T]$, system incurs lost sales cost after which the inventory is replenished, shortages are partially backlogged and the inventory cycle [0, T] repeats itself.

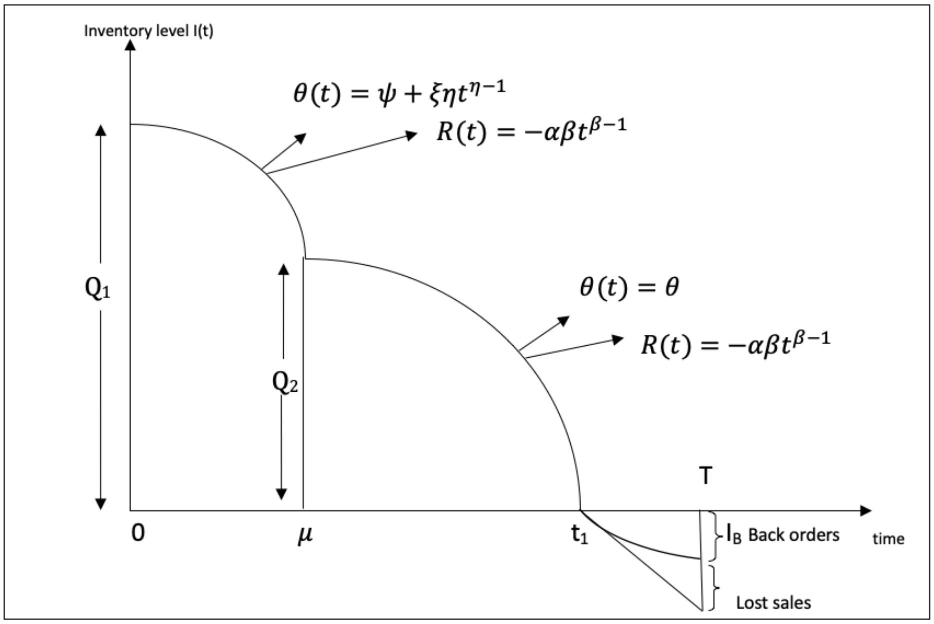


Fig. 1: Graphical Presentation for Inventory level-time relationship

The differential equations during the interval [0, T], where all the states of the inventory level are involved are given by

$$\frac{dI(t)}{dt} + (\psi + \xi\eta t^{\eta-1})I(t) = -\alpha\beta t^{\beta-1}; \qquad 0 \le t \le \mu \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -\alpha\beta t^{\beta-1}; \qquad \mu \le t \le t_1 \qquad (2)$$

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1} e^{-\delta(T-t)}; \qquad t_1 \le t \le T \qquad (3)$$

The solution of equation (1) using the boundary condition I (0) = Q_1 becomes

$$I(t) = Q_1(1 - \psi t - \xi t^{\eta}) - \alpha\left(t^{\beta} - \frac{\psi t^{\beta+1}}{\beta+1} - \frac{\xi\eta t^{\eta+\beta}}{(\eta+\beta)}\right) \qquad 0 \le t \le \mu \qquad (4)$$

The solution of equation (2) using the boundary condition $I(t_1) = 0$ becomes

$$I(t) = C_2(1 - \theta t) - \alpha\left(t^{\beta} - \frac{\theta t^{\beta+1}}{\beta+1}\right) \qquad \mu \le t \le t_1 \qquad (5)$$

The solution of equation (3) using the boundary condition I(t·) = 0 becomes

$$I(t) = \alpha\left(\left((1-\delta T)t_1^{\beta} + \left(\frac{\delta\beta}{\beta+1}\right)t_1^{\beta+1}\right) - \left((1-\delta T)t^{\beta} + \frac{\delta\beta t^{\beta+1}}{\beta+1}\right)\right) \qquad t_1 \le t \le T \qquad (6)$$

Inserting $I(\mu) = Q_2$ in equation (5),

$$Q_2 = C_2(1-\theta\mu) - \alpha\left(\mu^\beta - \frac{\theta\mu^{\beta+1}}{\beta+1}\right) \tag{7}$$

