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Short Communication

A Remark On The Fixed-Point Theorem of Mustafa

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Abstract *- We show that the main result (Theorem 2.11) due to Mustafa at. el.(J. Fixed Point Theory and Applications, 2019:16, https://doi.org/10.1186/s13663-019-0666-3) can be prove without taking continuity of the class of functions defined by Jleli et al.*

Keywords — *Fixed point, JS-contractions, Continuity*

I. INTRODUCTION AND PRELIMINARIES

Jleli et al.[3] introduced the class Θ_0 consisting of all functions $\theta:(0,\infty)\to(1,\infty)$ satisfying the following conditions:

(θ1) θ is nondecreasing.

(θ2) For each sequence $t_n \subseteq (0,\infty)$, $\lim_{n\to\infty}\theta(t_n)=1$ if and only if $\lim_{n\to\infty} t_n = 0$;

(θ3) There exists $r\in(0,1)$ and $l\in(0,\infty)$ such that $\lim_{t\to 0^+}\frac{\theta(t)-1}{t^r}=l;$.

(θ4) θ is continuous

Mustafa et.al. [1] redefined the class Θ_0 as following and denoted it by Θ. The set of all $\theta:[0,\infty)\to[1,\infty)$ satisfying the following conditions:

(θ1) θ is continuous and increasing.

(θ2) For each sequence $t_n \subseteq (0,\infty)$, $\lim_{n\to\infty}\theta(t_n)=1$ if and only if $\lim_{n\to\infty} t_n = 0$.

Definition 1.1. Let $(E, \preceq, \tilde{S})$ be an ordered S_p-metric space. A mapping $f:E\to E$ is called an S_p- rational JS contraction if

$$\theta\left(\Omega\left[2\tilde{S}\,(f\xi, f\eta, f\omega)\right]\right)\le\theta\left(M\left(\xi,\eta,\omega\right)\right)^k \tag{1.1}$$

for all mutually comparable elements $\xi, \eta, \omega \in E$, where $\theta \in \Theta$, $k \in [0, 1)$ and

$$M(\xi,\eta,\omega)=\max\left\{\tilde{S}(\xi,\eta,\omega),\frac{\tilde{S}(\xi,\xi,f\xi)\tilde{S}(\eta,\eta,f\eta)}{1+\tilde{S}(\xi,\eta,\eta)+\tilde{S}(\xi,\omega,\omega)},\frac{\tilde{S}(\eta,\eta,f\eta)\tilde{S}(\omega,\omega,f\omega)}{1+\tilde{S}(\eta,f\omega,f\omega)+\tilde{S}(\eta,\xi,\xi)}\right\},$$

Definition 1.2. [1] An ordered S_p- metric space $(E, \preceq, \tilde{S})$ is said to have the s.l.c. property if, whenever $\{\xi_n\}$ is an increasing sequence in E such that $\xi_n \to u \in E$, one has $\xi_n \preceq u$ for all $n \in N$.

Theorem 1.3. [1] Let $(E, \preceq, \tilde{S})$ be an ordered S_p-metric space. Let $f:E\to E$ be an increasing mapping with respect to $\preceq$

such that there exists an elements $\xi_0 \in E$ *with* $\xi_0 \preceq f\xi_0$. *Suppose that* f *is an* $S_p -$ *rational JS-contractive mapping. If*

I. f *is continuous, or*

II. $(E, \preceq, \tilde{S})$ *enjoys the s.l.c. property,*

then f *has a fixed point. Moreover, the set of fixed points of* f *is well ordered if and only if* f *has one and only one fixed point.*

In this paper, we show that the assumption of continuity of the class Θ is not necessary. So, hereby we prove above theorem (i.e. Theorem 2.11) of Mustafa without considering continuity.

II. MAIN RESULTS

We redefine the class Θ_0 and denote it by Θ. Θ consisting of all functions θ : [0,∞) → [1,∞) satisfying the conditions (θ_1) and (θ_2) defined by Jleli etal.[3].

