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Neighborhood, Locating Dominating and Almost Locating Dominating Set of a Ladder Graph

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Abstract: *In this paper, we have defined four variants that is simple, powerful, maximal, and foul of n-set, nl-set and ald-set. We have also obtained exact values of all these numbers for the ladder graph of order m.*

Keywords: *neighbourhood set, locating dominating set, almost locating dominating set.*

AMS Subject Classification Number: 05C20

I.Introduction

All the graphs considered in this paper are simple, undirected, finite and connected. In a given network, locating dominating set can be viewed as a set of monitors which can determine the exact location of an intruder (e.g., burglar, fire, etc.,). The concept of locating dominating set is studied in [2, 3, 4, 5]. For similar related work we refer [7, 8, 9, 10, 11, 12, 13]. The terms not defined here may found in [1].

Throughout this paper *G (V, E)* denotes a graph. For a vertex $v \in V$, *N(v)* denotes the set of all vertices of *G* which are adjacent to *v* and *N[v] = N(v) U {v}.* The concept of neighbourhood number for a graph was first introduced by E. Sampathkumar et al. [6]. A subset *D* of vertices in a graph *G* is a dominating set(*d*-set) if every vertex in *V-D* is adjacent to some vertex in *D*. The domination number d(G) is the minimum cardinality of a dominating set of *G*. A neighbourhood dominating set (or simply *nd-set*) of a graph *G* is a dominating set *D* with the property that each adjacent pair of vertices in *V(G)-D* is dominated by a common vertex in *D*. A set $S \subseteq V(G)$ is an independent set, if there are no edge between the vertices in S. The number of vertices in *S* is called independent number of *G*, denoted by *id(G).*

Definition1.1. [6] Let *G (V, E)* be a graph. For a vertex $v \in V$, *N(v)* denotes the set of all vertices of *G* which are adjacent to *v* and *N[v] = N(v)* ∪ {*v*}. A subset *S* of *V* is called a neighbourhood set or n-set of *G*, if $G = \cup_{v \in S} < N[v] >$ where, $< S >$ denotes the subgraph of *G* induced by the set *S*. An *n*-set *S* is called minimal if no proper subset of *S* is an *n*-set. The minimum cardinality of a minimal n-set is called the neighbourhood number of *G* and is denoted by *n(G).*

Definition1.2. A subset *S* of *V(G)* is called locating dominating set (*ld*-set) in a connected graph *G* if for every pair vertices $u, v \in V(G)-S$, $N_G[u] \cap S \neq N_G[v] \cap S \neq \emptyset$. The minimum cardinality of locating dominating set is called locating domination number of *G*, denoted by *ld(G).*

Definition 1.3. A subset *S* of *V(G)* is called almost locating dominating set in a connected graph *G*, if for every pair of vertices $u, v \in V(G)-S$, $N_G[u] \cap S \neq N_G[v] \cap S$. The minimum cardinality of almost locating dominating set in *G* is called the almost locating domination number of *G*, denoted by *ald(G).* Location of each vertices with respect to *S* in *V-S* should be distinct, that is $N_G[v_i] \cap S = l(v_i|S \neq l(v_j|S) = N_G[v_j] \cap S$ for all $v_i, v_j \in V-S$.

Definition 1.4. A subset *S* of *V* is called a neighbourhood locating dominating set (or *nld*-set) of *G*, if *S* is both neighbourhood and locating dominating set of *G*. The minimum cardinality of a minimal *nld*-set is called the neighbourhood locating domination number of *G* and is denoted by *nld(G).*

We recall the following for immediate reference:

Theorem 1.5 [6]. A set *S* of vertices of a graph *G* is an *n*-set if and only if every line *of* $< V(G) - S >$ belongs to a triangle one of whose vertices belong to *S.*

Remark 1.6 [6]. If *G* is a triangle free graph, then by Theorem 1.5 a set *S* is an *n*-set of *G* if and only if for each edge $e = v_i, v_j$ of *G* either $v_i \in$ S or $v_j \in S$.

Remark 1.7 [6]. If *G* has no triangles, then $n(G) = \alpha_o(G)$, where α_o (G) is the vertex cover number of *G.*

Remark 1.8. A set *S* is an *n*-set of a triangle free graph if and only if $\bar{S}$ is totally disconnected.

Remark 1.9 [13]. Let *S* be a subset of a connected graph *G* with |*V(G)*| = *m*. Then *S* is always an *n*-set whenever |*S*| ≥ *m - 1*.

Theorem1.10 [8]. For any integer *n*, d ($P_n \times P_2$)= $\left\lfloor \frac{n+2}{2} \right\rfloor$.

In this paper, we have obtained results of simple, powerful, maximal and foul of *n*-set, *ld*-set and *ald*-set. We have also computed exact values of all these numbers for the ladder graph of order *m*.

II. Results on Ladder graph

Throughout this paper, L_m denotes a Ladder graph of order m ≥ 4 and m is an even integer with a vertex set *U* ∪ *V*, where *V* = {v_i: *1* ≤ i ≤ $\frac{m}{2}$}, *U* = {u_i: *1* ≤ $i \le \frac{m}{2}$}, and an edge set *E* = {$v_i v_{i+1}, u_i u_{i+1}, v_j u_j$; $1 \le i \le \frac{m}{2} - 1$ and $1 \le j \le \frac{m}{2}$ }.