Inserting $I(\mu) = Q_2$ in equation (4),

$$Q_1 = \frac{C_2(1-\theta\mu) - \alpha\left(\mu^\beta - \frac{\theta\mu^{\beta+1}}{\beta+1}\right) + \alpha\left(\mu^\beta - \frac{\psi\mu^{\beta+1}}{\beta+1} - \frac{\xi\eta\mu^{\eta+\beta}}{\eta+\beta}\right)}{1-\psi\mu-\xi\mu^\beta} \tag{8}$$

Inserting $\mathrm{I(T)} = -\mathrm{I_B}$ in equation (6),

$$I_B = \alpha\left(\left((1-\delta T)T^\beta + \frac{\delta\beta T^{\beta+1}}{\beta+1}\right) - \left((1-\delta T)t_1^{\ \beta} + \frac{\delta\beta t_1^{\ \beta+1}}{\beta+1}\right)\right) \tag{9}$$

A. COST COMPONENTS:

For finding the total inventory costs, we consider the following cost elements:

1) ***Ordering Cost*:** The ordering cost OC = A (10)

2) ***Deterioration Cost* :**Deterioration cost DC over the period $[0, t_1]$ is defined by

$$DC = C_d\left[\int_0^\mu \left(\psi + \xi\eta t^{n-1}\right)I(t)dt + \int_\mu^{t_1} \theta I(t)dt\right]$$

$$DC = C_d\left[\left\{Q_1(\psi\mu + \xi\mu^\eta) - \alpha\left(\frac{\psi\mu^{\beta+1}}{\beta+1} + \frac{\xi\eta\mu^{\eta+\beta}}{\eta+\beta}\right)\right\} + \theta\left\{C_2(t_1-\mu) - \alpha\frac{t_1^{\ \beta+1} - \mu^{\beta+1}}{\beta+1}\right\}\right] \tag{11}$$

3) ***Shortage Cost* :** Shortage cost is accumulated during the time interval $[t_1, T]$ so the shortage cost SC is expressed as

$$SC = -C_s\int_{t_1}^{T} I(t)dt$$

$$SC = C_s\alpha\left[\begin{array}{l}\left\{(1-\delta T)\frac{T^{\beta+1} - t_1^{\ \beta+1}}{\beta+1} + \left(\frac{\delta\beta}{\beta+1}\right)\frac{T^{\beta+2} - t_1^{\ \beta+2}}{\beta+2}\right\} \\ -(T-t_1)\left\{(1-\delta T)t_1^{\ \beta} + \left(\frac{\delta\beta}{\beta+1}\right)t_1^{\ \beta+1}\right\}\end{array}\right] \tag{12}$$

4) ***Lost Sale Cost* :** Not all customers are willing to wait for the next lot size to arrive during the shortage period $[t_1, T]$ which may cause some loss in sales. Hence, lost sale cost LSC is: $LSC = C_l\int_{t_1}^{T} \alpha\beta t^{\beta-1}\left(1-e^{-\delta(T-t)}\right)dt$

$$LSC = C_l\alpha\beta\delta\left[T\left(\frac{T^\beta - t^\beta}{\beta}\right) - \left(\frac{T^{\beta+1} - t^{\beta+1}}{\beta+1}\right)\right] \tag{13}$$

5) ***Purchase Cost :*** Purchase cost PC is given by

$$PC = C_p\left[\begin{array}{l}\dfrac{C_2(1-\theta\mu)-\alpha\left(\mu^{\beta}-\dfrac{\theta\mu^{\beta+1}}{\beta+1}\right)+\alpha\left(\mu^{\beta}-\dfrac{\psi\mu^{\beta+1}}{\beta+1}-\dfrac{\xi\eta\mu^{\eta+\beta}}{\eta+\beta}\right)}{1-\psi\mu-\xi\mu^{\beta}} \\ +\alpha\left(\left((1-\delta T)T^{\beta}+\dfrac{\delta\beta T^{\beta+1}}{\beta+1}\right)-\left((1-\delta T)t_1^{\beta}+\dfrac{\delta\beta t_1^{\beta+1}}{\beta+1}\right)\right)\end{array}\right] \tag{14}$$

