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The (a, b)-Status Neighborhood Dakshayani Index

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Abstract: *Many distance based topological indices of a graph have been studied in the literature. In this paper, we introduce the F_1-status neighborhood Dakshayani index, the general status neighborhood Dakshayani index, symmetric division status neighborhood Dakshayani index, first and second status neighborhood Dakshayani-Gourava indices, (a, b)-status neighborhood Dakshayani index of a graph. Also we propose the F_1-status neighborhood Dakshayani polynomial, symmetric division status neighborhood Dakshayani polynomial, first and second status neighborhood Dakshayani-Gourava polynomials of a graph. We compute these newly defined status neighborhood Dakshayani indices for certain standard graphs and friendship graphs.*

Keywords: *status, F_1-status neighborhood Dakshayani index, symmetric division status neighborhood Dakshayani index, (a, b)-status neighborhood Dakshayani index, graph.*

Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*35.

I. Introduction

Let G be a simple, finite, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The distance $d(u, v)$ between any two vertices u and v is the length of shortest path containing u and v. The status $\sigma(u)$ of a vertex u in a connected graph G is the sum of distances of all other vertices from u in G. Let $N(u)=N_G(u)=\{v:uv\in E(G)\}$. Let $\sigma_n(u)=\sum_{v\in N(u)}\sigma(v)$. Let $\sigma_d(u)=\sigma(u)+\sum_{v\in N(u)}\sigma(v)$. Then $\sigma_d(u)$ is the status sum of closed neighborhood vertices u. We refer [1] for undefined term and notation.

A topological index is a numerical parameter mathematically derived from the graph structure. Topological indices have been found to be useful in chemical documentation, isomer discrimination, QSPR/QSAR study, see [2, 3]. Among distance based indices, Wiener index [4], Harary index [5], status [6, 7] are studied well in the literature. Several status indices of a graph can be found in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

The first and second status neighborhood Dakshayani indices of a graph were introduced by Kulli in [20], defined as

$$SD_1(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)\right], \qquad SD_2(G)=\sum_{uv\in E(G)}\sigma_d(u)\sigma_d(v).$$

Recently, some variants of status neighborhood Dakshayani indices were studied in [21, 22, 23].

Motivated by the work on distance based indices, we introduce the F_1-status neighborhood Dakshayani index of a graph, defined as

$$F_1SD(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)^2+\sigma_d(v)^2\right].$$

Considering the F_1-status neighborhood Dakshayani index, we introduce the F_1-status neighborhood Dakshayani polynomial of a graph G, defined as

$$F_1SD(G,x)=\sum_{uv\in E(G)}x^{\sigma_d(u)^2+\sigma_d(v)^2}.$$

We propose the general status neighborhood Dakshayani index of a graph G, defined it as

$$SD_a(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)^a+\sigma_d(v)^a\right],$$

where a is a real number.

We now introduce the symmetric division status neighborhood Dakshayani index of a graph G, defined as

$$SDSD(G)=\sum_{uv\in E(G)}\left(\frac{\sigma_d(u)}{\sigma_d(v)}+\frac{\sigma_d(v)}{\sigma_d(u)}\right).$$

Considering the symmetric division status neighborhood Dakshayani index, we define the symmetric division status neighborhood Dakshayani polynomial of a graph G as

$$SDSD(G,x)=\sum_{uv\in E(G)} x^{\left(\frac{\sigma_d(u)}{\sigma_d(v)}+\frac{\sigma_d(v)}{\sigma_d(u)}\right)}.$$

Also we introduce the first and second status neighborhood Dakshayani-Gourava indices of a graph G and they are defined as

$$SDGO_1(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)\right],$$

$$SDGO_2(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)+\sigma_d(v)\right]\sigma_d(u)\sigma_d(v).$$

Recently some Gourava indices were studied in [10, 12].