Remark 2.1. From the definition of *d*-set and *ald*-set, for any graph *G*, |*d(G)*| ≤ |*ald(G)*|.

Remark 2.2. If *G* contains an induced subgraph C_4 then any two adjacent vertices of C_4 must be in *S* to locate the vertices in *G*.

Remark 2.3. For a ladder graph L_m of order $m \ge 6$, we have |*ld(G)*| ≥ |*ald(G)*|.

Remark 2.4. For a ladder graph L_m of order *m* ≥ *6*, any neighbourhood set (*n*-set) is also an almost locating dominating set (*ald*-set) and hence |*nl(G)*| ≥ |*ald(G)*|.

Remark 2.5. For a ladder graph L_m of order *m* ≥ *6*, any neighbourhood set (*n*-set) is also dominating set (*d*-set) and hence |*nd(G)*| ≥ |*ald(G)*|.

Remark 2.6. For a ladder graph L_m of order *m* ≥ *6*, any neighbourhood set (*n*-set) is also locating dominating set (*nl*-set) and hence |*nld(G)*| ≥ |*ald(G)*|.

Lemma 2.7. Any independent set *S* of a ladder L_m with |*S*| ≥ $\frac{m}{2}$ is always an *n*-set.

III. Types of Neighbourhood sets

In this section we call an *n*-set defined above as a simple *n*-set.

Definition 2. 8. An *n*-set *S* of the graph *G* is called a powerful *n*-set if $\bar{S}$ is also an *n*-set of *G*.

Definition 2.9. An *n*-set *S* of the graph *G* is called a maximal *n*-set if $\bar{S}$ is not an *n*-set of *G*.

Definition 2.10. A set *S* of vertices of the graph *G* is called a foul *n*-set if neither *S* nor $\bar{S}$ is an *n*-set of *G*.

The minimum cardinality of a simple *n*-set, powerful *n*-set, maximal *n*-set and foul *n*-set are respectively, called simple *n*-number, powerful *n*-number, maximal *n*-number and foul *n*-number of *G* and are denoted by $sim_n(G)$, $pow_n(G)$, $max_n(G)$ and $fou_n(G)$ respectively. From the above definitions it is clear that for every graph *G*, $pow_n(G) \ge sim_n(G)$ and $max_n(G) \ge sim_n(G)$.

IV. Simple *n*-set and Powerful *n*-set of a ladder graph

Theorem 2.11. For any even integer $m \ge 4$, $sim_n(L_m) = pow_n(L_m) = \frac{m}{2}$.

Proof. Let *S* be an *n*-set of L_m. Then $\bar{S}$ is independent (Since $L_m$is triangle free graph then by Remark 1.8 $\bar{S}$ should be independent set). Therefore, $sim_n(L_m)$ = *V*(L_m) - | $\bar{S}$| ≥ *m* - *id* (L_m)= $m - \frac{m}{2} = \frac{m}{2}$. On the other hand, consider a set $S = \{v_1, v_3, ..., v_{\frac{m}{2}-1}\} \cup \{u_2, u_3, ..., u_{\frac{m}{2}}\}$, when $\frac{m}{2}$ is even and $S = \{v_1, v_3, ..., v_{\frac{m}{2}}\} \cup \{u_2, u_3, ..., u_{\frac{m}{2}-1}\}$ when $\frac{m}{2}$ is odd, we see that *S* as well as $\bar{S}$ are independent and hence by Remark 1.8 $\cup_{v \in S} \langle N[v] \rangle = L_m$. Hence, $\bar{S}$ is also an n-set. Therefore, $sim_n(L_m) \le pow_n(L_m) \le |S| = \frac{m}{2}$.

V.Maximal *n*-set of a ladder graph

Theorem 2.12. For any even integer m $\geq$ 4, $max_n(L_m) = \frac{m}{2}+1$.

