International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373/doi:10.14445/22315373/IJMTT-V67I10P504

Volume 67 Issue 10, 41-58, October, 2021

Original Article

Modeling the Treatment Effect on LDL-C and Atherosclerotic Blood Flow through Microchannel with Heat and Magnetic Field

K.W. Bunonyo [1], E. Amos [2] and C. Nwaigwe [3]

[1]*Department of Mathematics & Statistics, Federal University Otuoke, Nigeria*
[1, 2, 3] *Department of Mathematics, Rivers State University, Nigeria*

Abstract *- This article investigated the treatment effect on LDL-C and atherosclerotic blood flow through microchannel with heat and magnetic field. The study involved the formulation of mathematical models which represent the blood momentum equation, LDL-C concentration and Energy equation, we also remodeled the region of atherosclerosis in order to incorporate the treatment term for the control and prevention of excessive cholesterol bloodstream in order to improve healthy. The modeled (PDEs) equations governing the general flow were scaled to a system of dimensionless ordinary differential equations (ODEs) using perturbation technique and are solved directly using the method of undetermined coefficient. Wolfram Mathematica, version 10 was used to code the various flow profiles with some of the pertinent governing parameters varied. From the simulation, it is noticed that the blood velocity increased, as we varied the treamtnt, Soret number, heat radiation, Grashof number, solutal Grashof number and the permeability of the porous medium, while the velocity decreases for variation of Hartmann number, Schmidt number, chemical reaction and oscillatory frequency, mainwhile the fluid temperautre also rises for the variation of growth rate, heat radiation and oscillatory frequency, however, we noticed a decreases in temperautre for the increasing values of the Prandtl number. In conclusion, it is seen that, first, if we want to keep blood velocity in check, we need to control the impact of the Lorentz force, Soret number, the treatment and keep a watch on the intake of the Trans fats, secondly, Hartmann number control could be useful for early detection and treatment of termal ailments like tumour growth. If we must avoid Trans fat induced cardiovascular diseases, we must adhere to diet control and limit the intake of fatty substances.*

Keywords *- Modeling, Treatment, Heat, Blood, Atherosclerosis, atheroma, Cholesterol, Magnetic Field, LDL-C, ODEs and PDEs*

I. INTRODUCTION

Cholesterol and triglycerides cannot circulate loosely in the blood, so they travel in “round parcels” called lipoproteins. Lipoproteins contain a special mix of fats and proteins which allow them to flow freely in the blood. They are four main lipoproteins (sometimes called apo-lipoproteins). They vary in size, content and how tightly they are packed (density). However, problems occur when any one of these arteries becomes narrowed due to the slow build-up of fatty material (called plaque or atheroma) [20]. This process is called atherosclerosis, which causes coronary heart disease (CHD). Sometimes an artery can become so narrow that it cannot deliver enough oxygenated blood to the tissues and organs of the body. When these fatty deposits become very large or extended they may burst, and over time this damage may partly or completely block the artery, when this happens it is called acute coronary syndrome (ACS), unstable angina or heart attack. A heart attack is sometimes referred to as a myocardial infarction or MI. Fats that circulate in the blood are called lipids. Cholesterol and triglycerides are both lipids. They have essential roles in the body. In excess they are harmful. Cholesterol is needed to build cell walls and to make hormones and vitamin D. Some of the body cholesterol comes from the food we eat; but most is made in the liver. When broken down cholesterol is used to make bile acids which help us to digest our food [26].

Atherosclerosis is a disease of the cardiovascular system which involves a hardening of the arteries due to the deposition of plaque. Localized atherosclerotic constrictions in arteries, known as arterial stenosis, are predominantly found in the internal carotid artery which supplies blood to the brain, the coronary artery which supplies blood to the cardiac muscles, and the femoral artery which supplies blood to the lower limbs. Blockage of more than about 70 % (by area) of the artery is considered clinically significant, since it presents significant health risks for the patient [23, 25]. Complete closure of the artery can occur if a blood-clot becomes lodged in the stenosis, and this can lead to a stroke or a heart attack. Atherosclerosis is a very slow

process, happening over many years. It can start very early in life and results in the buildup of fatty material in the linings of your blood vessels. These fatty deposits start when the blood vessel lining becomes damaged. This makes it easier for cholesterol (carried on lipoproteins like LDL) to stick on and build up more rapidly. HDL lipoproteins can remove cholesterol from these deposits. Reducing your LDL cholesterol, increasing your HDL cholesterol and reducing other risk factors can help slow down the process of atherosclerosis.

The heart, blood vessels and blood make up our circulatory system. The heart is a muscle which never stops beating; it pumps blood around the body. The left side pumps oxygen and nutrient rich blood to the brain, muscles, organs, and every cell in the body. The right side of the heart is slightly smaller and returns blood to the lungs to be topped up with oxygen. The heart has its own blood supply which comes from the coronary arteries. These divide many times to provide oxygen and nutrients to every part of the heart muscle to help keep it healthy and pumping normally [3].

Many investigators have theoretically studied the flow of blood through permeable walls. Elshehawey and Husseny [1] studied the peristaltic transport of a magneto-fluid with porous boundaries. Fluid entering the flow region through one plate at the same rate as it left through the other plate was considered. Sinha and Misra [2] investigated the blood flow through an artery with permeable wall. Makinde and Osalusi [3] studied steady magnetohydrodynamics (MHD) flow in a 2 dimension channel with permeable boundaries. Steady MHD flow through a circular vertical pipe with permeable boundaries has been investigated by Elangovan and Ratchagar [4]. Makinde and Chinyoka [5] studied the unsteady MHD flow in a porous 2 dimension channel with one wall impermeable and the other porous. Recently, Sattar and Waheedullah [6] investigated the unsteady flow of a viscoelastic fluid through porous medium bounded by two porous plates. It was assumed that one plate is injected with certain constant velocity and that the other sucked it off with the same velocity. Xin-Hui Si *et al*. [7] studied the asymmetric laminar flow in a porous channel with expanding or contracting walls. Homotopy analysis method (HAM) was employed to obtain expressions for the velocity fields.

The study of pulsatile flow in a porous channel or porous pipe has recently becomes the object of scientific research because of its importance in some practical phenomena, such as transpiration cooling and gaseous diffusion. Particularly, the study of pulsatile flow in a porous channel is useful in understanding the process of dialysis of blood in artificial kidneys and in industrial applications in relation to heat exchange efficiency. Also, the pulsatile flow between permeable walls is important in understanding blood flow in the circulatory system, where the nutrients are supplied to tissues of various organs and waste products are removed. In 1971, Wang [14] studied the interesting problem of pulsatile flow in a porous channel bounded by rigid walls. Many researchers studied the effect of slip velocity at permeable boundaries [2-4]. Recently, Eldesoky [8] investigated the unsteady pulsatile flow of blood through porous medium in an artery under the influence of periodic body acceleration and slip condition in the presence of magnetic field, considering blood as an incompressible electrically conducting fluid.

Most of the researchers dealing with steady incompressible laminar flow with uniform injection or suction have attempted to determine the axial pressure variation, wall shear stress on the porous walls, and shapes of the velocity profiles within the tube. The unsteady suction problem was considered by Tsangaris *et al.* [15]. The case of periodic suction for flow through parallel plates was considered by Ramanamurthy *et al.* [16]. The flow of blood can be controlled by applying the appropriate magnetic field. Many researchers have shown that blood is an electrically conducting fluid. The Lorentz force will act on the constituent particles of blood, and this force will oppose the motion of blood and thus reduce its velocity. This decelerated blood flow may help in the treatment of certain cardiovascular diseases and in diseases with accelerated blood circulation, such as hypertension, hemorrhage etc. Therefore, it is essential to study the blood flow in presence of a magnetic field. Much work has been done in this field by various investigators [8, 13, 21, and 22].

The main objective of the present paper is to study the treatment effect on LDL-C and atherosclerotic blood flow through a microchannel with heat and magnetic field. The walls are permeable and considering blood as an incompressible electrically conducting fluid. The governing modified Navier-Stokes equations are solved by perturbation technique. In the following sections, the problem is formulated, analyzed and discussed.

The research questions are as follows:

i. How does Lorentz force enhance blood velocity horizontally?
ii. How does the increase in atheroma (plaque) affects blood temperature distribution?
iii. How does treatment helps in improving blood velocity?
iv. How does the chemical reaction affects blood flow and temperature profile?
v. Does heat applied to the surface improve oxygenated blood circulation?
vi. Does Grashof number and solutal Grashof number accelerates oxygenated blood circulation?
vii. Does the atheroma concentration affects oxygenated blood circulation?