Considering the first and second status neighborhood Dakshayani-Gourava indices, we introduce the first and second status neighborhood Dakshayani-Gourava polynomials of a graph G and they are defined as

$$SDGO_1(G,x)=\sum_{uv\in E(G)} x^{\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)},$$

$$SDGO_2(G,x)=\sum_{uv\in E(G)} x^{\left[\sigma_d(u)+\sigma_d(v)\right]\sigma_d(u)\sigma_d(v)}.$$

We now introduce the (a, b)-status neighborhood Dakshayani index of a graph G, defined as

$$SD_{a,b}(G)=\sum_{uv\in E(G)}\left[\sigma_d(u)^a\sigma_d(v)^b+\sigma_d(u)^b\sigma_d(v)^a\right]$$

where a and b are real numbers.

II. Observations

We observe the following relations between (a, b)-status neighborhood Dakshayani index with some other neighborhood Dakshayani indices.

(i) $SD_1(G) = SD_{1,0}(G)$.
(ii) $F_1SD(G) = SD_{2,0}(G)$.
(iii) $SD_a(G) = SD_{a,0}(G)$.
(iv) $SDSD(G) = SD_{1,-1}(G)$.
(v) $SDSD_2(G) = SD_{2,1}(G)$.

III. Results for Complete Graphs

Theorem 1. The (a, b)-status neighborhood Dakshayani index of a complete graph K_n with n vertices is given by

$$SD_{a,b}(K_n)=\left[n(n-1)\right]^{a+b+1}.$$

Proof: If K_n is a complete graph with n vertices, then it has $\frac{n(n-1)}{2}$ edges. Then $d_{K_n}(u)=n-1$ and $\sigma(u)=n-1$ for every vertex u in K_n. Thus $\sigma_n(u)=(n-1)^2$ for every vertex u in K_n. Therefore $\sigma_d(u)=n-1+(n-1)^2=n(n-1)$. Thus

$$SD_{a,b}(K_n)=\sum_{uv\in E(K_n)}\left[\sigma_d(u)^a\sigma_d(v)^b+\sigma_d(u)^b\sigma_d(v)^a\right]$$

$$=\frac{n(n-1)}{2}\left[\{n(n-1)\}^a\{n(n-1)\}^b+\{n(n-1)\}^b\{n(n-1)\}^a\right]$$

$$=\left[n(n-1)\right]^{a+b+1}.$$

We establish the following results from Theorem 1.

Corollary 1.1. Let K_n be a complete with n vertices. Then

(i) $SD_1(K_n)=SD_{1,0}(K_n)=\left[n(n-1)\right]^2=n^4-2n^3+n^2.$

(ii) $F_1SD(K_n)=SD_{2,0}(K_n)=n^3(n-1)^3.$

(iii) $SD_a(K_n)=SD_{a,0}(K_n)=\left[n(n-1)\right]^{a+1}.$

(iv) $SDSD(K_n)=SD_{1,-1}(K_n)=n(n-1).$

(v) $SDGO_2(K_n) = SD_{2,1}(K_n) = n^4(n-1)^4.$

Theorem 2. The first status neighborhood Dakshayani-Gourava index of a complete graph K_n is given by

$$SDGO_1(K_n) = \frac{1}{2}n^2(n-1)^2(n^2-n+2).$$

Proof: We have $\sigma_d(u) = n(n-1)$ for every vertex u in K_n. By definition, we have

$$SDGO_1(K_n) = \sum_{uv\in E(K_n)} \left[\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)\right]$$
$$= \frac{n(n-1)}{2}\left[n(n-1)+n(n-1)+n(n-1)n(n-1)\right]$$
$$= \frac{1}{2}n^2(n-1)^2(n^2-n+2).$$