6) ***Inventory Holding Cost :*** Inventory holding cost IHC is computed for time interval $[0, t_1]$ because only during this time period inventory is available in the system. So, IHC is defined as follows:

$$IHC = \int_0^{t_1} C_h I(t)dt \quad IHC = \int_0^{\mu}(h+rt)I(t)dt + \int_{\mu}^{t_1}(h+rt)I(t)dt$$

$$IHC = \left[\begin{array}{l} h\left\{Q_1\left(\mu-\psi\dfrac{\mu^2}{2}-\xi\dfrac{\mu^{\eta+1}}{\eta+1}\right)-\alpha\left(\dfrac{\mu^{\beta+1}}{\beta+1}-\psi\dfrac{\mu^{\beta+2}}{(\beta+1)(\beta+2)}-\xi\dfrac{\eta\mu^{\eta+\beta+1}}{(\eta+\beta)(\eta+\beta+1)}\right)\right\} \\ +r\left\{Q_1\left(\dfrac{\mu^2}{2}-\psi\dfrac{\mu^3}{3}-\xi\dfrac{\mu^{\eta+2}}{\eta+2}\right)-\alpha\left(\dfrac{\mu^{\beta+2}}{\beta+2}-\psi\dfrac{\mu^{\beta+3}}{(\beta+1)(\beta+3)}-\xi\dfrac{\eta\mu^{\eta+\beta+2}}{(\eta+\beta)(\eta+\beta+2)}\right)\right\} \\ +h\left\{C_2\left((t_1-\mu)-\dfrac{\theta}{2}(t_1^2-\mu^2)\right)-\alpha\left(\left(\dfrac{t_1^{\beta+1}-\mu^{\beta+1}}{\beta+1}\right)-\dfrac{\theta}{\beta+1}\left(\dfrac{t_1^{\beta+2}-\mu^{\beta+2}}{\beta+2}\right)\right)\right\} \\ +r\left\{C_2\left(\dfrac{(t_1^2-\mu^2)}{2}-\theta\dfrac{(t_1^3-\mu^3)}{3}\right)-\alpha\left(\left(\dfrac{t_1^{\beta+2}-\mu^{\beta+2}}{\beta+2}\right)-\dfrac{\theta}{\beta+1}\left(\dfrac{t_1^{\beta+3}-\mu^{\beta+3}}{\beta+3}\right)\right)\right\}\end{array}\right] \tag{15}$$

Total Cost

Hence, the total cost of the system per time unit is denoted by TC and defined as