II. MATHEMATICAL FORMULATION

Let consider blood as viscous fluid which is made up of 45% formed elements and 55% plasma, it is also an incompressible, and electrically conducting fluid flowing through a porous microchannel saturated with an atheroma. The arteries where flow takes place is assumed to be vessel, which is three dimensional where the radius of the vessel it is too small and as such can be approximated to be a channel, Misra & Adhikar [28]. We considered drugs as medications used to treat and control the excess cholesterol in bloodstream in order to keep the flow in check. If not treated the excessive cholesterol derived from the Trans fat and the under utilization of fats produced by the liver could lead to morbidity and mortality. We also assumed that blood is pumped to circulation through microchannel, where back flow is unlikely or prevented by the valves in the body. Let us also assume that the flow is unidirectional with a velocity $\vec{w}^* = \left(0, 0, w^*\right)$ in a horizontal direction x^*. Following the aformnetioned literature, we sketched diagram showing an atherosclerotic microchannel with magnetic field applied perdicularly to the direction of flow with intensity $\vec{B} = \left(0, 0, B_0\right)$, (see Figure 2.1).

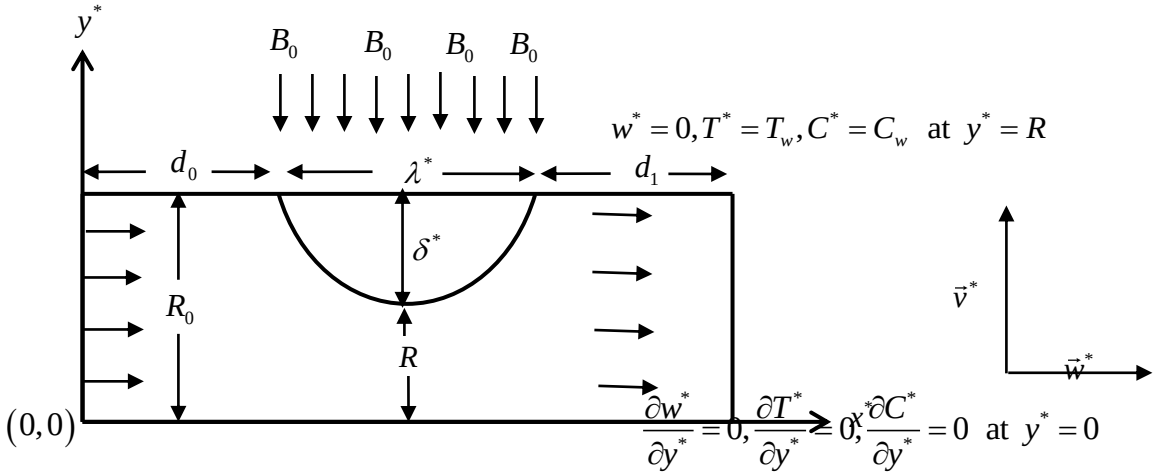

Figure 2.1: Schematic Diagram Showing Atherosclerotic Channel with Applied Magnetic Intensity

Following the Figure 2.1, and considering the modification of the region in Amos and Ogulu [21], Midya *et. al* [18], Bunonyo and Amos [20], the geometry of atherosclerosis with treatment and growth rate is modeled as:

$$\frac{R}{R_0} = \begin{cases} 1 - R_T + \dfrac{\delta e^{at}}{R_0}\left(cos2\dfrac{\pi x^*}{\lambda^*}\right) & \text{at} \quad \dfrac{d_0}{\lambda^*} \le x \le 1 \\ 1 & \text{at} \quad 0 \le x \le \dfrac{d_0}{\lambda^*} \end{cases} \tag{3.1}$$

where $x^* = \left(d_0 + \dfrac{\lambda^*}{2}\right)$

III. GOVERNING EQUATIONS

Following Bunonyo and Amos [20], we present the models governing blood flow through a microchannel, which is:

A. Blood Momentum Equation

$$\rho_b \frac{\partial w^*}{\partial t^*} = -\frac{\partial P^*}{\partial x^*} + \mu_b \frac{\partial^2 w^*}{\partial y^{*2}} - \sigma_e B_0^2 w^* - \frac{\mu_b \varphi}{k^*} w^* + \rho_b g \beta_T \left(T^* - T_\infty\right) + \rho_b g \beta_C \left(C^* - C_\infty\right) \tag{3.2}$$

B. Energy Equation

$$\rho_b c_{bp} \frac{\partial T^*}{\partial t^*} = k_{bT} \frac{\partial^2 T^*}{\partial y^{*2}} + Q_0 \left(T^* - T_\infty\right) \tag{3.3}$$

C. LDL-C Concentration Equation

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - k_0 \left(C^* - C_\infty\right) + \frac{D_T k_{bT}}{T_m} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{3.4}$$

The flow equations in (3.1)–(3.4) are subject to the boundary conditions:

$$\left.\begin{aligned} &\frac{\partial w^*}{\partial y^*} = 0, \frac{\partial T^*}{\partial y^*} = 0, \frac{\partial C^*}{\partial y^*} = 0 \ \text{ at } \ y^* = 0 \\ &w^* = 0, T^* = T_w, C^* = C_w \quad \text{ at } \ y^* = R \end{aligned}\right\} \tag{3.5}$$

where Q_0 is the dimensional heat source of the fluid, k_{bT} is the blood thermal conductivity, D_m is the molecular diffusivity, T_∞ is far field temperature of the fluid, D_T is the thermal diffusivity of the fluid, ρ_b is the density of blood, σ_e is electrical conductivity, B_0 is the magnetic intensity, k_0 is the chemical term, c_{bp} is the specific heat capacity of blood, β_T is the volumetric expansion, β_C is the volumetric expansion due to concentration, μ_b is the dynamic viscosity of blood, φ is the porosity, k^* is the permeability of the porous medium, T_m is the mean temperature of the fluid.

D. DIMENSIONLESS PARAMETERS

Equations (3.2) to (3.5) are made dimensionless using the following quantities:

$$\left.\begin{aligned} &\theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, Gr = \frac{g\beta_T \left(T_w - T_\infty\right) R_0^3}{\upsilon^2}, Gc = \frac{g\beta_C \left(C_w - C_\infty\right) R_0^3}{\upsilon^2}, x = \frac{x^*}{\lambda^*}, \\ &Rd_1 = \frac{Q_0 R_0^2}{\mu_b c_{bp}}, Rd_3 = \frac{k_0 R_0^2}{\upsilon}, M = B_0 R_0 \sqrt{\frac{\sigma_e}{\mu_b}}, S_0 = \frac{D_T k_b \left(T_w - T_\infty\right)}{\upsilon T_m \left(C_w - C_\infty\right)}, Sc = \frac{\upsilon}{D_m}, \\ &y = \frac{y^*}{R_0}, w = \frac{w^* R_0}{\upsilon}, t = \frac{\upsilon t^*}{R_0^2}, \alpha_1 = 1 - R_T, \delta^* = -\frac{\delta}{R_0}, \frac{1}{k} = \frac{\varphi R_0^2}{k^*}, Pr = \frac{\mu_b c_{bp}}{k_b}, g = 1 \end{aligned}\right\} \tag{3.6}$$

Using the dimensionless quantities in equation (3.6), the governing models in equations (3.2)- (3.4) and the corresponding boundary condition, equation (3.5) reduces to:

$$\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 w}{\partial y^2} - M^2 w - \frac{1}{k} w + Gr\theta + Gc\phi \tag{3.7}$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Rd_1 Pr\theta \tag{3.8}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Rd_3\phi + S_0\frac{\partial^2 \theta}{\partial y^2} \tag{3.9}$$

The flow equations in equation (3.7) to equation (3.9) are subject to the following boundary conditions:

$$\left.\begin{aligned} &\frac{\partial w}{\partial y} = 0, \frac{\partial \theta}{\partial y} = 0, \frac{\partial \phi}{\partial y} = 0 \ \text{ at } \ y = 0 \\ &w = 0, \theta = 1, \phi = 1 \qquad \text{at } \ y = \frac{R}{R_0} \end{aligned}\right\} \tag{3.10}$$