Theorem 3. Let K_n be a complete graph with n vertices. Then

(i) $F_1SD(K_n,x) = \frac{n(n-1)}{2}x^{2n^2(n-1)^2}.$

(ii) $SDSD(K_n,x) = \frac{n(n-1)}{2}x^2.$

(iii) $SDGO_1(K_n,x) = \frac{n(n-1)}{2}x^{n(n-1)(n^2-n+2)}.$

(iv) $SDGO_2(K_n,x) = \frac{n(n-1)}{2}x^{2n^3(n-1)^3}.$

Proof: We have $\sigma_d(u) = n(n-1)$ for every vertex u in K_n. Thus

(i) $$F_1SD(K_n,x) = \sum_{uv\in E(K_n)} x^{\sigma_d(u)^2+\sigma_d(v)^2} = \frac{n(n-1)}{2}x^{n^2(n-1)^2+n^2(n-1)^2}$$
$$= \frac{n(n-1)}{2}x^{2n^2(n-1)^2}.$$

(ii) $$SDSD(K_n,x) = \sum_{uv\in E(K_n)} x^{\frac{\sigma_d(u)}{\sigma_d(v)}+\frac{\sigma_d(v)}{\sigma_d(u)}} = \frac{n(n-1)}{2}x^{\frac{n(n-1)}{n(n-1)}+\frac{n(n-1)}{n(n-1)}}$$
$$= \frac{n(n-1)}{2}x^2.$$

(iii) $$SDGO_1(K_n,x) = \sum_{uv\in E(K_n)} x^{\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)} = \frac{n(n-1)}{2}x^{n(n-1)+n(n-1)+n(n-1)n(n-1)}$$
$$= \frac{n(n-1)}{2}x^{n(n-1)(n^2-n+2)}.$$

(iv) $$SDGO_2(K_n,x) = \sum_{uv\in E(K_n)} x^{[\sigma_d(u)+\sigma_d(v)]\sigma_d(u)\sigma_d(v)} = \frac{n(n-1)}{2}x^{[n(n-1)+n(n-1)]n(n-1)n(n-1)}$$
$$= \frac{n(n-1)}{2}x^{2n^3(n-1)^3}.$$

IV. Results for Complete Bipartite Graphs

Theorem 4. The (a, b)-status neighborhood Dakshayani index of a complete bipartite graph $K_{p,q}$ is given by

$$SD_{a,b}(K_{p,q}) = pq\Big[\{(1+p)(q+2p-2)\}^a\{(1+q)(p+2q-2)\}^b +$$

$$\left\{(1+p)(q+2p-2)\right\}^{b}\left\{(1+q)(p+2q-2)\right\}^{a}\Big].$$

Proof: The vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that u□V_1 and v□V_2 for any edge uv in $K_{p,q}$. Let $K= K_{p,q}$. We have $d_K(u) = q$ and $d_K(v) = p$. Then □$(u)=q+2p – 2$ and □$(v)=p+2q – 2$. Thus □$_d(u)=p(q+2p – 2)$ and □$_d(v)=q(p+2q – 2)$. Hence □$_d(u)=(1+p)(q+2p – 2)$ and □$_d(v)= (1+q)(p+2q – 2)$. Therefore

$$SD_{a,b}\left(K_{p,q}\right)=\sum_{uv\in E(K_n)}\left[\sigma_d(u)^a\,\sigma_d(v)^b+\sigma_d(u)^b\,\sigma_d(v)^a\right]$$

$$=pq\Big[\left\{(1+p)(q+2p-2)\right\}^{a}\left\{(1+q)(p+2q-2)\right\}^{b}$$

$$+\left\{(1+p)(q+2p-2)\right\}^{b}\left\{(1+q)(p+2q-2)\right\}^{a}\Big].$$

We obtain the following results from Theorem 4 and observations.