Proof. Let *S* be minimal *maximal n-set* of the graph L_m. Then *S* is an *n*-set and $\bar{S}$ is not an *n*-set. For the case *m = 4*, then suppose $|S| \leq 3$, say $|S| = 2$. In case of *S = {u, v}* where *u* and *v* are adjacent, both *S* and $\bar{S}$ are not *n*-sets, a contradiction. In case of *S = {u, v}* where *u* and *v* are antipodal vertices, both S and $\bar{S}$ are *n*-set, a contradiction to maximal *n*-set. Hence $|S| \geq 3$. Let us consider a set with $|S| = 3$, then *S* is an *n*-set and $\bar{S}$ is not an *n*-set (Since $\cup_{v\in\bar{S}} \langle N[v]\rangle \neq L_m$). For the case $m \geq 6$, for the maximal *n*-set, then both *S* and $\bar{S}$ are independent (Since *S* is an *n*-set), $|S| \geq \frac{m}{2}$ (by Theorem 2.11) and *S* is not independent (by Remark 1.8). For every independent set $\bar{S}$, *S* is also an independent set. Hence, $|S| \geq \frac{m}{2} + 1$. On the other hand, a set $S = \{v_1, v_3, ..., v_{\frac{m}{2}-1}\} \cup \{u_1\} \cup \{u_2, u_4, ..., u_{\frac{m}{2}}\}$, when $\frac{m}{2}$ is even and $S = \{v_1, v_3, ..., v_{\frac{m}{2}}\} \cup \{u_1\} \cup \{u_2, u_4, ..., u_{\frac{m}{2}-1}\}$, when $\frac{m}{2}$ is odd is a maximal *n*-set (Since *S* is an *n*-set by Theorem 2.11 and an edge $v_1 u_1 \notin \cup_{v\in\bar{S}} \langle N[v]\rangle$, $\bar{S}$ is not an *n*-set). Hence $|S| \leq \frac{m}{2} + 1$. Therefore, $max_n(L_m) = \frac{m}{2} + 1$.

VI. Foul *n*-set of a ladder graph

Theorem 2.13. For any even integer m $\geq$ 4, $fou_n(L_m) = 2$.

Proof. Let *S* be a minimal foul *n*-set of L_m. Then both *S* and $\bar{S}$ are not *n*-sets. If possible, let |*S*| = 1, then by Theorem 2.11 *S* is not an *n*-set. But $|\bar{S}|$ = m-1 is an *n*-set (by Remark 1.9), a contradiction. Thus, $2 \leq |S| \leq m - 2$. On the other hand, let $S_1 = \{u_j, v_j\}$ where $1 \leq j \leq \frac{m}{2}$. The set S_1 and $\overline{S_1}$ are not n-set (since $\cup_{v\in S_1} \langle N[v]\rangle \neq L_m$ and $u_j\, v_j$ are not an edge of $\cup_{v\in\overline{S_1}} \langle N[v]\rangle$). Since *S* is minimal |*S*| = 2. Therefore, $fou_n(L_m\ )$.

VII. Types of Locating Dominating sets

In this section we call an *ld*-set defined above as a simple *ld*-set.

Definition 2.14. A *ld*-set *S* of the graph *G* is called a powerful *ld*-set if $\bar{S}$ is also an *ld*-set of *G*.

Definition 2.15. A *ld*-set *S* of the graph *G* is called a maximal *ld*-set if $\bar{S}$ is not an *ld*-set of *G*.

Definition 2.16. A set *S* of vertices of the graph *G* is called a foul *ld*-set if neither *S* nor $\bar{S}$ is an *ld*-set of *G*.

The minimum cardinality of a simple *ld*-set, powerful *ld*-set, maximal *ld*-set and foul *ld*-set are respectively, called simple *ld*-number, powerful *ld*-number, maximal *ld*-number and foul *ld*-number of *G* and are denoted by $sim_{ld}(G)$, $pow_{ld}(G)$, $max_{ld}(G)$ and $fou_{ld}(G)$ respectively. From the above definitions it is clear that for every graph *G*, $pow_{ld}(G) \geq sim_{ld}(G)$ and $max_{ld}(G) \geq sim_{ld}$(G).

VIII. Simple *ld*-set and Powerful *ld*-set of a ladder graph

Theorem 2.17. For any even integer $m \geq 4$,

$$sim_{ld}(L_m) = pow_{ld}(L_m) = \begin{cases} 3, & for\ m = 6 \\ \frac{m}{2}, & for\ m \geq 4, m \neq 6\ and\ \frac{m}{2} = even \\ \frac{m}{2} - 1, & for\ m \geq 10\ and\ \frac{m}{2} = odd \end{cases}$$

Proof. For *m = 6* result follows fr re 1

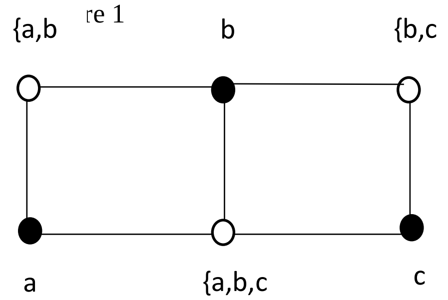

Figure 1: Locating Dominating set of L_6.

Let S be minimal simple ld-set of $G = L_m$ for $m \geq 4$.

Case(i): When $m \geq 4$ and $\frac{m}{2}$ even except for $m = 6$. Let $S_i = \{u_{2i-1}, v_{2i-1}, u_{2i}, v_{2i}\}$, $1 \leq i \leq \frac{m}{4}$. Since S_i are the partitions of $V(G)$ then by Theorem 1.5 we have $S \cap S_i = 2 \neq \emptyset$. Thus $|S| \geq 2\left(\frac{m}{4}\right) = \frac{m}{2}$. Conversely, let $S = V \cup U$ where, $V = \{v_2, v_4, v_6, \dots, v_{\frac{m}{2}}\}$ and $U = \{u_2, u_4, u_6, \dots, u_{\frac{m}{2}}\}$ then S is clearly an ld-set(because $l(v_1|S) \neq l(v_3|S) \neq \dots \neq l(v_{\frac{m}{2}-1}|S) \neq l(u_1|S) \neq l(u_3|S) \neq \dots \neq l(u_{\frac{m}{2}-1}|S)$. We also see that the chosen set S holds the Definition 2.14. Therefore, both S and $\bar{S}$ are ld-set. Hence $|S| \leq |S \cap V| + |S \cap U| = \left(\frac{m}{4}\right) + \left(\frac{m}{4}\right) = \frac{m}{2}$. Thus $sim_{ld}(L_m) = pow_{ld}(L_m) = \frac{m}{2}$.