IV. METHOD OF SOLUTIONS

The flow is purely oscillatory due to the pumping action of the heart, thus, it is appropriate to seek an oscillatory (periodic) solution, the coupled partial differential equation (3.7)-(3.9) are reduced to system of ordinary differential equations and the boundary conditions in equation (3.10) is also reduced Bunonyo and Amos [20], the solution can be presented in the following form:

$$\left.\begin{aligned} &w(y,t) = w_0(y)e^{i\omega t} \\ &\theta(y,t) = \theta_0(y)e^{i\omega t} \\ &\phi(y,t) = \phi_0(y)e^{i\omega t} \\ &-\frac{\partial P}{\partial x} = P_0 e^{i\omega t} \\ &\chi = \frac{y}{h} \end{aligned}\right\} \tag{3.11}$$

We also assumed long wavelength approximation in equation (3.11), Noreen and Qasim [27] so that the dimensionless governing equations (3.7)-(3.9) and the corresponding boundary condition in equation (3.10) are reduced to:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 + hGr\theta_0 + hGc\phi_0 = P_0 h \tag{3.12}$$

$$\frac{\partial^2 \theta_0}{\partial \chi^2} + \beta_2 \theta_0 = 0 \tag{3.13}$$

$$\frac{\partial^2 \phi_0}{\partial \chi^2} - \beta_3\phi_0 + S_0 Sc\frac{\partial^2 \theta_0}{\partial \chi^2} = 0 \tag{3.14}$$

where $\beta_1 = h\left(M^2 + \frac{1}{k} + i\omega\right)$, $\beta_2 = h\left(Rd_1 - i\omega\right)Pr$ and $\beta_3 = h\left(Rd_3 + i\omega\right)Sc$

The boundary conditions become:

$$\left.\begin{aligned} &\frac{\partial w_0}{\partial \chi}=0,\frac{\partial \theta_0}{\partial \chi}=0,\frac{\partial \phi_0}{\partial \chi}=0 \quad \text{at } \chi=0\\ &w_0=0,\theta_0=e^{-i\omega t},\phi_0=e^{-i\omega t} \quad \text{at } \chi=1 \end{aligned}\right\} \tag{3.15}$$

We solve equation (3.13) in order to obtain the temperature distribution in the blood, in which the solution becomes:

$$\theta_0(\chi)=A_1 sin\left(\sqrt{\beta_2}\chi\right)+B_1 cos\left(\sqrt{\beta_2}\chi\right) \tag{3.16}$$

Using the boundary conditions in equation (3.15) to obtain the constant coefficients, we obtained:

$$\theta_0(\chi)=\frac{e^{-i\omega t}}{cos\left(\sqrt{\beta_2}\right)}cos\left(\sqrt{\beta_2}\chi\right) \tag{3.17}$$

where $B_1=\dfrac{e^{i\omega t}}{cos\left(\sqrt{\beta_2}\right)}$

Differentiating equation (3.17) twice, and substitute the result into equation (3.14), we have the LDL-C equation as:

$$\frac{\partial^2\phi_0}{\partial\chi^2}-\beta_3\phi_0=\frac{\beta_2 S_0 Sce^{-i\omega t}}{cos\left(\sqrt{\beta_2}\right)}cos\left(\sqrt{\beta_2}\chi\right) \tag{3.18}$$

Let $\beta_4=\dfrac{\beta_2 S_0 Sce^{-i\omega t}}{cos\left(\sqrt{\beta_2}\right)}$, equation (3.18) reduces to:

$$\frac{\partial^2\phi_0}{\partial\chi^2}-\beta_3\phi_0=\beta_4 cos\left(\sqrt{\beta_2}\chi\right) \tag{3.19}$$

The homogenous solution of equation (3.19) is:

$$\phi_{0h}(\chi)=A_2 sinh\left(\sqrt{\beta_3}\chi\right)+B_2 cosh\left(\sqrt{\beta_3}\chi\right) \tag{3.20}$$

The particular solution of equation (3.19) reduces to:

$$\phi_{0p}(\chi)=-\left(\frac{\beta_4}{(\beta_2+\beta_3)}\right)cos\left(\sqrt{\beta_2}\chi\right) \tag{3.21}$$

where $B_3=-\dfrac{\beta_4}{(\beta_2+\beta_3)}$

The general solution of equation (3.19) is:

$$\phi_0(\chi)=A_2 sinh\left(\sqrt{\beta_3}\chi\right)+B_2 cosh\left(\sqrt{\beta_3}\chi\right)-\left(\frac{\beta_4}{(\beta_2+\beta_3)}\right)cos\left(\sqrt{\beta_2}\chi\right) \tag{3.22}$$

Using the boundary conditions in equation (3.15) in order to solve for the constant coefficients in equation (3.22), we have:

$$\phi_0(\chi)=B_2 cosh\left(\sqrt{\beta_3}\chi\right)-\left(\frac{\beta_4}{(\beta_2+\beta_3)}\right)cos\left(\sqrt{\beta_2}\chi\right) \tag{3.23}$$

where $B_2=\dfrac{e^{-i\omega t}}{cosh\left(\sqrt{\beta_3}\right)}+\left(\dfrac{\beta_4}{(\beta_2+\beta_3)}\right)\dfrac{cos\left(\sqrt{\beta_2}\right)}{cosh\left(\sqrt{\beta_3}\right)}$

in order to investigate the impact of the temperature profile and LDL-C on blood velocity, we substitute equation (3.23) and (3.17) into the momentum equation (3.12), we have:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 = P_0 h - \left(\frac{Grhe^{i\omega t}}{\cos\left(\sqrt{\beta_2}\right)} \cos\left(\sqrt{\beta_2}\chi\right) \right) - \left(B_2 hGc \cosh\left(\sqrt{\beta_3}\chi\right) - \left(\frac{\beta_4 hGc}{(\beta_2+\beta_3)} \right) \cos\left(\sqrt{\beta_2}\chi\right) \right) \quad (3.24)$$

Simplifying equation (3.24) we have:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 = P_0 h - \beta_5 \cos\left(\sqrt{\beta_2}\chi\right) - B_2 hGc \cosh\left(\sqrt{\beta_3}\chi\right) \quad (3.25)$$

where $\beta_5 = \left(\frac{Grhe^{i\omega t}}{\cos\left(\sqrt{\beta_2}\right)} - \left(\frac{\beta_4 hGc}{(\beta_2+\beta_3)} \right) \right)$

The particular solution of equation (3.25) is:

$$w_{0p} = B_0 + A_4 \cos\left(\sqrt{\beta_2}\chi\right) + B_4 \cosh\left(\sqrt{\beta_3}\chi\right) \quad (3.26)$$

Applying the boundary conditions (3.15) on equation (3.26), we have:

$$w_{0p} = -\frac{P_0 h}{\beta_1} + \frac{\beta_5}{(\beta_2+\beta_1)} \cos\left(\sqrt{\beta_2}\chi\right) + \frac{B_2 hGc}{(\beta_1-\beta_2)} \cosh\left(\sqrt{\beta_3}\chi\right) \quad (3.27)$$

where $A_4 = \frac{\beta_5}{(\beta_2+\beta_1)}, B_4 = \frac{B_2 hGc}{(\beta_1-\beta_2)}, B_0 = -\frac{P_0 h}{\beta_1}$

The homogenous solution of equation (3.25), we have:

$$w_{0h}(\chi) = A_5 \sinh\left(\sqrt{\beta_1}\chi\right) + B_5 \cosh\left(\sqrt{\beta_1}\chi\right) \quad (3.28)$$

The general solution of equation (3.25) is:

$$w_0(\chi) = A_5 \sinh\left(\sqrt{\beta_1}\chi\right) + B_5 \cosh\left(\sqrt{\beta_1}\chi\right) - \frac{P_0 h}{\beta_1} + \frac{\beta_5}{(\beta_2+\beta_1)} \cos\left(\sqrt{\beta_2}\chi\right) + \frac{B_2 hGc}{(\beta_1-\beta_2)} \cosh\left(\sqrt{\beta_3}\chi\right) \quad (3.29)$$

Solving for the constant coefficients in equation (3.29) using the boundary condition in equation (3.15), we have:

$$w_0(\chi) = B_5 \cosh\left(\sqrt{\beta_1}\chi\right) - \frac{P_0 h}{\beta_1} + \frac{\beta_5}{(\beta_2+\beta_1)} \cos\left(\sqrt{\beta_2}\chi\right) + \frac{B_2 hGc}{(\beta_1-\beta_2)} \cosh\left(\sqrt{\beta_3}\chi\right) \quad (3.30)$$

where $B_5 = \frac{P_0 h}{\beta_1 \cosh\left(\sqrt{\beta_1}\right)} - \frac{\beta_5}{(\beta_2+\beta_1)} \frac{\cos\left(\sqrt{\beta_2}\right)}{\cosh\left(\sqrt{\beta_1}\right)} + \frac{B_2 hGc}{(\beta_2-\beta_1)} \frac{\cosh\left(\sqrt{\beta_3}\right)}{\cosh\left(\sqrt{\beta_1}\right)}$