$$TC = \frac{1}{T}[OC + DC + SC + LSC + PC + IHC]$$

$$TC=\frac{1}{T}\left[\begin{array}{l} A+C_d\left[\left\{Q_1\left(\psi\mu+\xi\mu^{\eta}\right)-\alpha\left(\frac{\psi\mu^{\beta+1}}{\beta+1}+\frac{\xi\eta\mu^{\eta+\beta}}{\eta+\beta}\right)\right\}+\theta\left\{C_2\left(t_1-\mu\right)-\alpha\frac{t_1^{\beta+1}-\mu^{\beta+1}}{\beta+1}\right\}\right] \\ +C_s\alpha\left[\begin{array}{l}\left\{(1-\delta T)\frac{T^{\beta+1}-t_1^{\beta+1}}{\beta+1}+\left(\frac{\delta\beta}{\beta+1}\right)\frac{T^{\beta+2}-t_1^{\beta+2}}{\beta+2}\right\} \\ -(T-t_1)\left\{(1-\delta T)t_1^{\beta}+\left(\frac{\delta\beta}{\beta+1}\right)t_1^{\beta+1}\right\}\end{array}\right] \\ +C_l\alpha\beta\delta\left[T\left(\frac{T^{\beta}-t^{\beta}}{\beta}\right)-\left(\frac{T^{\beta+1}-t^{\beta+1}}{\beta+1}\right)\right] \\ +C_p\left[\begin{array}{l}\frac{C_2(1-\theta\mu)-\alpha\left(\mu^{\beta}-\frac{\theta\mu^{\beta+1}}{\beta+1}\right)+\alpha\left(\mu^{\beta}-\frac{\psi\mu^{\beta+1}}{\beta+1}-\frac{\xi\eta\mu^{\eta+\beta}}{\eta+\beta}\right)}{1-\psi\mu-\xi\mu^{\beta}} \\ +\alpha\left(\left((1-\delta T)T^{\beta}+\frac{\delta\beta T^{\beta+1}}{\beta+1}\right)-\left((1-\delta T)t_1^{\beta}+\frac{\delta\beta t_1^{\beta+1}}{\beta+1}\right)\right)\end{array}\right] \\ +\left[\begin{array}{l} h\left\{Q_1\left(\mu-\psi\frac{\mu^2}{2}-\xi\frac{\mu^{\eta+1}}{\eta+1}\right)-\alpha\left(\frac{\mu^{\beta+1}}{\beta+1}-\psi\frac{\mu^{\beta+2}}{(\beta+1)(\beta+2)}-\xi\frac{\eta\mu^{\eta+\beta+1}}{(\eta+\beta)(\eta+\beta+1)}\right)\right\} \\ +r\left\{Q_1\left(\frac{\mu^2}{2}-\psi\frac{\mu^3}{3}-\xi\frac{\mu^{\eta+2}}{\eta+2}\right)-\alpha\left(\frac{\mu^{\beta+2}}{\beta+2}-\psi\frac{\mu^{\beta+3}}{(\beta+1)(\beta+3)}-\xi\frac{\eta\mu^{\eta+\beta+2}}{(\eta+\beta)(\eta+\beta+2)}\right)\right\} \\ +h\left\{C_2\left((t_1-\mu)-\frac{\theta}{2}\left(t_1^2-\mu^2\right)\right)-\alpha\left(\left(\frac{t_1^{\beta+1}-\mu^{\beta+1}}{\beta+1}\right)-\frac{\theta}{\beta+1}\left(\frac{t_1^{\beta+2}-\mu^{\beta+2}}{\beta+2}\right)\right)\right\} \\ +r\left\{C_2\left(\frac{\left(t_1^2-\mu^2\right)}{2}-\theta\frac{\left(t_1^3-\mu^3\right)}{3}\right)-\alpha\left(\left(\frac{t_1^{\beta+2}-\mu^{\beta+2}}{\beta+2}\right)-\frac{\theta}{\beta+1}\left(\frac{t_1^{\beta+3}-\mu^{\beta+3}}{\beta+3}\right)\right)\right\}\end{array}\right]\end{array}\right] \tag{16}$$

The optimum values μ_*, t_1^* and T^* of μ, t_1 and T respectively minimize the average total cost TC. An appropriate mathematical software gives these optimum values that can be obtained by solving equations $\frac{\partial TC}{\partial \mu}=0, \frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial TC}{\partial T}=0$ which satisfy the sufficient conditions

$$\left|\begin{matrix} \left[\frac{\partial^2 TC}{\partial \mu^2}\right] \\ \left[\left(\frac{\partial^2 TC}{\partial \mu^2}\right)\left(\frac{\partial^2 TC}{\partial {t_1}^2}\right) - \left(\frac{\partial^2 TC}{\partial \mu \partial t_1}\right)^2\right] \\ \begin{vmatrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_1} & \frac{\partial^2 TC}{\partial \mu \partial T} \\ \frac{\partial^2 TC}{\partial t_1 \partial \mu} & \frac{\partial^2 TC}{\partial {t_1}^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial \mu} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{vmatrix} \end{matrix}\right|_{\mu=\mu^*, t_1={t_1}^*, T=T^*} > 0 \qquad (17)$$

IV. NUMERICAL EXAMPLE

To illustrate the proposed method, we take a numerical example of an inventory system with the following parametric values in appropriate unit:

$C_d = 6, C_s = 20, C_l = 2.1, C_p = 1.2, A = 1500, h = 15, r = 0.5, \alpha = 0.0001, \beta = 2.1$
$\theta = 0.002, \psi = 0.0001, \xi = 0.0001, \eta = 5, \delta = 0.0002$

To find optimal value of the average total cost TC, we solve the equation (16) such that μ^*, t_1^* and T^* satisfy the conditions stated in equations (17). An appropriate software gives the value of Optimum Inventory Level (Q) = 0.53285434780 units and Optimal Total Cost per unit time
(TC) = 22.96085928 units.