Case(ii): When $m \geq 10$ and $\frac{m}{2}$ odd: Let $S_i = \{u_{2i-1}, v_{2i-1}, u_{2i}, v_{2i}\}$, $1 \leq i \leq \frac{m-2}{4}$. Since S_i are the partitions of V(G) then by Theorem 1.5 we have $S \cap S_i = 2 \neq \emptyset$ Thus $|S| \geq 2\left(\frac{m-2}{4}\right) = \frac{m}{2} - 1$. Conversely, let $S = V \cup U$ where, $V = \{v_2, v_4, v_6, \dots, v_{\frac{m}{2}-1}\}$ and $U = \{u_2, u_4, u_6, \dots, u_{\frac{m}{2}-1}\}$ be a subset of $V(G)$ then S is clearly an ld-set (because $l(v_1|S) \neq l(v_3|S) \neq \dots \neq l(v_{\frac{m}{2}}|S) \neq l(u_1|S) \neq l(u_3|S) \neq \dots \neq l(u_{\frac{m}{2}}|S))$. So S is an ld-set. We also see that the chosen set S holds the Definition2.14. Therefore, both S and $\bar{S}$ are ld-set. Hence $|S| \leq |S \cap V| + |S \cap U| = \left(\frac{m-2}{4}\right) + \left(\frac{m-2}{4}\right) = \frac{m}{2} - 1$. Thus $sim_{ld}(L_m) = pow_{ld}(L_m) = \frac{m}{2} - 1$.

IX. Maximal *ld*-set of a ladder graph

Theorem 2.18. For any even integer $m \geq 4$, $max_{ld}(L_m) = \frac{m}{2} + 1$.

Proof. For $m = 4,6$ result follows from Figure 2

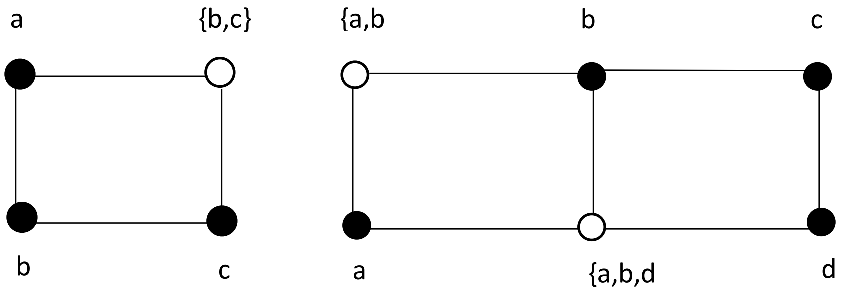

Figure 2: Maximal Locating Dominating set of L_4 and L_6.

Let S be minimal *maximal* ld-set of $G = L_m$ for $m \geq 4$.

Case i : When $m \geq 8$ and $\frac{m}{2}$ even. Let $S_i = \{u_{2i-1}, v_{2i-1}\}$, $1 \leq i \leq \frac{m-2}{4}$, $T_i = \{v_{2i}, u_{2i}\}$, $1 \leq i \leq \frac{m-2}{4}$, and $W = \{v_{\frac{m}{2}}, u_{\frac{m}{2}}\}$. Since S_i, T_i and W are the partitions of $V(G)$ then by Theorem 1.5 we have $S \cap S_i \neq \emptyset$, $S \cap T_i \neq \emptyset$ and $S \cap W \neq \emptyset$. Thus $|S| \geq \left(\frac{m-2}{4}\right) + \left(\frac{m-2}{4}\right) + 2 = \frac{m}{2} + 1$. Conversely, let $S = V \cup U \cup W$, where $V = \{v_1, v_3, v_5, \dots, v_{\frac{m}{2}-1}\}$, $U = \{u_2, u_4, u_6, \dots, u_{\frac{m}{2}-2}\}$ and $W = \{v_{\frac{m}{2}}, u_{\frac{m}{2}}\}$ then S is clearly an ld-set (because $l(v_2|S) \neq l(v_4|S) \neq \dots \neq l(v_{\frac{m}{2}-1}|S) \neq l(u_1|S) \neq l(u_3|S) \neq \dots \neq l(u_{\frac{m}{2}-1}|S))$. so S is an ld-set. We also see that the chosen set S holds the Definition 2.15. Therefore, both S and $\bar{S}$ are ld-set. Hence $|S| \leq |S \cap V| + |S \cap U| + |S \cap W| = \left(\frac{m}{4}\right) + \left(\frac{m-4}{4}\right) + 2 = \frac{m}{2} + 1$. Thus $max_{ld}(L_m) = \frac{m}{2} + 1$.