Corollary 4.1. Let $K_{p,q}$ be a complete bipartite. Then

(i) $SD_1\left(K_{p,q}\right)=pq\left[2\left(p^2+q^2\right)+(p+q)+2pq-4\right].$

(ii) $F_1SD\left(K_{p,q}\right)=pq\left[(1+p)^2(q+2p-2)^2+(1+q)^2(p+2q-2)^2\right].$

(iii) $SD_a\left(K_{p,q}\right)=pq\left[(1+p)^a(q+2p-2)^a+(1+q)^a(p+2q-2)^a\right].$

(iv) $SDSD\left(K_{p,q}\right)=pq\left[\frac{(1+p)(q+2p-2)}{(1+q)(p+2q-2)}+\frac{(1+q)(p+2q-2)}{(1+p)(q+2p-2)}\right].$

(v) $SDGO_2\left(K_{p,q}\right)=pq\Big[\left\{(1+p)(q+2p-2)+(1+q)(p+2q-2)\right\}$
$(1+p)(1+q)(q+2p-2)(p+2q-2)\Big].$

Theorem 5. The first status neighborhood Dakshayani-Gourava index of a complete bipartite graph $K_{p,q}$ is

$$SDGO_1\left(K_{p,q}\right)=pq\Big[\left\{2\left(p^2+q^2\right)+(p+q)+2pq-4\right\}+(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}\Big].$$

Proof: We have $\sigma_d(u)=(1+p)(q+2p-2)$ and $\sigma_d(v)=(1+p)(q+2p-2)$ for every edge uv in $K_{p,q}$. Therefore

$$SDGO_1\left(K_{p,q}\right)=\sum_{uv\in E(K)}\left[\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)\right]$$

$$=pq\left[(1+p)(q+2p-2)+(1+q)(p+2q-2)+(1+p)(q+2p-2)(1+q)(p+2q-2)\right]$$

$$=pq\Big[\left\{2\left(p^2+q^2\right)+(p+q)+2pq-4\right\}+(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}\Big].$$

Theorem 6. Let $K_{p,q}$ be a complete bipartite graph. Then

(i) $F_1SD\left(K_{p,q},x\right)=pqx^{(1+p)^2(q+2p-2)^2+(1+q)^2(p+2q-2)^2}.$

(ii) $SDSD\left(K_{p,q},x\right)=pqx^{\frac{(1+p)(q+2p-2)}{(1+q)(p+2q-2)}+\frac{(1+q)(p+2q-2)}{(1+p)(q+2p-2)}}.$

(iii) $SDGO_1\left(K_{p,q},x\right)=pqx^{\left\{2\left(p^2+q^2\right)+(p+q)+2pq-4\right\}+(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}}.$

(iv) $SDGO_2\left(K_{p,q},x\right)=pqx^{\left\{2\left(p^2+q^2\right)+(p+q)+2pq-4\right\}(1+p)(1+q)\left\{2\left(p^2+q^2\right)-6(p+q)+5pq+4\right\}}.$

Proof: We have $\sigma_d(u)=(1+p)(q+2p-2)$ and $\sigma_d(v)=(1+q)(p+2q-2)$ for every edge uv in $K_{p,q}$. Therefore

(i) $F_1SD\left(K_{p,q},x\right)=\sum_{uv\in E(K)}x^{\sigma_d(u)^2+\sigma_d(v)^2}=pqx^{(1+p)^2(q+2p-2)^2+(1+q)^2(p+2q-2)^2}.$

(ii) $SDSD\left(K_{p,q},x\right)=\sum_{uv\in E(K)}x^{\frac{\sigma_d(u)}{\sigma_d(v)}+\frac{\sigma_d(v)}{\sigma_d(u)}}=pqx^{\frac{(1+p)(q+2p-2)}{(1+q)(p+2q-2)}+\frac{(1+q)(p+2q-2)}{(1+p)(q+2p-2)}}.$

(iii) $SDGO_1\left(K_{p,q},x\right)=\sum_{uv\in E(K_n)}x^{\left[\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)\right]}$

$$= pqx^{(1+p)(q+2p-2)+(1+q)(p+2q-2)+(1+p)(q+2p-2)(1+q)(p+2q-2)}.$$

After simplifying, we get the desired result.

