# Effect of Couple Stress On Thermal Instability of Visco-Elastic Walter's (Modal B') Nanofluid Layer Through A Porous Medium

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**Abstract** - The effect of couple stress on thermal instability of visco-elastic Walter's (modal B') nanofluid layer through a porous medium is analytically and graphically discussed in this paper. The effects of the physical parameters such as nano particle Rayleigh number, Lewis number, modified diffusivity ratio, and couple stress on the stationary deportation have been observed by applying perturbation method and normal mode analysis. It is found that modified diffusivity ratio, nano particle Rayleigh number and Lewis number have stabilizing effect on stationary deportation whereas couple stress shows a destabilizing effect for stationary convection.

Keywords - Rayleigh number; couple stress, Lewis number; modified diffusivity ratio.

# I. INTRODUCTION

The visco-elastic Walter's (modal B') is a fluid that does not obey the Maxwell's constitutive relations. Hydromagnetic instability of visco-elastic Walter's (modal B') nanofluid layer heated from below has been studied by D.Kapil and S. Kumar [1] and resulted that magnetic field shows the stabilizing effect for stationary deportation. Thermal instability of Newtonian fluid under the consideration of hydromagnetics and hydrodynamics has been investigated broadly by Chandrasekhar [2]. Sharma and Aggarwal [3] discussed thermal instability of visco-elastic Walter's (modal B') fluid in hydromagnetics under the effect of suspended particles and compressibility. S.Pundir, D.Kapil and R.Pundir [4] investigated the effect of suspended particles on Rayleigh Benard convection in visco-elastic Walter's (modal B') nanofluid in porous medium and resulted that suspended particles have stabilizing effect. The flow through porous medium having a large number of application in the field of production of crude oil from the pores of reservoir rocks, petroleum engineering and geophysics. The effect of suspended particles on thermal instability of Oldroydian visco-elastic fluid in hydromagnetics through porous medium has been investigated by Sharma and Sunil [5]. D.Kapil, S.Pundir and R.Pundir [6] investigated hydro-magnetic impermanence of visco-elastic Rivlin-Ericksen nanofluid saturated by a Darcy-Brinkman porous medium and found that magnetic field has stabilizing effect on stationary deportation. A porous medium is that medium in which a solid with pores and characterized by the manner in which the holes are imbedded. The flow through the porous medium is governed by Darcy's law which states that the usual viscous term in the equation of motion of Walter's (modal B') nanofluid is replaced by resistance term  $\left[-\frac{1}{k_1}\left(\Omega - \Omega'\frac{\partial}{\partial t}\right)\right]\nabla^2 q_f$ , where  $\Omega$  and  $\Omega'$  are the viscosity and visco-elasticity of incompressible Walter's (model B') fluid,  $k_1$  is the medium permeability and q is the Darcian velocity of the fluid. Stokes [7] explained broadly couple-stress assumption. Couple-stress theory having a large number of applications in various industrial fields, medical field such as lubrication mechanism, functioning of synovial joints and opened new ways in several fields of scientific research. Sharma and Sharma [8] have discussed the influence of suspended particles on couple-stress fluid heated from below in the presence of vertical rotation and vertical magnetic field and resulted that the rotation has stabilize effect on the system whereas suspended particles have destabilizing effects. Shiva Kumar et al.[9] discussed the influence of rotation on thermal convection in a couple-stress fluid saturated rotating rigid porous layer.

The present paper attempts to discuss the effect of couple-stress on visco-elastic Walter's (modal B') nanofluid layer through porous medium.

## **II. MATHEMATICAL FORMULATION**

Suppose an infinite horizontal layer of couple-stress visco-elastic Walter's (modal B') nanofluid of thickness is  $d^{\#}$  bounded by z = 0 and  $z = d^{\#}$  and heated from below. The gravity force g (0, 0,-g) is acting on fluid layer in upward direction.  $T_0$  and  $\varphi_0$  are the temperature and volumetric fraction of nano particles at z = 0 and  $T_1$ ,  $\varphi_1$  are temperature and volumetric fraction at  $z = d^{\#}$  respectively.