In order to obtain the blood velocity, we substitute equation (3.30) into equation (3.11), we have:

$$w(\chi,t) = \left(B_5 \cosh\left(\sqrt{\beta_1}\chi\right) - \frac{P_0 h}{\beta_1} + \frac{\beta_5}{(\beta_2+\beta_1)} \cos\left(\sqrt{\beta_2}\chi\right) + \frac{B_2 hGc}{(\beta_1-\beta_2)} \cosh\left(\sqrt{\beta_3}\chi\right) \right) e^{i\omega t} \quad (3.31)$$

To obtain the blood temperature profile, we substitute equation (3.17) into equation (3.11), which is:

$$\theta(\chi,t) = \left(\frac{e^{-i\omega t}}{\cos\left(\sqrt{\beta_2}\right)} \cos\left(\sqrt{\beta_2}\chi\right) \right) e^{i\omega t} \quad (3.32)$$

The LDL-C effect on blood velocity, we substitute equation (3.23) into equation (3.11). so we have:

$$\phi(\chi,t) = \left(B_2 \cosh\left(\sqrt{\beta_3}\chi\right) - \left(\frac{\beta_4}{(\beta_2+\beta_3)} \right) \cos\left(\sqrt{\beta_2}\chi\right) \right) e^{i\omega t} \quad (3.33)$$

V. RESULTS

The study involved mathematical formulation, proffering of analytical solution and numerical simulation using Mathematica, version 10. Having solved the formulated problem, we've performed the numerical computation of equation (3.31) to equation (3.33) by varying some of the biophysical parameters such as: the treatment parameter R_T, radiation parameter Rd_1, chemical parameter Rd_3, Grashof number Gr, solutal Grashof number Gc, Hartmann number M, Schmidt number Sc, oscillatory frequency ω, pulse rate f. We considered the parameters values within a specific standard range as: $0 \le \omega \le 5$, $0 \le Sc \le 10$, $k_{bT} = 2.2\times10^{-3}$J/ms ˚K, $0 \le \delta \le 1$, $c_{bp} = 14.65$J/kg˚K, $0 \le M \le 5$, $Pr = 21, 0 \le R_T \le 2, 0 \le Rd_1 \le 5$, $x = 0.5, 0 \le k \le 1$, $\mu_b = 3.2\times10^3$. The dataset used in this research are obtained from the Bunonyo and Amos [25] Chato [29] and Anderson & Valvano [30], also considering the Prandtl number $Pr = 21, 22, 23, 24, 25$ [20]. The results are:

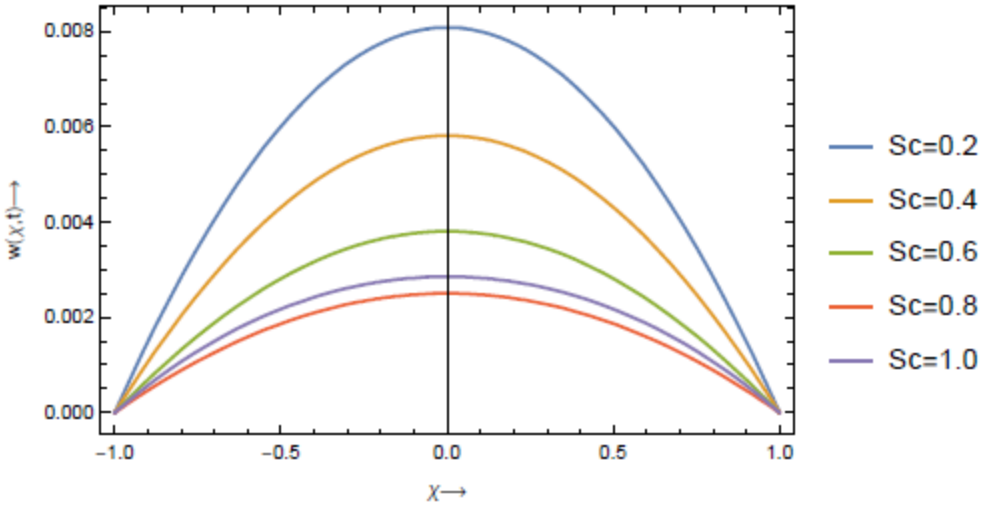

Figure 4.1: Effect of Schmidt number Sc on axial velocity $w(\chi,t)$ with variation of χ

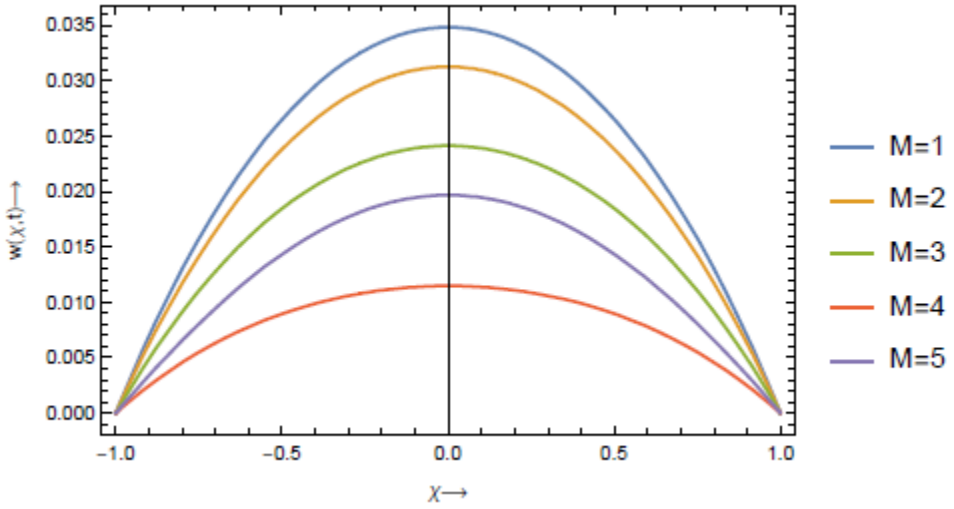

Figure 4.2: Effect of Hartmann number M on axial velocity $w(\chi,t)$ with variation of χ

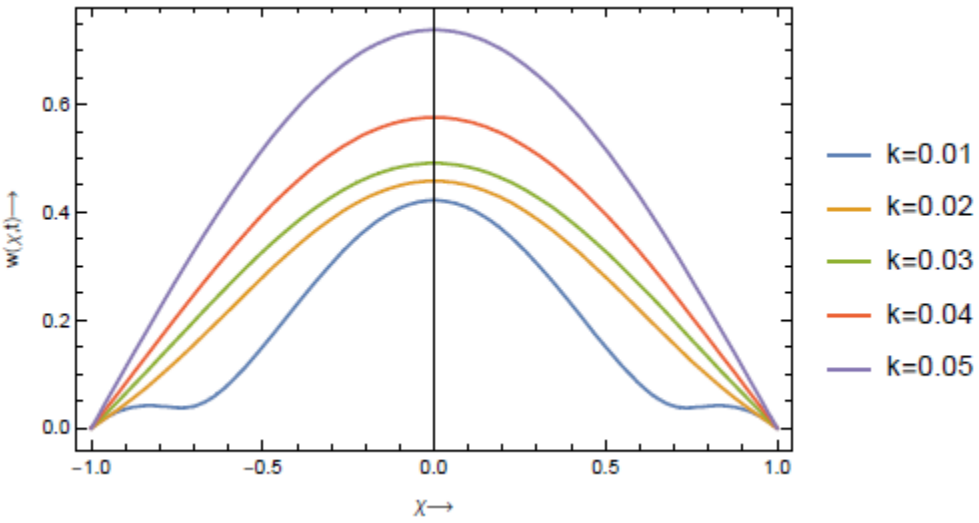

Figure 4.3: Effect of permeability parameter k on axial velocity $w(\chi,t)$ with variation of χ