(iv) $$SDGO_2\left(K_{p,q},x\right) = \sum_{uv\in E(K_n)} x^{\left[\sigma_d(u)+\sigma_d(v)\right]\sigma_d(u)\sigma_d(v)}$$

$$= pqx^{[(1+p)(q+2p-2)+(1+q)(p+2q-2)](1+p)(q+2p-2)(1+q)(p+2q-2)}.$$

After simplifying, we obtain the desired result.

V. Results for Wheel Graphs

A wheel graph W_n is the join of C_n and K_1. A wheel graph W_n is shown in Figure 1.

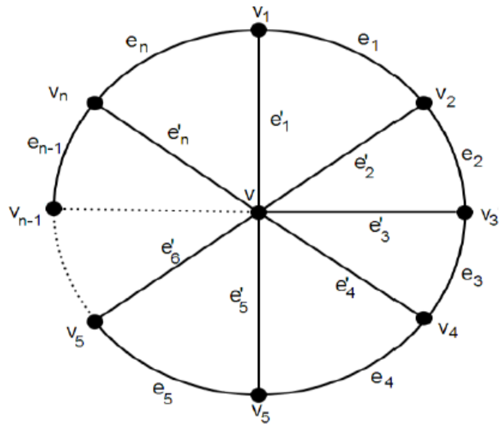

Figure 1. Wheel graph W_n

A graph W_n has n+1 vertices and 2n edges. In this graph, there are two types of vertices as follows:

$V_1 = \{u \in V(W_n) \mid \sigma(u) = n\}$, $|V_1| = 1$.

$V_2 = \{u \in V(W_n) \mid \sigma(u) = 2n-3\}$, $|V_2| = n$.

By calculation, we obtain that there are two types of status neighborhood vertices as follows:

$V_1 = \{u \in V(W_n) \mid \sigma_n(u) = n(2n-3)\}$, $|V_1| = 1$.

$V_2 = \{u \in V(W_n) \mid \sigma_n(u) = 5n-6\}$, $|V_2| = n$.

By calculation, we obtain that there are two types of status neighborhood Dakshayani vertices as given in Table 1.

$\sigma_d(u) \setminus u \in V(W_n)$	$2(2n-2)$	$7n-9$
Number of edges	n	n

Table 1. Status neighborhood Dakshayani vertex partition of W_n

By calculation, we obtain that there are two types of status neighborhood Dakshayani edges as given in Table 2.

$\sigma_d(u), \sigma_d(v) \setminus uv \in V(W_n)$	$(7n-9, 7n-9)$	$(7n-9, n(2n-2))$
Number of edges	n	n

Table 2. Status neighborhood Dakshayani edge partition of W_n

Theorem 7. The (a, b) status neighborhood Dakshayani index of a wheel graph W_n is given by

$$SD_{a,b}(W_n) = 2n(7n-9)^{a+b} + n\left[(7n-9)^a(2n^2-2n)^b + (7n-9)^b(2n^2-2n)^a\right].$$

Proof : Using definition and Table 2, we deduce

$$SD_{a,b}(W_n) = \sum_{uv\in E(W_n)} \left[\sigma_d(u)^a \sigma_d(v)^b + \sigma_d(u)^b \sigma_d(v)^a\right]$$

$$= n\left[(7n-9)^a(7n-9)^b + (7n-9)^b(7n-9)^a\right] + n\left[(7n-9)^a(2n-2n)^b + (7n-9)^b(2n-2n)^a\right]$$

$$= 2n(7n-9)^{a+b} + n\left[(7n-9)^a(2n^2-2n)^b + (7n-9)^b(2n^2-2n)^a\right].$$

Corollary 7.1. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $SD_1(W_n) = SD_{1,0}(W_n) = 2n^3 + 19n^2 - 27n.$