Case ii: When $m \geq 10$ and $\frac{m}{2}$ odd. Let $S_i = \{u_{2i-1}, v_{2i-1}\}$, $1 \leq i \leq \frac{m}{4}$, $T_i = \{v_{2i}, u_{2i}\}$, $1 \leq i \leq \frac{m}{4} - 1$ and $W = \{v_{\frac{m}{2}}, u_{\frac{m}{2}}\}$. Since S_i, T_i and W are the partitions of $V(G)$ then by Theorem 1.5 we have $S \cap S_i \neq \emptyset$, $S \cap T_i \neq \emptyset$ *and* $S \cap W \neq \emptyset$. Thus $|S| \geq \left(\frac{m}{4}\right) + \left(\frac{m}{4} - 1\right) + 2 = \frac{m}{2} + 1$. Conversely, let $S = V \cup U \cup W$, where $V = \{v_1, v_3, v_5, \dots, v_{\frac{m}{2}-2}\}$, $U = \{u_2, u_4, u_6, \dots, u_{\frac{m}{2}-1}\}$ and $W = \{v_{\frac{m}{2}}, u_{\frac{m}{2}}\}$ then S is clearly an ld-set (because $l(v_2|S) \neq l(v_4|S) \neq \dots \neq l(v_{\frac{m}{2}-1}|S) \neq l(u_1|S) \neq$

$l(u_3|S) \neq ... \neq l(u_{\frac{m}{2}-2}|S))$. so S is an ld-set. We also see that the chosen set S holds the Definition 2.15. Therefore, both S and $\bar{S}$ are ld-set. Hence $|S| \leq |S \cap V| + |S \cap U| + |S \cap W| = (\frac{m-2}{4}) + (\frac{m-2}{4}) + 2 = \frac{m}{2} + 1$. Thus $max_{ld}(L_m) = \frac{m}{2} + 1$.

X. Foul *ld*-set of a ladder graph

Theorem 2.19. For any even integer $m \geq 4$, $fou_{ld}(L_m) = 3$.

Proof. Let S be minimal foul ld-set. Then S is not an ld-set and $\bar{S}$ is also not an ld-set of L_m. For $m \geq 4$. If possible, let $|S|=1$, then S is not an ld-set. But $|\bar{S}| = m-1$ is an ld-set, which contradicts the Definition 2.16. If $S = \{u, v\}$ where $u, v \in G$ then $|\bar{S}| = m - 2$ is an ld-set, which contradicts the Definition 2.16. Thus $3 \leq |S| \leq m - 3$. On the other hand, let $S_1 = \{u_1, v_1, v_2\}$. The set S_1 and $\bar{S_1}$ are not ld-sets (because $l(u_2|S_1) = l(v_3|S_1)$ and $l(u_1|V\text{-}S_1) = l(v_2|V\text{-}S_1)$ respectively). Since S is minimal, $|S| = 3$. Therefore $fou_{ld}(L_m) = 3$.

XI. Types of Almost Locating Dominating sets

In this section we call an *ald*-set defined above as a simple *ald*-set.

Definition 2.20. An *ald*-set S of the graph G is called a powerful *ald*-set if $\bar{S}$ is also an *ald*-set of G.

Definition 2.21. An *ald*-set S of the graph G is called a maximal *ald*-set if $\bar{S}$ is not an *ald*-set of G.

Definition 2.23. A set S of vertices of the graph G is called a foul *ald*-set if neither S nor $\bar{S}$ is an *ald*-set of G.

The minimum cardinality of a simple *ald*-set, powerful *ald*-set, maximal *ald*-set and foul *ald*-set are respectively, called simple *ald*-number, powerful *ald*-number, maximal *ald*-number and foul *ald*-number of G and are denoted by $sim_{ald}(G)$, $pow_{ald}(G)$, $max_{ald}(G)$, and $fou_{ald}(G)$. From the above definitions it is clear that for every graph G, $pow_{ald}(G) \geq sim_{ald}(G)$ and $max_{ald}(G) \geq sim_{ald}(G)$.

XII. Simple *ald*-set and Powerful ald-set of a ladder graph

Theorem 2.23. For any even integer $m \geq 4$, $sim_{ald}(L_m) = pow_{ald}(L_m) = \begin{cases} 2, & for\ m = 4, 6 \\ \frac{m}{2} - 1, & for\ m \geq 8 \end{cases}$

Proof. When $m = 4$ and 6 result follows from the Figure 3.

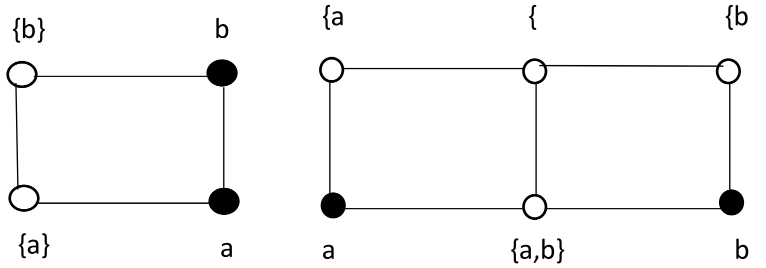

Figure 3: Almost Locating Dominating set of L_4 and L_6.