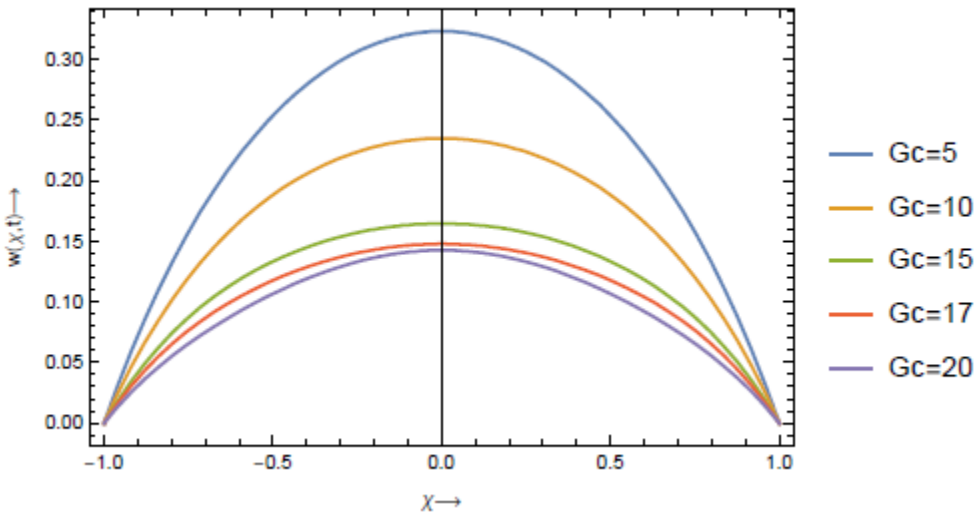

Figure 4.4: Effect of solutal Grashof parameter Gc on axial velocity $w(\chi,t)$ with variation of χ

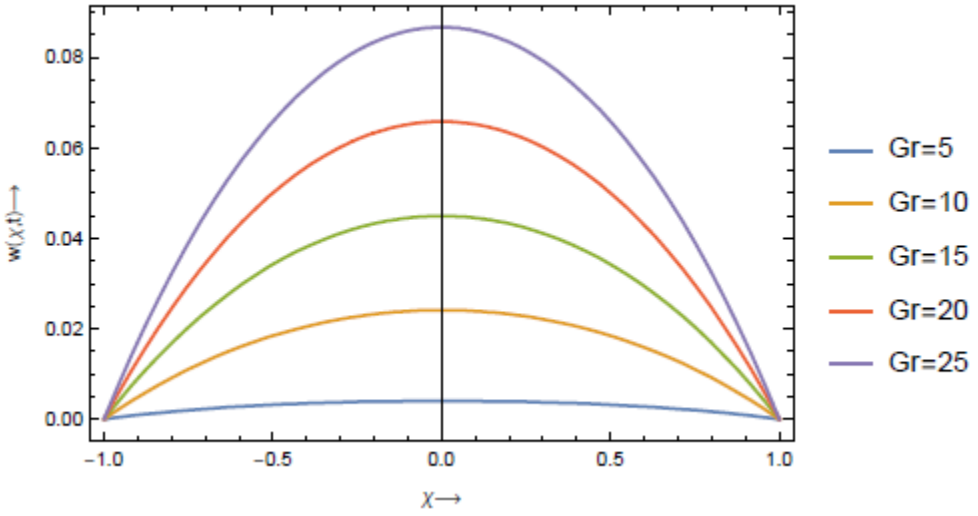

Figure 4.5: Effect of Grashof parameter Gr on axial velocity $w(\chi,t)$ with variation of χ

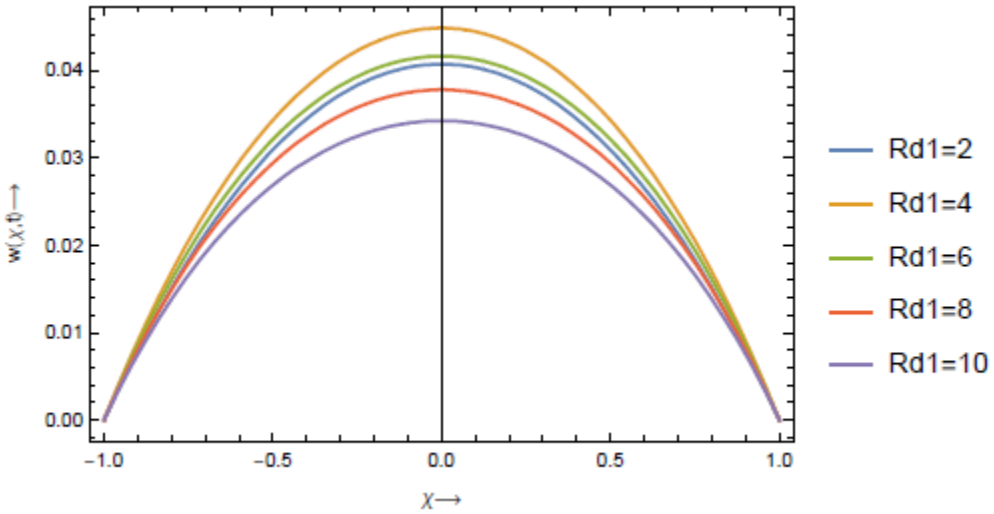

Figure 4.6: Effect of Radiation parameter Rd_1 on axial velocity $w(\chi,t)$ with variation of χ

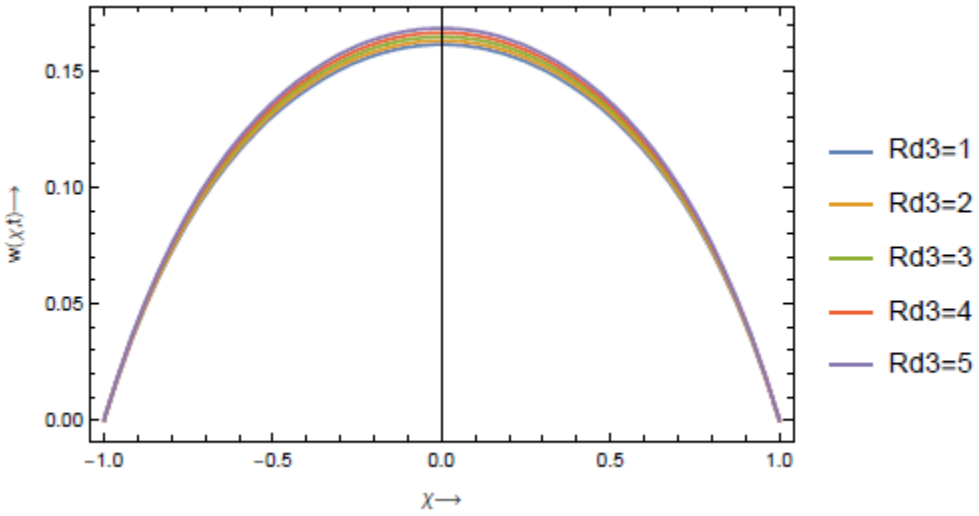

Figure 4.7: Effect of Chemical parameter Rd_3 on axial velocity $w(\chi,t)$ with variation of χ

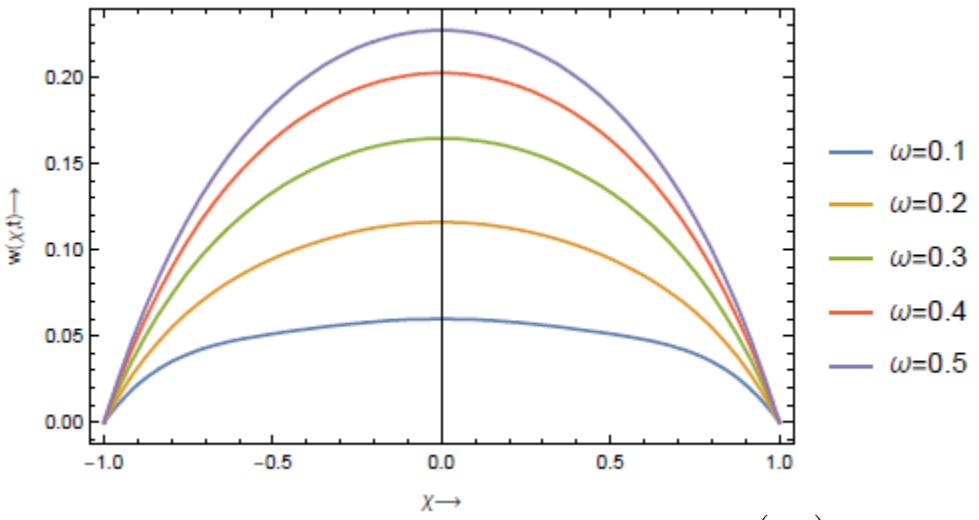

Figure 4.8: Effect of Oscillatory frequency ω on axial velocity $w(\chi,t)$ with variation of χ