(ii) $F_1SD(W_n) = SD_{2,0}(W_n) = 3(7n-9)^2 + 4n^3(n^2 - 2n + 1).$

(iii) $SD_a(W_n) = SD_{a,0}(W_n) = 2n(7n-9)^a + n\left[(7n-9)^a + (2n^2 - 2n)^a\right].$

(iv) $SDSD(W_n) = SD_{1,-1}(W_n) = 2n\dfrac{(7n-9)^2 + 4n^2(n-1)^2}{2(n-1)(7n-9)}.$

(v) $SDGO_2(W_n) = SD_{2,1}(W_n) = 2n(7n-9)^3 + n(2n^2 + 5n - 9)(7n-9)(2n^2 - 2n).$

Theorem 8. The first status neighborhood Dakshayani-Gourava index of a wheel graph W_n is

$$SDGO_1(W_n) = 14n^4 + 19n^3 - 89n^2 + 54n.$$

Proof: Using definition and Table 2, we derive

$$SGO_1(W_n) = \sum_{uv \in E(W_n)} \left[\sigma_d(u) + \sigma_d(v) + \sigma_d(u)\sigma_d(v)\right]$$

$$= n\left[(7n-9) + (7n-9) + (7n-9)(7n-9)\right] + n\left[(7n-9)(2n^2 - 2n) + (7n-9)(2n^2 - 2n)\right]$$

$$= 14n^4 + 19n^3 - 89n^2 + 54n.$$

Theorem 9. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $F_1SD(W_n, x) = nx^{2(7n-9)^2} + nx^{(7n-9)^2 + (2n^2-2n)^2}.$

(ii) $SDSD(W_n, x) = nx^2 + nx^{\frac{(7n-9)^2 + (2n^2-2n)^2}{2n(n-1)(7n-9)}}.$

(iii) $SDGO_1(W_n, x) = nx^{49n^2 - 112n + 63} + nx^{14n^3 - 30n^2 + 23n - 9}.$

(iv) $SDGO_2(W_n, x) = nx^{2(7n-9)^3} + nx^{(2n^2+5n-9)(7n-9)(2n^2-2n)}.$

Proof: From definitions and Table 2, we derive

(i) $$F_1SD(W_n, x) = \sum_{uv \in E(W_n)} x^{\sigma_d(u)^2 + \sigma_d(v)^2}$$

$$= nx^{(7n-9)^2 + (7n-9)^2} + nx^{(7n-9)^2 + (2n^2-2n)^2}$$

$$= nx^{2(7n-9)^2} + nx^{(7n-9)^2 + (2n^2-2n)^2}.$$

(ii) $$SDSD(W_n, x) = \sum_{uv \in E(W_n)} x^{\frac{\sigma_d(u)}{\sigma_d(v)} + \frac{\sigma_d(v)}{\sigma_d(u)}}$$

$$= nx^{\frac{7n-9}{7n-9} + \frac{7n-9}{7n-9}} + nx^{\frac{7n-9}{2n^2-2n} + \frac{2n^2-2n}{7n-9}}$$

$$= nx^2 + nx^{\frac{(7n-9)+(2n^2-2n)^2}{2n(n-1)(7n-9)}}.$$

(iii) $$SDGO_1(W_n, x) = \sum_{uv \in E(W_n)} x^{\sigma_d(u) + \sigma_d(v) + \sigma_d(u)\sigma_d(v)}$$

$$= nx^{7n-9+7n-9+(7n-9)(7n-9)} + nx^{7n-9+2n^2-2n+(7n-9)(2n^2-2n)}$$

$$= nx^{49n^2 - 112n + 63} + nx^{14n^3 - 30n^2 + 23n - 9}.$$

(iv) $$SDGO_2(W_n, x) = \sum_{uv \in E(W_n)} x^{\left[\sigma_d(u) + \sigma_d(v)\right]\sigma_d(u)\sigma_d(v)}$$

$$= nx^{(7n-9+7n-9)(7n-9)(7n-9)} + nx^{(7n-9+2n^2-2n)(7n-9)(2n^2-2n)}.$$

$$= nx^{2(7n-9)^3} + nx^{(2n^2+5n-9)(7n-9)(2n^2-2n)}.$$

VI. Results for Friendship Graphs

A friendship graph F_n is the graph obtained by taking n □ 2 copies of C_3 with vertex in common. A friendship graph F_4 is shown in Figure 2.