V. SENSITIVITY ANALYSIS

Sensitivity analysis helps study the effect of changes in the input parameter values on the optimal solution of the model. The sensitivity is checked by considering 10% and 20% increase and decrease in each of the parameters keeping values of all other parameters fixed at a time. Here, the sensitivity for times μ, t_1, Cycle time T and total cost per time per unit (TC) with respect to the changes in the parameters C_s, A, α, β, C_p, h, r and θ The results are as shown in the table below.

Table I : Partial Sensitivity Analysis

Parameters	%Change	μ	t_1	T	Q	TC
C_s	-20%	0.0614	39.5177	100.2997	0.4752	21.7254
	-10%	0.0690	40.6664	97.1857	0.5054	22.4185
	10%	0.0845	42.5754	92.2805	0.5579	23.6093
	20%	0.0923	43.3800	90.3060	0.5808	23.9596
A	-20%	0.0617	39.5664	88.1682	0.4765	19.6787
	-10%	0.0690	40.6678	91.4794	0.5055	21.3484
	10%	0.0848	42.6104	97.4146	0.5589	24.5235
	20%	0.0934	43.4792	100.1083	0.5837	26.0422
α	-20%	0.0641	44.3100	101.4558	0.4864	21.3718
	-10%	0.0703	42.8988	97.7486	0.5104	22.1968
	10%	0.0832	40.6041	91.7443	0.5541	23.6740
	20%	0.0900	39.6497	89.2577	0.5743	24.3438
β	-20%	0.0353	79.2601	200.3977	0.3408	11.3835
	-10%	0.0514	55.9067	133.7363	0.4309	16.6478
	10%	0.1164	32.2499	69.9635	0.6379	30.3069
	20%	0.1806	25.6716	53.7201	0.7385	38.6358

C_p	-20%	0.0684	41.6800	94.5557	0.5329	22.9572
	-10%	0.0725	41.6786	94.5520	0.5328	22.9590
	10%	0.0807	41.6758	94.5446	0.5328	22.9626
	20%	0.0849	41.6744	94.5409	0.5327	22.9644
h	-20%	0.1238	44.0233	95.1802	0.5995	22.7502
	-10%	0.09622	42.8146	94.8506	0.5646	22.8604
	10%	0.06235	40.6055	94.2706	0.5038	23.0526
	20%	0.05158	39.5943	94.0149	0.4772	23.1366
r	-20%	0.0928	43.4358	95.0604	0.5824	22.8299
	-10%	0.0839	42.5228	94.7910	0.5564	22.8981
	10%	0.0706	40.8906	94.3282	0.5114	23.0189
	20%	0.0654	40.1559	94.1276	0.4918	23.0727
θ	-20%	0.0730	41.6336	94.5837	0.5259	22.9935
	-10%	0.0749	41.6549	94.5658	0.5294	22.9772
	10%	0.0783	41.7007	94.5311	0.5363	22.9444
	20%	0.0799	41.7252	94.5142	0.5398	22.9278