Let S be a minimal simple *ald*-set of $G = L_m$ for $m \geq 8$.

Case i: When $m \equiv 0 \pmod 3$:

Subcase (i): When $\frac{m}{2} \equiv 0 \pmod 6$: Let $S_i = \{v_{2i}, u_{2i}\}$, $1 \leq i \leq \frac{m}{4} - 1$ and $T_i = \{v_{2i-1}, u_{2i-1}\}$, $1 \leq i \leq \frac{m}{4}$. Then $S_1, S_2, ..., T_1 T_2, ...$ are the partitions of $V(G)$ and by the Theorem 1.5 we have $S \cap S_i \neq \emptyset$ and $S \cap T_i \neq \emptyset$. Therefore, $|S| \geq (\frac{m}{4} - 1) + (\frac{m}{4}) = \frac{m}{4} - 1$. Conversely, let $S = V \cup U$ where, $V = \{v_1, v_3, v_5, ..., v_{\frac{m}{2}-1}\}$ and $U = \{u_2, u_4, u_6, ..., u_{\frac{m}{2}-2}\}$ then both S and $\bar{S}$ are ald-set(because $l(v_2|S) \neq l(v_4|S) \neq l(v_6|S) \neq ... \neq l(v_{\frac{m}{2}}|S) \neq l(u_1|S) \neq l(u_3|S) \neq ... \neq l(u_{\frac{m}{2}}|S))$. Hence $|S| \leq |S \cap V| + |S \cap U| = (\frac{m}{4}) + (\frac{m}{4} - 1) + 2 = \frac{m}{2} - 1$. Thus $sim_{ald}(L_m) = pow_{ald}(L_m) = \frac{m}{2} - 1$.

Subcase(ii): When $\frac{m}{2} \neq 0$ (mod 6): Let $S_i = \{v_{2i}, u_{2i}\}$, $1 \leq i \leq \frac{m-2}{4}$ and $T_i = \{v_{2i-1}, u_{2i-1}\}$, $1 \leq i \leq \frac{m-2}{4}$. Then $S_1, S_2, \ldots, T_1 T_2, \ldots$ are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i \neq \emptyset$ and $S \cap T_i \neq \emptyset$. Therefore, $|S| \geq (\frac{m-2}{4}) + (\frac{m-2}{4}) = \frac{m}{4} - 1$. Conversely, let $S = V \cup U$ where, $V = \{v_1, v_3, v_5, \ldots, v_{\frac{m}{2}-2}\}$ and $U = \{u_2, u_4, u_6, \ldots, u_{\frac{m}{2}-1}\}$ then both S and $\bar{S}$ are *ald*-set(because $l(v_2|S) \neq l(v_4|S) \neq l(v_6|S) \neq \ldots \neq l(v_{\frac{m}{2}}|S) \neq l(u_1|S) \neq l(u_3|S) \neq \ldots \neq l(u_{\frac{m}{2}}|S))$. Hence, $|S| \leq |S \cap V| + |S \cap U| = (\frac{m-2}{4}) + (\frac{m-2}{4}) + 2 = \frac{m}{2} - 1$. Thus $sim_{ald}(L_m) = pow_{ald}(L_m) = \frac{m}{2} - 1$.

Case(ii): When m ≡ 2 (mod 3): Let $S_i = \{u_{3i-2}, v_{3i-2}, u_{3i-1}, v_{3i-1}\}$, $1 \leq i \leq \frac{m-2}{6}$ and $T_i = \{u_{3i}, v_{3i}\}$, $1 \leq i \leq \frac{m-2}{6}$. Then $S_1, S_2, \ldots, T_1 T_2, \ldots$ are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i = 2 \neq \emptyset$ and $S \cap T_i \neq \emptyset$. Since S_i is a set containing 2 elements of an *ald*-set. Therefore, total number of elements in S_i sets which are the elements of *ald*-set is $2(\frac{m-2}{6})$. Thus, $|S| \geq 2(\frac{m-2}{6}) + (\frac{m-2}{6}) = \frac{m}{2} - 1$. Conversely, let $S = V_1 \cup V_2 \cup U$ where, $V_1 = \{v_2, v_5, v_8, \ldots, v_{\frac{m}{2}-2}\}$, $V_2 = \{v_3, v_6, v_9, \ldots, v_{\frac{m}{2}-1}\}$ and $U = \{u_2, u_5, u_8, \ldots, u_{\frac{m}{2}-2}\}$ then both S and $\bar{S}$ are *ald*-set(*because* $l(v_1|S) \neq l(v_4|S) \neq l(v_7|S) \neq \ldots \neq l(v_{\frac{m}{2}}|S) \neq l(u_1|S) \neq l(u_3|S) \neq \ldots \neq l(u_{\frac{m}{2}}|S))$. Hence, $|S| \leq |S \cap V_1| + |S \cap V_2| + |S \cap U| = (\frac{m-2}{6}) + (\frac{m-2}{6}) + \frac{m-2}{6} = \frac{m}{2} - 1$. Thus $sim_{ald}(L_m) = pow_{ald}(L_m) = \frac{m}{2} - 1$.