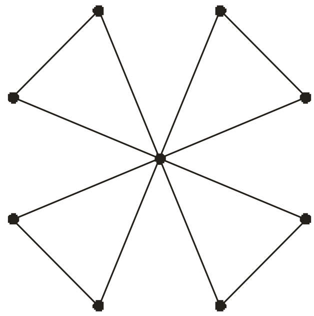

Figure 2. Friendship graph F_4

A graph F_n has $2n$+1 vertices and $3n$ edges. In F_n, there are two types of status vertices as follows:

$V_1 = \{u \in V(F_n) \mid \sigma(u) = 2n\}$, $|V_1| = 1$.

$V_2 = \{u \in V(F_n) \mid \sigma(u) = 4n - 2\}$, $|V_2| = 2n$.

By Calculation, there are two types of status neighborhood vertices as follows:

$V_1 = \{u \in V(F_n) \mid \sigma_n(u) = 2n(4n - 2)\}$, $|V_1| = 1$.

$V_2 = \{u \in V(F_n) \mid \sigma_n(u) = 6n - 2\}$, $|V_2| = 2n$.

By calculation, we obtain that there are two types of status neighborhood Dakshayani vertices as given in Table 3.

$\sigma_d(u) \setminus u \in V(F_n)$	$2n(4n - 1)$	$10n - 4$
Number of edges	1	$2n$

Table 3. Status neighborhood Dakshayani vertex partition of F_n

By calculation, we obtain that there are two types of status neighborhood Dakshayani edges as given in Table 4.

$\sigma_d(u), \sigma_d(v) \setminus uv \in E(F_n)$	$(10n - 4, 10n - 4)$	$(10n - 4, 2n(4n - 1))$
Number of edges	n	$2n$

Table 4. Status neighborhood Dakshayani edge partition of F_n

Theorem 10. The (a, b)-status neighborhood Dakshayani index of a friendship graph F_n is

$$SD_{a,b}(F_n) = 2n(10n-4)^{a+b} + 2n\left[(10n-4)^a(8n^2-2n)^b + (10n-4)^b(8n^2-2n)^a\right].$$

Proof : Using definition and Table 4, we derive

$$S_{a,b}(F_n) = \sum_{uv \in E(F_n)} \left[\sigma_d(u)^a \sigma_d(v)^b + \sigma_d(u)^b \sigma_d(v)^a\right]$$

$$= n\left[(10n-4)^a(10n-4)^b + (10n-4)^b(10n-4)^a\right] + 2n\left[(10n-4)^a(8n^2-2n)^b + (10n-4)^b(8n^2-2n)^a\right]$$

$$= 2n(10n-4)^{a+b} + 2n\left[(10n-4)^a(8n^2-2n)^b + (10n-4)^b(8n^2-2n)^a\right].$$

Corollary 10.1. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(i) $SD_1(F_n) = SD_{1,0}(F_n) = 16n^3 + 36n^2 - 16n.$

(ii) $F_1SD(F_n) = SD_{2,0}(F_n) = 128n^5 - 64n^4 + 408n^3 - 320n^2 + 64n^2.$

(iii) $SD_a(F_n) = SD_{a,0}(F_n) = 4n(10n-4)^a + 2n(8n^2-2n)^a.$

(iv) $SDSD(F_n) = SD_{1,-1}(F_n) = 2n + 2n\dfrac{(5n-2)^2 + (4n^2-n)^2}{(4n^2-n)(5n-2)}.$