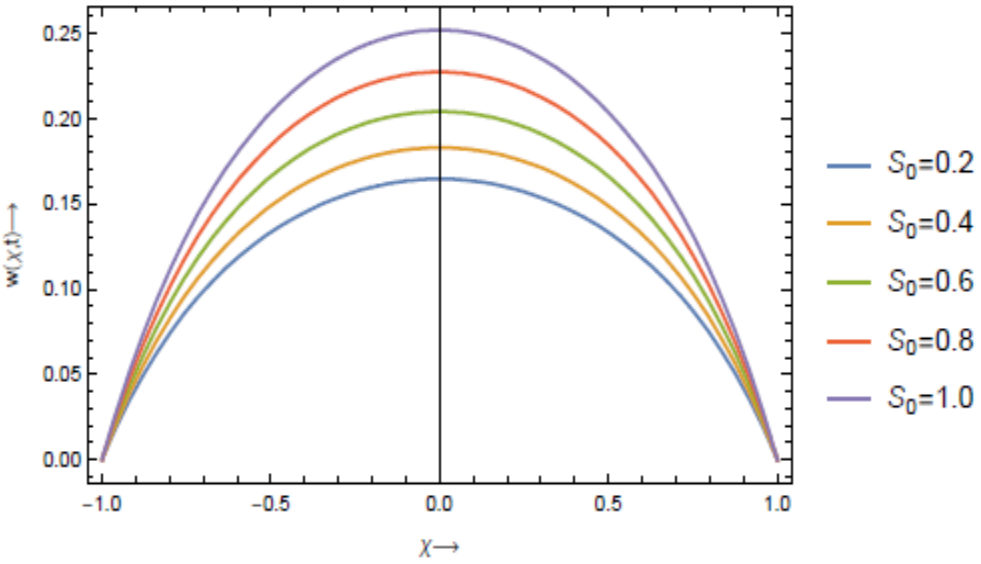

Figure 4.9: Effect of Soret number S_0 on axial velocity $w(\chi,t)$ with variation of χ

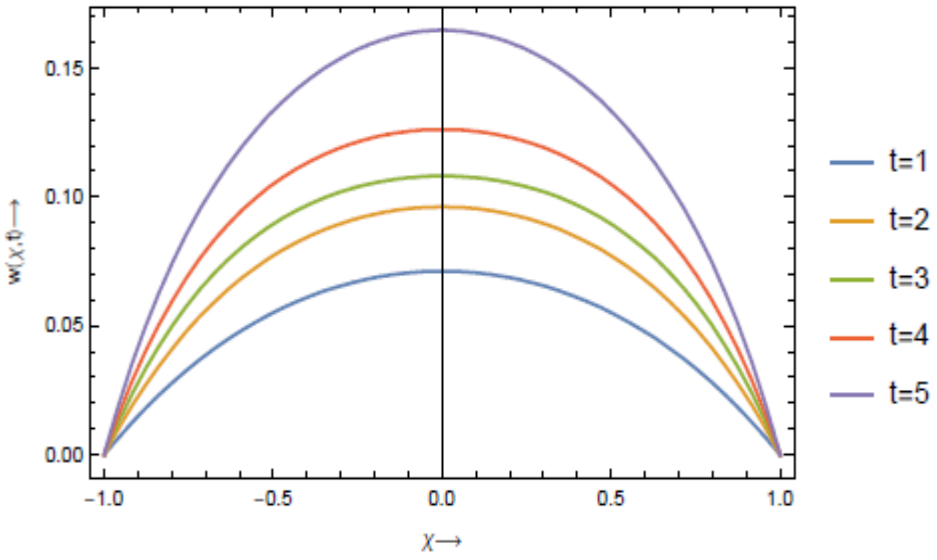

Figure 4.10: Effect of time parameter t on axial velocity $w(\chi,t)$ with variation of χ

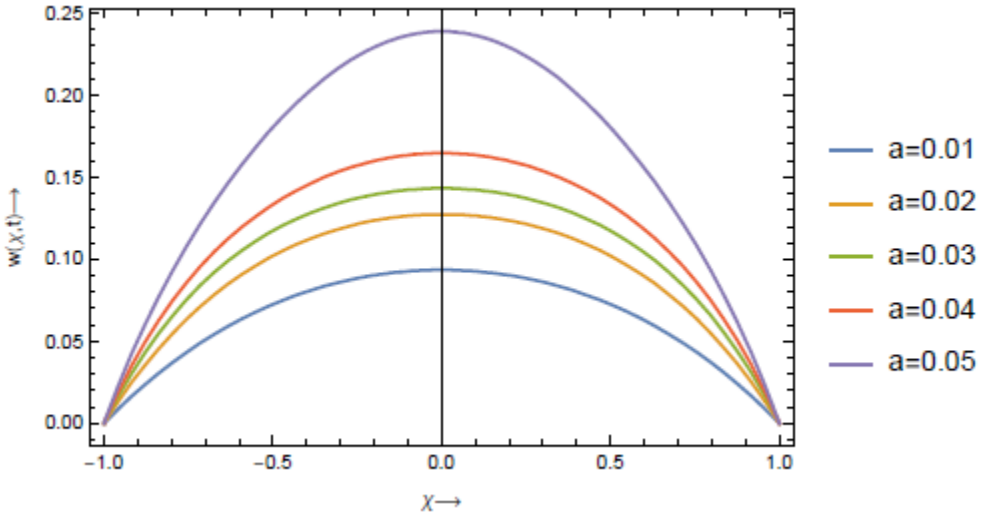

Figure 4.11: Effect of Growth rate a on axial velocity $w(\chi,t)$ with variation of χ

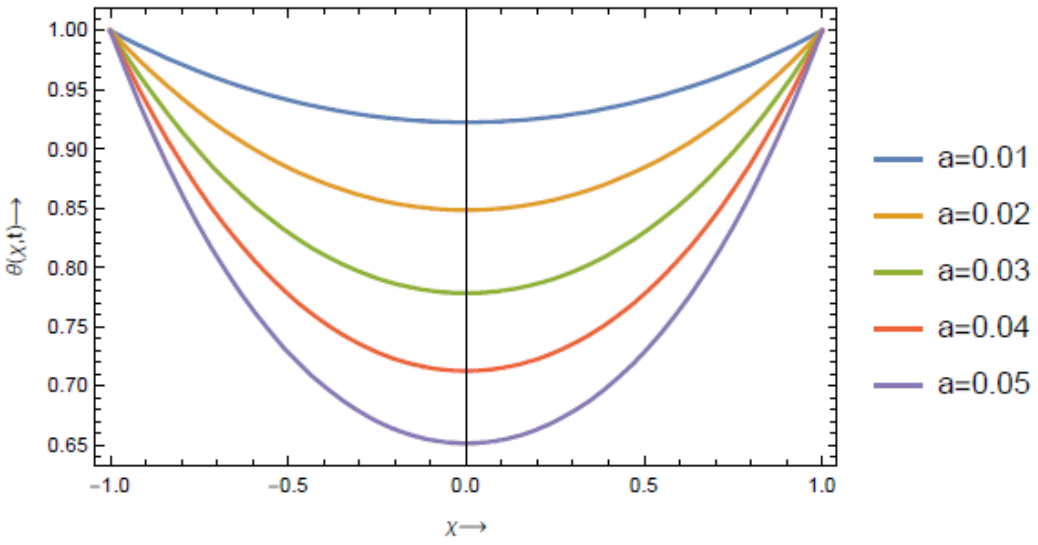

Figure 4.12: Effect of Growth rate a on Temperature $\theta(\chi,t)$ with variation of χ