The governing equation for couple-stress visco-elastic Walter's (modal B') nanofluid

$$\nabla \boldsymbol{q}_d = 0 \tag{1}$$

$$\frac{\rho}{\epsilon} \frac{d\boldsymbol{q}_d}{dt} = -\nabla \mathbf{p} + \rho \mathbf{g} - \frac{1}{k_1} \left( \mu - \mu' \frac{\partial}{\partial t} \right) \nabla^2 \boldsymbol{q}_d - \frac{1}{k_1} \left( \delta - \delta' \nabla^2 \right) \boldsymbol{q}_d$$

where  $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{q}_d \cdot \nabla)$  stands for convection derivative,  $\mathbf{q}_d(\mathbf{u}, \mathbf{v}, \mathbf{w})$  is the velocity vector, p is the hydrostatic pressure,  $\mu$  and  $\mu'$  are the viscosity and visco-elasticity  $\delta$  and  $\delta'$  are fluid viscosity and couple-stress fluid viscosity and g(0, 0, -g) is acceleration due to gravity. The density  $\rho$  of nanofluid can be written as

(2)

$$\rho = \varphi \,\rho_p + (1 - \varphi)\rho_f \tag{3}$$

where  $\rho_p$  and  $\rho_f$  are the densities of nano particles and base fluid of nano particles and  $\varphi$  is the volume fraction.

The equation of motion for couple-stress visco-elastic Walter's (modal B') nanofluid is given as:

$$\frac{\rho}{\varepsilon}\frac{d\boldsymbol{q}_d}{dt} = -\nabla \mathbf{p} + \left(\varphi\,\rho_p + (1-\varphi)\left\{\rho\left(1-\alpha(T-T_0)\right)\right\}\right)\mathbf{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{q}_d - \frac{1}{k_1}\left(\delta - \delta'\nabla^2\right)\boldsymbol{q}_d \tag{4}$$

where  $\alpha$  is the coefficient of thermal expansion.

The continuity equation for the nano particles is

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} \boldsymbol{q}_d \,\nabla \varphi = D_B \,\nabla^2 \,\varphi + \frac{D_T}{T_1} \nabla^2 \,T \tag{5}$$
where  $D_B$  is the Brownian diffusion

coefficient and  $D_T$  is the Thermoporetic diffusion coefficient of the nano particles.

The energy equation in nanofluid is

$$\rho_c \left(\frac{\partial T}{\partial t} + \boldsymbol{q}_d \,\nabla T\right) = k_m \nabla^2 T + \varepsilon (\rho_c)_p (D_B \nabla \varphi . \,\nabla T + \frac{D_T}{T_1} \,\nabla T . \,\nabla T) \tag{6}$$

Where  $\rho_c$  is the heat capacity of fluid,  $(\rho_c)_p$  is the heat capacity of nano particles and  $k_m$  is the thermal conductivity.

Introducing non-dimensional variables as:

$$(x', y', z') = \left(\frac{x, y, z}{d^{\#}}\right),$$

$$q_{d'}(u', v', w') = q_{d}\left(\frac{u, v, w}{k}\right)d^{\#}, t' = \frac{tk}{d^{\#^{2}}},$$

$$p' = \frac{p}{\rho k^{2}} d^{\#^{2}}, \phi' = \frac{\varphi - \varphi_{0}}{\varphi_{1} - \varphi_{0}},$$

$$T' = \frac{T - T_{0}}{T_{0} - T_{1}},$$

where  $\frac{k_m}{\rho_c} = k$  is the thermal diffusivity of the fluid.

Equations (1), (4), (5) and (6), in non-dimensional form can be written as:

$$\nabla \boldsymbol{q}_d = 0 \tag{7}$$

$$\frac{1}{p_{r\varepsilon}}\frac{\partial q_{d}}{\partial t} = -\nabla \mathbf{p} - \mathbf{R}_{a_{m}}\hat{\mathbf{e}}_{z} - \mathbf{R}_{a_{m}}\varphi\hat{\mathbf{e}}_{z} - \mathbf{R}_{a}T\hat{\mathbf{e}}_{z} - \frac{1}{k_{1}}\left(1 - \mathbf{n}F\right)\nabla^{2}\boldsymbol{q}_{d} - \frac{1}{p_{l}}\left(1 - \gamma\nabla^{2}\right)\boldsymbol{q}_{d}$$
(8)  
$$\frac{\partial\varphi}{\partial q} + \frac{1}{2}\boldsymbol{q}_{d} \nabla q_{d} - \frac{1}{2}\nabla^{2}\boldsymbol{q}_{d} + \frac{N_{A}}{2}\nabla^{2}T$$
(9)