Theorem 2.1. *Let* $(E, \preceq, \tilde{S})$ *be an ordered* S_p*-metric space. Let* $f : E \to E$ *be an increasing mapping with respect to* $\preceq$ *such that there exists an elements* $\xi_0 \in E$ *with* $\xi_0 \preceq f\xi_0$. *Suppose that* f *is an* $S_p -$ *rational JS-contractive mapping. If*

I. f *is continuous, or*

II. $(E, \preceq, \tilde{S})$ *enjoys the s.l.c. property,*

then f *has a fixed point. Moreover, the set of fixed points of* f *is well ordered if and only if* f *has one and only one fixed point.*

Proof. Let us define $\xi_n = f^n\xi_0$.

Step 1. From the Theorem 2.11 of [1], we conclude that

$$\lim_{n\to\infty} \tilde{S}(\xi_n, \xi_{n+1}, \xi_{n+1}) = 0$$

Step 2. Again from the Theorem 2.11 of [1], we have

$$\frac{1}{2}\Omega^{-1}(\in) \le \limsup_{i\to\infty} \tilde{S}(\xi_{m_i+1}, \xi_{n_i}, \xi_{n_i}) \tag{2.1}$$

And

$$\limsup_{i\to\infty} M\left(\xi_{m_i}, \xi_{n_i-1}, \xi_{n_i-1}\right) \le \in \tag{2.2}$$

Since, θ is nondecreasing , One can write from the equation (1.1),

$$\Omega\left[2\tilde{S}(f\xi, f\eta, f\omega)\right] \le \left(M(\xi,\eta,\omega)\right)^k$$

Put, $\xi = \xi_{m_i}, \eta = \xi_{n_i-1}, \omega = \xi_{n_i-1}$ in above equation. We have,

$$\Omega\left[2\tilde{S}\left(\xi_{m_i+1}, \xi_{n_i}, \xi_{n_i}\right)\right] \le \left(M\left(\xi_{m_i}, \xi_{n_i-1}, \xi_{n_i-1}\right)\right)^k$$

Taking limit supremum both side,

$$\limsup_{i\to\infty}\left(\Omega\left[2\tilde{S}\left(\xi_{m_i+1}, \xi_{n_i}, \xi_{n_i}\right)\right]\right) \le \limsup_{i\to\infty}\left(M\left(\xi_{m_i}, \xi_{n_i-1}, \xi_{n_i-1}\right)\right)^k$$

Again using the condition (θ_1). i.e θ is nondecreasing, we obtain

$$\theta\left\{\limsup_{i\to\infty}\left(\Omega\left[2\tilde{S}(\xi_{m_i+1},\xi_{n_i},\xi_{n_i})\right]\right)\right\}\le\theta\left\{\limsup_{i\to\infty}\left(M\left(\xi_{m_i},\xi_{n_i-1},\xi_{n_i-1}\right)\right)^k\right\}$$

Hence, from the equations (2.1),(2.2) and above equation, one arrive at

$$\theta\left(\Omega\left[2.\frac{1}{2}\Omega^{-1}(\in)\right]\right)\le\theta\left\{\Omega\left[2\limsup_{i\to\infty}\tilde{S}\left(\xi_{m_i+1},\xi_{n_i},\xi_{n_i}\right)\right]\right\}$$

$$\le\theta\left\{\limsup_{i\to\infty}\ \Omega\left[2\tilde{S}\left(\xi_{m_i+1},\xi_{n_i},\xi_{n_i}\right)\right]\right\}$$

$$\le\theta\left\{\limsup_{i\to\infty}\left(M\left(\xi_{m_i},\xi_{n_i-1},\xi_{n_i-1}\right)\right)^k\right\}$$

$$\le\theta(\in)^k$$

Hence,

$$\theta(\in)\le\theta(\in)^k$$

which possible only if $\in=0$, a contradiction.
The rest proof is as per the theorem 2.11 of [1].

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