IV. GRAPHICAL PRESENTATION OF SENSITIVITY ANALYSIS

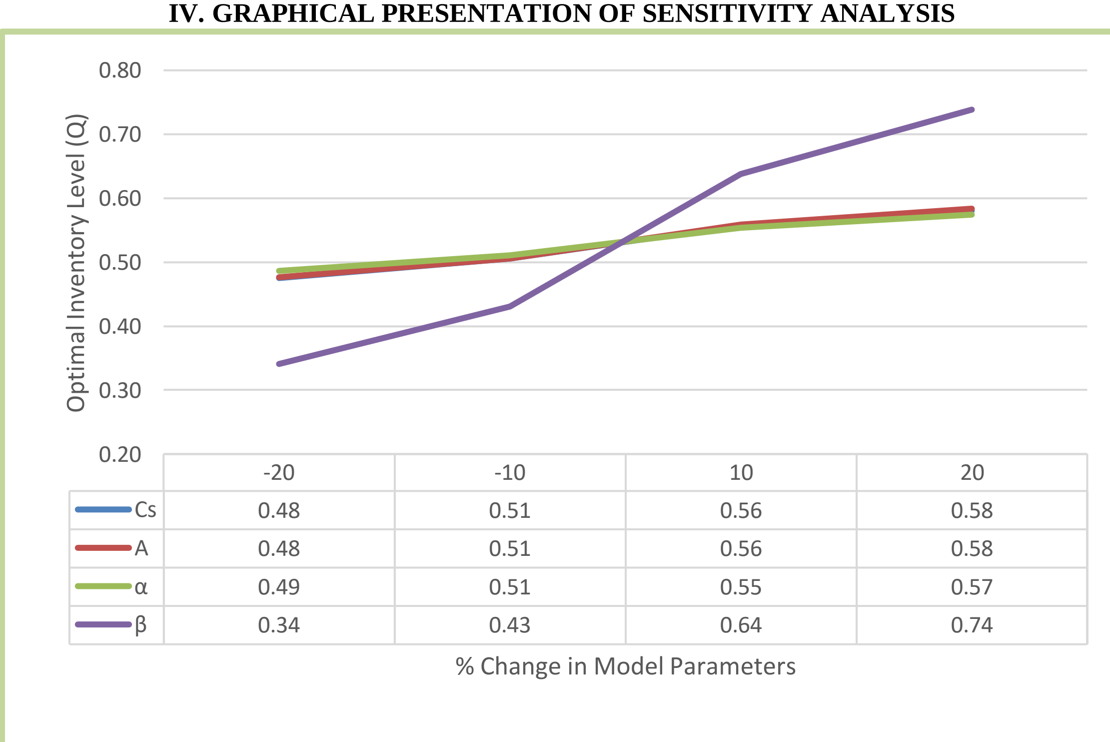
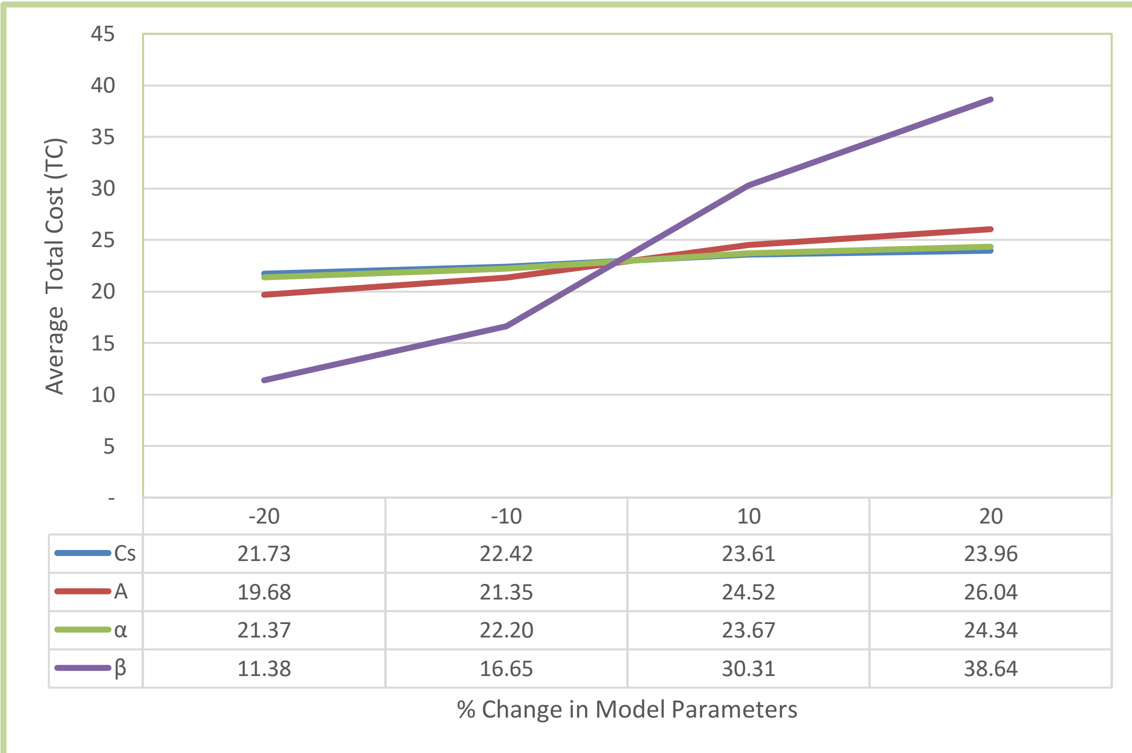