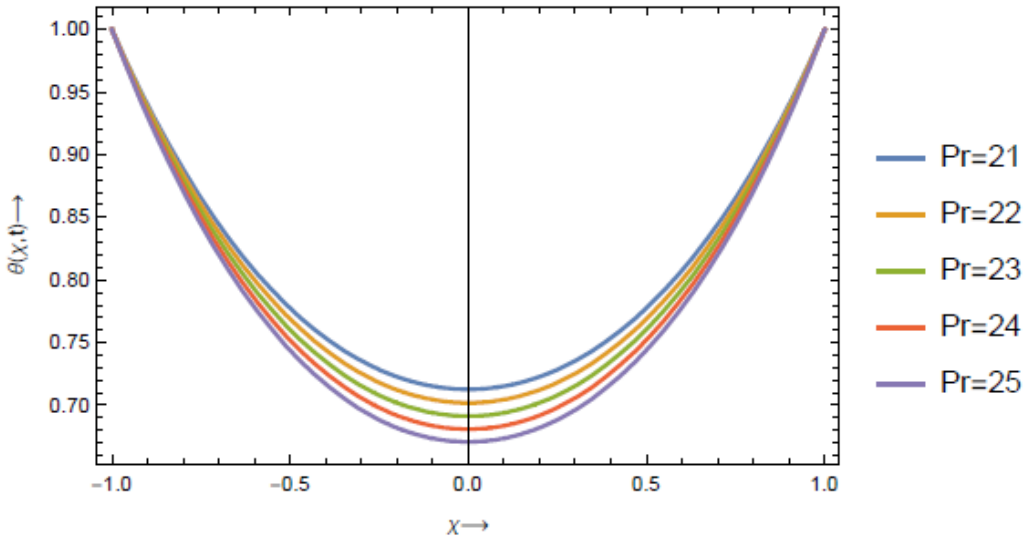

Figure 4.3: Effect of Prandtl number Pr on Temperature $\theta(\chi,t)$ with variation of χ

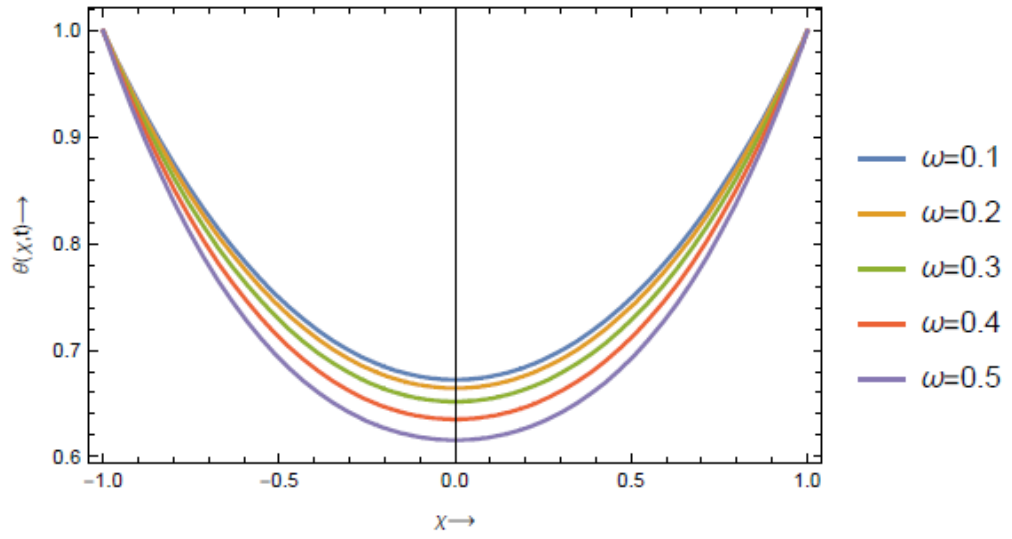

Figure 4.14: Effect of oscillatory frequency ω on Temperature $\theta(\chi,t)$ with variation of χ

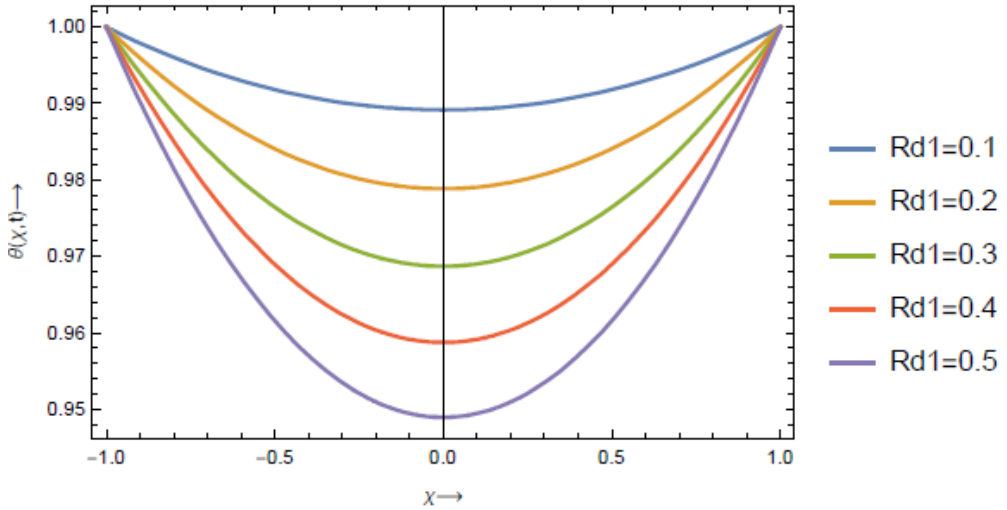

Figure 4.15: Effect of radiation parameter value Rd_1 on Temperature $\theta(\chi,t)$

χ

IV. DISCUSSION AND CONCLUSIONS

This study investigates mathematical model of the treatment effect on LDL-C and atherosclerotic blood flow microchannel with radiative heat. The coupled partial differential equations were scaled, reduced to system of ordinary differential equations and solved analytically within the set boundary conditions; the analytical profiles obtained are blood velocity, LDL-C concentration and blood temperature profiles respectively. We developed Mathematica codes to perform the numerical computations, where we varied the pertinent parameter values. $Rd_1 = 0.5, Gr = 10, Gr = 15, \delta = 0.5, R_T = 0.5,$ $Rd_3 = 03, Sc = 0.5, Pr = 21$ and $\omega = 0.2$ the following analysis was reached as $\chi \to 1$ as $y \to h$. The discussions are as follows:

Figure 4.1 illustrates that the axial blood velocity decreases for an increase in Schmidt number $Sc = 0.2, 0.4, 0.6, 0.8, 1.0$. This shows that mass diffusivity is greater than momentum diffusivity, and if that happen the blood velocity decreases. The figure clearly indicates maximum blood velocity at the point when $\chi = 0$, however, the velocity began to decrease as the boundary layer of the stenosis increases until it is zero when $\chi = 1$. The effect of the magnetic field was also investigated, and Figure 4.2 depicts a decrease in blood velocity, for various numbers of the magnetic field $M = 1, 2, 3, 4, 5$. Figure 4.2 is of the view that if a magnetic field is perpendicularly applied on an electrically conducting fluid such as blood, it generates a force called the Lorentz force which opposes flow and inhibits the movement of the fluid in the vessel. The figure also showed that the velocity is at its peak when $\chi = 0$ before it reduces at the boundary layer gets bigger $\chi = 1$. The permeability effect in allowing blood velocity was investigated using Figure 4.3: this figure depicts an increase in blood velocity for the increase in permeability parameter $k = 0.01, 0.02, 0.03, 0.04, 0.05$ as blood flows axially. Permeability measures the rate of ease with which cholesterol saturated channel will permit the passage of blood through it. It is also illustrated in Figures 4.4 to 4.5, where the blood velocity increases for the increase in Grashof number $Gr = 5, 10, 15, 20, 25$ and $Gc = 5, 10, 15, 20, 25$. These results are of the view that the viscous force is greater than the buoyancy force due to high cholesterol in circulation; however, the cholesterol did not inhibit the flow because of the presence of the treatment $R_T = 0.5$. Treatment drugs like pentoxifylline is used to improve blood flow in patients with circulation problem to reduce aching, cramping, and tiredness in hands and feet. It works by decreasing the thickness (viscosity) of blood, and this change allows the blood to flow more easily, especially in the small blood vessels of the hands and feet.

Figure 4.6 showed an increase in blood velocity for the various increase in radiation parameter $Rd_1 = 2, 4, 6, 8, 10,$ this result is of the view that heat therapy increases blood flow to a specific area and also improved circulation. This is because heat

on an inflame area causes the blood vessels to dilate, and allows blood to flow to the injured area. The application of heat to an affected area can provide comfort and increase muscle flexibility, as well as heal damaged tissue.