(v) $SDGO_2(F_n) = SD_{2,1}(F_n) = 2n(10n-4)^3 + 2n(8n^2+8n-4)(10n-4)(8n^2-2n).$

Theorem 11. The first status neighborhood Dakshayani-Gourava index of a friendship graph F_n is

$$SDGO_1(F_n) = 160n^4 + 12n^3 - 44n^2.$$

Proof: Using definition and Table 4, we deduce

$$SDGO_1(F_n) = \sum_{uv\in E(F_n)} \left[\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)\right]$$

$$= n\left[(10n-4)+(10n-4)+(10n-4)(10n-4)\right]+2n\left[(10n-4)+(8n^2-2n)+(10n-4)(8n^2-2n)\right]$$

$$= 160n^4+12n^3-44n^2.$$

Theorem 12. Let F_n be a friendship graph with $2n$+1 vertices and $3n$ edges. Then

(i) $F_1SD(F_n, x) = nx^{2(10n-4)^2} + 2nx^{(10n-4)^2+(8n^2-2n)^2}$.

(ii) $SDSD(F_n, x) = nx^2 + 2nx^{\frac{(5n-2)^2+(4n^2-n)^2}{(4n^2-n)(5n-2)}}$.

(iii) $SDGO_1(F_n, x) = nx^{100n^2-60n+8} + 2nx^{80n^3-44n^2+8n-4}$.

(iv) $SDGO_2(F_n, x) = nx^{2(10n-4)^3} + 2nx^{(8n^2+8n-4)(10n-4)(8n^2-2n)}$.

Proof: Using definitions and Table 4, we obtain

(i) $$F_1SD(F_n, x) = \sum_{uv\in E(F_n)} x^{\sigma_d(u)^2+\sigma_d(v)^2}$$

$$= nx^{(10n-4)^2+(10n-4)^2} + 2nx^{(10n-4)^2+(8n^2-2n)^2}$$

$$= nx^{2(10n-4)^2} + 2nx^{(10n-4)^2+(8n^2-2n)^2}.$$

(ii) $$SDSD(F_n, x) = \sum_{uv\in E(F_n)} x^{\frac{\sigma_d(u)}{\sigma_d(v)}+\frac{\sigma_d(v)}{\sigma_d(u)}}$$

$$= nx^{\frac{10n-4}{10n-4}+\frac{10n-4}{10n-4}} + nx^{\frac{10n-4}{8n^2-2n}+\frac{8n^2-2n}{10n-4}}$$

$$= nx^2 + 2nx^{\frac{(5n-2)^2+(4n^2-n)^2}{(4n^2-n)(5n-2)}}.$$

(iii) $$SDGO_1(F_n, x) = \sum_{uv\in E(F_n)} x^{\sigma_d(u)+\sigma_d(v)+\sigma_d(u)\sigma_d(v)}$$

$$= nx^{10n-4+10n-4+(10n-4)(10n-4)} + nx^{10n-4+8n^2-2n+(10n-4)(8n^2-2n)}$$

$$= nx^{100n^2-60n+8} + 2nx^{80n^3-44n^2+8n-4}.$$

(iv) $$SDGO_2(F_n, x) = \sum_{uv\in E(F_n)} x^{\left[\sigma_d(u)+\sigma_d(v)\right]\sigma_d(u)\sigma_d(v)}$$

$$= nx^{(10n-4+10n-4)(10n-4)(10n-4)} + 2nx^{(10n-4+8n^2-2n)(10n-4)(8n^2-2n)}.$$

$$= nx^{2(10n-4)^3} + 2nx^{(8n^2+8n-4)(10n-4)(8n^2-2n)}.$$

CONCLUSION

In this paper, we have introduced some new status neighborhood Dakshayani indices and polynomials of a graph. Also we have determined these newly defined the status neighborhood Dakshayani indices for complete graphs, complete bipartite graphs, wheel graphs and friendship graphs.

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