Fig. 2



	-20	-10	10	20
Cs	21.73	22.42	23.61	23.96
A	19.68	21.35	24.52	26.04
α	21.37	22.20	23.67	24.34
β	11.38	16.65	30.31	38.64

Fig. 3

VII. CONCLUSIONS

1. Total cost is highly sensitive to the changing ordering cost A which means that to reduce overall inventory cost, we need to order optimum. (Refer Fig. (3)). Total cost is also sensitive to the shortage cost indicating that not being able to meet the demand may lead to cancellation of orders and heavy losses which affects the overall average costs of inventory management.
2. In Fig. (2), it is seen that the optimal order quantity Q is mostly affected by the change in the values of the shape and scale parameters of the demand function. Increase in demand leads to increase in order quantity also leading to an increase in the average total cost of the inventory as seen in figure (2).

REFERENCES

[1] Alin Rosca and Natalia Rosca R. S. (2013) On (T, Si) policy inventory model for deteriorating items with time proportional demand, ISSN: 1582-5329 No. 35/2013, pp. 229-244.

[2] Chandra, Sujan. (2017) An inventory model with ramp type demand, time varying holding cost and price discount on backorders. *Uncertain Supply Chain Management*. 5. 51-58.

[3] Covert, R. P., & Philip, G. S. (1973) An EOQ model for items with Weibull distribution deterioration. *AIIE Transactions*, 5, 323–326.

[4] Dye, C.Y., Hsieh, T.P., and Ouyang, L.Y., (2007), Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging, *European Journal of Operational Research*, 181, 668- 678.

[5] Ghare, P.M., and Schrader, G.H., (1963), A model for exponentially decaying inventory system, *Journal of industrial Engg*.14, 238-243.

[6] Giri,B.C.,Goswami,A.,&Chaudhuri,K.S.(1996).An EOQ model for deteriorating items with time-varying demand and costs, *The Journal of the Operational Research Society*, 47(11), 1398–1405.

[7] G.P. Samanta, Jhuma Bhowmick (2010), A deterministic inventory system with Weibull distribution deterioration and ramp type demand rate, *Electronic Journal of Applied Statistical Analysis EJASA*, 3(2), 92 – 114.

[8] K. J. Chung and P. S. Ting, (1993) A Heuristic for Replenishment for Deteriorating Items with a Linear Trend in Demand, *Journal of the Operational Research Society*, Vol. 44, 1993, pp. 1235-1241.

[9] Mishra,V. K., Singh, L. S., & Kumar, R.(2013), An inventory model for deteriorating items with time dependent demand and time-varying holding cost under partial backlogging, *Journal of Industrial Engineering International*. doi:10.1186/2251-712X-9-4.

[10] Parmar, K.C., & Gothi, U.B. (2015), EPQ model for deteriorating items under three-parameter Weibull distribution and time dependent IHC with shortages, *American Journal of Engineering Research*, Vol. 4, No. 7, pp. 246-255.

[11] T. M. Whitin, (1957) Theory of Inventory Management, *Princeton University Press*, Princeton, pp. 62-72.

[12] Yadav,R.K.,&Vats,A.K.(2014).A deteriorating inventory model for quadratic demand and constant holding cost with partial backlogging and inflation. *IOSR Journal of Mathematics* (IOSR-JM), 10(3), 47–52.