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon} \boldsymbol{q}_d \nabla \varphi = \frac{1}{L_e} \nabla^2 \varphi + \frac{N_A}{L_e} \nabla^2 T$$
(9)

$$\frac{\partial T}{\partial t} + \boldsymbol{q}_d \,\nabla T = \nabla^2 \mathbf{T} + \frac{\mathbf{N}_B}{L_e} \nabla \varphi. \,\nabla T + \frac{N_A N_B}{L_e} \,\nabla T \,. \,\nabla T \tag{10}$$

[The dashes (`) have been dropped for simplicity]

Here non-dimensional parameters are:

Lewis number  $L_e = \frac{k}{D_B}$ , Prandtl number  $p_r = \frac{\mu}{\rho k}$ , Thermal Rayleigh number  $R_a = \frac{\rho g \alpha d^{\#^3}}{\mu k} (T_0 - T_1)$ , Basic- density Rayleigh number  $R_{a_m} = \frac{[\rho_p \phi_0 + \rho (1 - \phi_0)]g d^{\#^3}}{\mu k}$ , Nano particle Rayleigh number  $R_{a_n} = \frac{(\rho_p - \rho)(\phi_1 - \phi_0)g d^{\#^3}}{\mu k}$ , Kinematic visco-elasticity parameter  $F = \frac{\mu'}{\rho d^{\#^2}}$ , Modified diffusivity ratio  $N_A = \frac{D_T}{D_B T_1(\phi_1 - \phi_0)} (T_0 - T_1)$ , Modified particle density increment  $N_B = \frac{\varepsilon(\rho_c)_P (\phi_1 - \phi_0)}{(\rho_c)_f}$ , Modified couple-stress parameter  $= \frac{\mu'}{\mu d^{\#^2}}$ , dimensionless medium permeability  $p_l = \frac{k}{d^{\#^2}}$ .

We assume that temperature and volumetric fraction of nano particles are constant on boundaries. Thus the dimensionless boundaries conditions are

 $w = 0, T = 1, \ \varphi = 0 \ at \ z = 0$ And  $w = 0, T = 0, \ \varphi = 1 \ at \ z = 1$  (11)

#### A. Basic States and its solution

The basic state of nanofluid is supposed to be time independent of time and can be written as  $q_d'(u, v, w) = 0$ , p' = p(z),  $T' = T_b(z)$ ,  $\varphi' = \varphi_b(z)$ , Equations (7) to (10) using boundary conditions (11) give solution as:

$$T_b = 1 - z \text{ and } \varphi_b = z \tag{12}$$

## **B.** Perturbation solution

The stability of the system can be studied by introducing small perturbations to primary flow, and written as

 $q_{d}(u, v, w) = 0 + q_{d}(u, v, w), T' = T_{b} + T', \varphi' = \varphi_{b} + \varphi'', p' = p_{b} + p'', with T_{b} = 1 - z \text{ and } \varphi_{b} = z$ (13)
Using equation (13) in equation
(7) to (10) and linearize by neglecting the product of the prime quantities, we obtain the following equations:

$$\nabla \boldsymbol{q}_d = 0 \tag{14}$$

by operating Eq. (15) with  $\hat{e}_z$ .curl curl, we get:

$$\frac{1}{p_{r}\varepsilon}\frac{\partial w}{\partial t}\hat{\mathbf{e}}_{z} = -\nabla \mathbf{p} - \mathbf{R}_{a_{m}}\hat{\mathbf{e}}_{z} - \mathbf{R}_{a_{n}}\varphi\hat{\mathbf{e}}_{z} - \mathbf{R}_{a}T\hat{\mathbf{e}}_{z} - \frac{1}{k_{1}}(1 - \mathbf{n}F)\nabla^{2}w\hat{\mathbf{e}}_{z} - \frac{1}{p_{l}}(1 - \gamma\nabla^{2})w\hat{\mathbf{e}}_{z}$$
(15)  
$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}w = \frac{1}{L_{e}}\nabla^{2}\varphi + \frac{N_{A}}{L_{e}}\nabla^{2}T$$
(16)

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{L_e} \left( \frac{\partial T}{\partial z} - \frac{\partial \phi}{\partial z} \right) - 2 \frac{N_A N_B}{L_e} \frac{\partial T}{\partial z}$$
(17) The dashes (``) have been dropped for simplicity.