The effect of chemical parameter $Rd_3 = 1, 2, 3, 4, 5$ on blood velocity in an axial direction was investigated as see in Figure 4.7, and the result showed a decrease of the velocity, for various values of chemical reaction parameters. Figure 4.8 depicts that blood velocity decrease for various input of the oscillatory frequency $\omega = \frac{2\pi}{T_c} = 0.1, 0.2, 0.3, 0.4, 0.5$ as it flows towards the axial direction. Cardiac output measures blood flow from the heart through the ventricles, it is in liters per minute. The factors that cause cardiac output to increase, by elevating heart rate or stroke volume, will elevate blood pressure and promote blood flow. However, the decrease is as a result of the increase in cholesterol level. Using Figure 4.9, the ratio of temperature difference to concentration difference was investigated. The figure depicts that the blood velocity increases for various increase in Soret or metabolic heat parameter $S_0 = 0.2, 0.4, 0.6, 0.8, 1.0$, it is seen that the blood velocity attained the peak at the centerline of the vessel $\chi = 0$ and began to decrease for each of the profiles as the boundary thickness increases until it gets to $\chi = 1$.

Radiation causes the affected blood vessels to narrow and this limits the blood flow to the area. Therefore, patients who have had radiation treatment are at an increased risk of developing the condition. It is seen in Figure 4.15, that increasing the radiation level beyond a threshold could be injurious to health of the patient. The article also looked at the effect of time parameter, growth rate and Prandtl number on both blood velocity and temperature profiles as depicted in Figures 4.10 – Figure 15.

In conclusion, in our investigation we can conclude that the blood velocity is improved with the help of an appropriate treatment, magnetic field intensity increase caused the blood velocity to decrease and the height of stenosis impede the blood velocity. The novelty is this research is the researchers ability to remodel the region of atheorslcerosis, where the treatment was incorporated and the growth rate that was looked into too, as it affect the general flow behavior.

REFERENCES

[1] El-Shehawey, E. F., & Husseny, S. Z., Peristaltic transport of a magneto-fluid with porous boundaries, Applied Mathematics and Computation, 129(2-3) (2002) 421-440.

[2] Sinha, A., & Misra, J. C., Influence of Slip Velocity on blood flow through an artery with Permeable Wall: A Theoretical Study, International Journal of Biomathematics, 5(05) (2012) 1250042.

[3] Makinde, O. D., & Osalusi, E., MHD steady flow in a channel with slip at the permeable boundaries, Romanian Journal of Physics, 51(3/4) (2006) 319.

[4] Elangovan, K., & Ratchaga, N. P., Steady flow through a circular vertical pipe with slip at the permeable boundaries with an applied magnetic field. Appl. Math. Sci, 4 (2010) 2445-52.

[5] Makinde, O. D., & Chinyoka, T., Unsteady MHD flow in a porous channel with an exponentially decreasing suction. J. Pure Appl. Math, 1(1) (2001) 1-13.

[6] Sattar, A., & Waheedullah, A., Unsteady flow of a visco-elastic fluid through porous medium bounded by two porous plates. Int. J. Eng. Sci. Tech, 5 (2013) 329-34.

[7] Si, X. H., Zheng, L. C., Zhang, X. X., & Chao, Y., Homotopy analysis solutions for the asymmetric laminar flow in a porous channel with expanding or contracting walls. Acta Mechanica Sinica, 27(2) (2011) 208-214.

[8] Eldesoky, I. M., Slip effects on the unsteady MHD pulsatile blood flow through porous medium in an artery under the effect of body acceleration, International Journal of Mathematics and Mathematical Sciences, (2012).

[9] Beavers, G. S., & Joseph, D. D., Boundary conditions at a naturally permeable wall, Journal of fluid mechanics, 30(1) (1967) 197-207.

[10] El-Shehawy, E. F., El-Dabe, N. T., & El-Desoky, I. M., Slip effects on the peristaltic flow of a non-Newtonian Maxwellian fluid. Acta mechanica, 186(1) (2006) 141-159.

[11] Eldesoky, I. M., Influence of slip condition on peristaltic transport of a compressible Maxwell fluid through porous medium in a tube, International Journal of Applied Mathematics and Mechanics, 8(2) (2012) 99-117.

[12] Chu, W. K. H., & Fang, J., Peristaltic transport in a slip flow, The European Physical Journal B-Condensed Matter and Complex Systems, 16(3) (2000) 543-547.

[13] Eldesoky, I. M., Effect of relaxation time on MHD pulsatile flow of blood through porous medium in an artery under the effect of periodic body acceleration, Journal of Biological Systems, 21(02) (2013) 1350011.

[14] Wang, C. Y., Pulsatile flow in a porous channel, (1971).

[15] Tsangaris, S., Kondaxakis, D., & Vlachakis, N. W., Exact solution for flow in a porous pipe with unsteady wall suction and/or injection, Communications in Nonlinear Science and Numerical Simulation, 12(7) (2007) 1181-1189.

[16] Ramanmurthy, J. V., Srinivasacharyulu, N., & Odelu, O., Viscous fluid flow between two parallel plates with periodic suction and injection, AMSE J. Model. B: Mech. Thermic, 50 (2007) 29- 37.

[17] Midya, C., Layek, G. C., Gupta, A. S., & Mahapatra, T. R., Magnetohydrodynamic viscous flow separation in a channel with constrictions. J. Fluids Eng., 125(6) (2003) 952-962.
[18] Bunonyo, K. W., & Amos, E., Lipid Concentration Effect on Blood Flow Through an Inclined Arterial Channel with Magnetic Field, Mathematical Modelling and Applications, 5(3) (2020) 129.
[19] Amos, E., & Ogulu, A., Magnetic effect on pulsatile flow in a constricted axis-symmetric tube. Indian Journal of Pure & Applied Mathematics, 34(9) (2003) 1315-1326.
[20] Jain, M., Sharma, G. C., & Singh, A., Mathematical Analysis of MHD Flow of Blood in Very Narrow Capillaries (RESEARCH NOTE), (2009).
[21] Mittal, R., Simmons, S. P., & Najjar, F., Numerical study of pulsatile flow in a constricted channel. Journal of Fluid Mechanics, 485 (2003) 337-378.
[22] Bunonyo, K. W., & Eli, I. C., Oscillatory Flow of LDL-C and Blood Fluid through a Slanted Channel with Heat within the Sight of Magnetic Field, European Journal of Applied Physics, 3(5) (2021) 37-44.
[23] Bunonyo, K. W., & Okardi, B., (2021). IJMTT Call for Paper June-2021 UGC Approved Journal in 2017.
[24] Noreen, S., Qasim, M., & Khan, Z. H., MHD pressure driven flow of nanofluid in curved channel, Journal of Magnetism and Magnetic Materials, 393 (2015) 490-497.
[25] Misra, J. C., & Adhikary, S. D., MHD oscillatory channel flow, heat and mass transfer in a physiological fluid in presence of chemical reaction, Alexandria Engineering Journal, 55(1) (2016) 287-297.
[26] Valvano, J. W., Nho, S., & Anderson, G. T., Analysis of the Weinbaum-Jiji model of blood flow in the Canine Kidney cortex for self-heated thermistors, (1994).
[27] Chato, J. C., Heat transfer to blood vessels, Journal of biomechanical engineering, 102(2) (1980) 110-118.

NOMENCLATURE

x^*	Dimensional coordinate along the channel
y^*	Dimensional coordinate perpendicular to the channel
R	Radius of an abnormal channel
R_0	Radius of normal channel
R_T	Treatment parameter
d_0	Onset of stenosis
Rd_1	Heat radiation parameter
Rd_3	Chemical reaction parameter
k_b	Thermal conductivity of blood
k	Permeability term
w^*	Dimensional blood velocity
w	Dimensionless blood velocity
w_0	Perturbed blood velocity
h	Atherosclerotic region
C	LDL-Concentration of lipoprotein-C
C_w	LDL-Concentration at the wall
C_∞	LDL-Concentration at far field
c_{bp}	The specific heat capacity of blood
t^*	Dimensionless time
T	Blood Temperature
T_∞^*	Far field blood temperature
T_w^*	Temperature at the wall
T_c	The periodic cardiac cycle

D_m Molecular diffusivity of lipoprotein-C concentration

B_0 Magnetic intensity

Q_0 Dimensional heat source term

Greek Symbols

υ	Kinematic viscosity of blood
μ_b	Dynamic viscosity of blood
Pr	Prandtl number for blood
g	Acceleration due to gravity
δ^*	Height of stenosis
σ_e	Electrical conductivity
λ^*	Length of stenosis
$\omega = \frac{2\pi}{T_c}$	Oscillatory frequency
ϕ	Dimensionless LDL-Concentration
ϕ_0	Perturbed LDL-Concentration
θ	Dimensionless Blood Temperature
θ_0	Perturbed Blood Temperature
ρ_b	Density of the fluid
χ	Boundary layer thickness

Subscripts

w Wall

b Blood

p Pulse