Case(iii): When m ≡ 1(mod 3): Let $S_i = \{u_{3i-1}, v_{3i-1}, u_{3i}, v_{3i}\}$, $1 \leq i \leq \frac{m-4}{6}$ and $T_i = \{v_{3i-2}, u_{3i-2}\}$, $1 \leq i \leq \frac{m+2}{6}$. Then $S_1, S_2, \ldots, T_1 T_2, \ldots$ are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i = 2 \neq \emptyset$ and $S \cap T_i \neq \emptyset$. Since S_i is a set containing *2* elements of an *ald*-set. Therefore, total number of elements in S_i sets which are the elements of *ald*-set is $2(\frac{m-4}{6})$. Thus, $|S| \geq 2(\frac{m-4}{6}) + (\frac{m+2}{6}) = \frac{m}{2} - 1$. Conversely, let $S = V_1 \cup V_2 \cup U$ where, $V_1 = \{v_1, v_4, v_7, \ldots, v_{\frac{m}{2}-1}\}$, $V_2 = \{v_3, v_6, v_9, \ldots, v_{\frac{m}{2}-2}\}$ and $U = \{u_3, u_6, u_9, \ldots, u_{\frac{m}{2}-2}\}$ then both S and $\bar{S}$ are *ald*-set(because $l(v_2|S) \neq l(v_5|S) \neq l(v_8|S) \neq \ldots \neq l(v_{\frac{m}{2}}|S) \neq l(u_1|S) \neq l(u_2|S) \neq \ldots \neq l(u_{\frac{m}{2}}|S))$. Hence, $|S| \leq |S \cap V_1| + |S \cap V_2| + |S \cap U| = (\frac{m+2}{6}) + (\frac{m-4}{6}) + \frac{m-4}{6} = \frac{m}{2} - 1$. Thus $sim_{ald}(L_m) = pow_{ald}(L_m) = \frac{m}{2} - 1$.

XIII. Maximal *ald*-set of a ladder graph

Theorem 2.24. For any even integer $m \geq 4$, $max_{ald}(L_m) = \begin{cases} 3, & for\ m = 4 \\ \frac{m}{2}, & for\ m \geq 6 \end{cases}$

Proof. Let S be minimal *maximal ald-set* of $G = L_m$ for $m \geq 4$. For $m = 4$ if $|S| = 1$ then S is not an *ald*-set (because $l(v_2|S) = l(u_1|S))$ and $\bar{S}$ is an ald-set, which contradicts Definition 2.21. If $|S| = 2$ and $S = \{u, v\}$, if u is adjacent to v then both S and $\bar{S}$ are *ald*-set (because $l(u_1|S) \neq l(v_1|S)$ and $l(u\ |V - S) \neq l(v\ |V - S)$ respectively), which contradicts Definition 2.21. Suppose if u is not adjacent to v then S is not an *ald*-set (because $l(u_1|S) = l(v_1|S))$. If $|S| = 3$ then S is an *ald*-set and $\bar{S}$ is not an ald-set (because $l(v_2|S) = l(u_1|S))$. Hence the result and is as shown in Figure *4*.

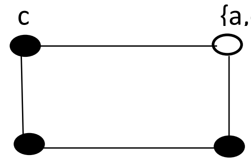

Figure 4: Maximal ald-set for m = 4.

Case(i). For $\frac{m}{2} \equiv 1$(mod 3): Let $S_i = \{v_{3i-1}, u_{3i-1}\}$, $1 \leq i \leq \frac{m-2}{6}$, $T_i = \{v_{3i}, u_{3i}, v_{3i+1}, u_{3i+1}\}$, $1 \leq i \leq \frac{m-2}{6}$ and $W = \{u_1, v_1\}$. Since S_i, T_{2i} *and* W are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i \neq \emptyset$, $S \cap T_i \neq \emptyset$ and $S \cap W \neq \emptyset$. Therefore, $|S| \geq (\frac{m-2}{6}) + 2(\frac{m-2}{6}) + 1 = \frac{m}{2}$. Conversely, let $S = V \cup U \cup W$ where, $V = \{v_3, v_6, v_9, \ldots, v_{\frac{m}{2}-4}, v_{\frac{m}{2}-1}\}$, $U = \{u_2, u_3, u_5, u_6, u_8, u_9, \ldots, u_{\frac{m}{2}-2}, u_{\frac{m}{2}-1}\}$ and $W = \{v_1\}$. We See that the set S chosen in this case holds the Definition 2.21. Therefore, S is *ald*-set of G. Hence, $|S| \leq |S \cap V| + |S \cap U| + |S \cap W| = (\frac{m-2}{6}) + 2(\frac{m-2}{6}) + 1 = \frac{m}{2}$. Thus $max_{ald}(L_m) = \frac{m}{2}$.

Case(ii). For $\frac{m}{2} \equiv 2(\text{mod } 3)$: Let $S_i = \{v_{3i-1}, u_{3i-1}\}$, $1 \leq i \leq \frac{m+2}{6}$, $T_i = \{v_{3i}, u_{3i}, v_{3i+1}, u_{3i+1}\}$, $1 \leq i \leq \frac{m-4}{6}$ and $W = \{u_1, v_1\}$. Since S_i, T_{2i} *and* W are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i \neq \emptyset$, $S \cap T_i \neq \emptyset$ and $S \cap W \neq \emptyset$. Therefore, $|S| \geq (\frac{m+2}{6}) + 2(\frac{m-4}{6}) + 1 = \frac{m}{2}$. Conversely, let $S = V \cup U \cup W$ where, $V = \{v_3, v_6, v_9, \ldots, v_{\frac{m}{2}-5}, v_{\frac{m}{2}-2}\}$, $U = \{u_2, u_3, u_5, u_6, u_8, u_9, \ldots, u_{\frac{m}{2}-3}, u_{\frac{m}{2}-2}, u_{\frac{m}{2}}\}$ and $W = \{v_1\}$. We See that the set *S* chosen in this case holds the Definition 2.21. Therefore, *S* is maximal *ald*-set of *G*. Hence, $|S| \leq |S \cap V| + |S \cap U| + |S \cap W| = (\frac{m-4}{6}) + 2(\frac{m-1}{6}) + 1 = \frac{m}{2}$. Thus $max_{ald}(L_m) = \frac{m}{2}$.

Case(iii). For $\frac{m}{2} \equiv 0$ (mod 3): Let $S_i = \{v_{3i-1}, u_{3i-1}\}$, $1 \leq i \leq \frac{m-6}{6}$, $T_i = \{v_{3i}, u_{3i}, v_{3i+1}, u_{3i+1}\}$, $1 \leq i \leq \frac{m-6}{6}$, $W = \{u_1, v_1\}$ and $W_1 = \{u_{\frac{m}{2}}, v_{\frac{m}{2}}, u_{\frac{m}{2}-1}, v_{\frac{m}{2}-1}\}$. Since S_i, T_i, W *and* W_1 are the partitions of *V(G)* and by the Theorem 1.5 we have $S \cap S_i \neq \emptyset$, $S \cap T_i \neq \emptyset$, $S \cap W \neq \emptyset$ and $S \cap W_1 \neq \emptyset$. *Therefore*, $|S| \geq (\frac{m-6}{6}) + 2(\frac{m-6}{6}) + 1 + 2 = \frac{m}{2}$. Conversely, let $S = V \cup U \cup W \cup W_1$ where, $V = \{v_3, v_6, v_9, \ldots, v_{\frac{m}{2}-3}\}$, $U = \{u_2, u_3, u_5, u_6, u_8, u_9, \ldots, u_{\frac{m}{2}-4}, u_{\frac{m}{2}-3}\}$, $W = \{v_1\}$ and $W_1 = \{u_{\frac{m}{2}-1}, u_{\frac{m}{2}}\}$. We See that the set *S* chosen in this case holds the Definition 2.21. Therefore, *S* is maximal *ald*-set of *G*. Hence, $|S| \leq |S \cap V| + |S \cap U| + |S \cap W| + |S \cap W_1| = (\frac{m-6}{6}) + 2(\frac{m-6}{6}) + 1 + 2 = \frac{m}{2}$. Thus $max_{ald}(L_m) = \frac{m}{2}$.

XIV. Foul *ald*-set of a ladder graph

Theorem 2.25. For any even integer $m \geq 4$, $fou_{ald}(L_m) = \begin{cases} 2, & for\ m = 4 \\ 3, & for\ m \geq 6 \end{cases}$

Let *S* be minimal foul *ald*-set, then *S* is not an *ald*-set and $\bar{S}$ is also not an *ald*-set of L_m. For *m = 4*, result follows from Figure *5*.

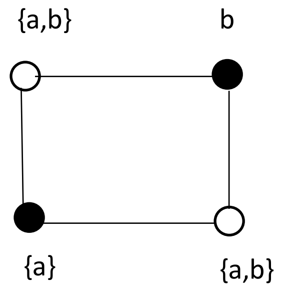

Figure5: Foul ald-set for m = 4.

For $m \geq 6$. If possible, let $|S| = 1$, then S is not an *ald*-set. But $|\bar{S}| = m - 1$ is an *ald*-set, which contradicts the Definition 2.22. If $S = \{u, v\}$ where $u, v \in G$ then $|\bar{S}| = m - 2$ is an *ald*-set, which contradicts the Definition 2.22. Thus $3 \leq |S| \leq m - 3$. On the other side, let $S_1 = \{v_1, u_2, u_3\}$. The set S_1 and $\bar{S_1}$ are not *ald*-sets (because $l(v_2|S_1) = l(u_1|S_1)$ *and* $l(v_1|V - S_1) = l(u_2|V - S_1)$ respectively). Since *S* is minimal, $|S| = 3$. Therefore $fou_{ald}(L_m) = 3$.

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