Since  $R_{a_m}$  is just a measure of basic static pressure gradient so it is not involved in these and subsequent equations. Now

$$\left[\frac{1}{k_1}\left(1-\mathrm{nF}\right)-\frac{\gamma}{p_l}\right]\nabla^4 w + \frac{n}{p_r\varepsilon}\nabla^2 w + \frac{1}{p_l}w = \mathrm{R}_a\nabla_\mathrm{H}^2\mathrm{T} - \mathrm{R}_{a_n}\nabla_\mathrm{H}^2\phi \tag{18}$$

where  $\nabla_{\rm H}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  is the two dimensional Laplacian operator on horizontal plane.

## **III. NORMAL MODE OBSERVATION**

For observing the disturbances in to normal modes and assuming that the perturbed quantities are of the form:

$$[W, T, \varphi] = [W(z), T(z), \varphi(z)] \exp(ik_x x + ik_y y + nt)$$
<sup>(19)</sup>

Where  $k_x$  and  $k_y$  are wave numbers in x and y directions respectively, while n is growth rate of disturbances.

Using eq. (19), eq. (16), (17) and (18) become:

$$\frac{1}{\varepsilon}W - \frac{N_A}{L_e}(D^2 - a^2)T - \left[\frac{1}{L_e}(D^2 - a^2) - n\right]\varphi = 0 \quad (20)$$
$$W + \left[(D^2 - a^2) - n + \frac{N_B}{L_e}D - \frac{2N_A N_B}{L_e}D\right]T - \frac{N_B}{L_e}D\varphi = 0 \quad (21)$$

$$\left[\left\{\frac{1}{k_1}\left(1 - nF\right) - \frac{\gamma}{p_l}\right\}\left(D^2 - a^2\right)^2 + \frac{n}{p_r\varepsilon}\left(D^2 - a^2\right) + \frac{1}{p_l}\right]W + a^2R_aT - a^2R_{a_n}\varphi = 0$$
(22)

Where  $D = \frac{d}{dz}$  and  $a = \sqrt{k_x^2 + k_y^2}$  is the dimensionless the resultant wave number. The boundary conditions of the problem in view of normal mode are written as

 $W = 0, D^2 W = 0, T = 0, \varphi = 0 \text{ at } z = 0 \text{ and } W = 0, D^2 W = 0, T = 0, \varphi = 0 \text{ at } z = 1$  (23)

## IV. LINEAR STABILITY OBSERVATION

Consider the solution in the form w, T,  $\varphi$  is given as:

 $w=w_0\,\sin\pi z$  ,  $T=\,T_0\,\sin\pi z\,$  ,  $arphi=\,arphi_0\,\sin\pi z$ 

Equations (20),(21) and (22) reduced as

$$\frac{1}{\varepsilon}w_0 + \frac{N_A}{L_e}JT_0 + \left[\frac{1}{L_e}J + n\right]\varphi_0 = 0$$
(24)

$$w_0 - (J+n)T_0 = 0 (25)$$

$$\left[\left\{\frac{1}{k_{1}}\left(1-\mathrm{nF}\right)-\frac{\gamma}{p_{l}}\right\}J^{2}-\frac{n}{p_{r}\varepsilon}J+\frac{1}{p_{l}}\right]w_{0}-a^{2}\mathrm{R}_{a}\mathrm{T}-a^{2}\mathrm{R}_{a_{n}}\varphi=0$$
(26)

From equation (24) & (25), we get

$$\left[\frac{1}{\varepsilon}(J+n) + \frac{N_A}{L_e}J\right]T_0 + \left(\frac{1}{L_e}J + n\right)\varphi_0 = 0$$
(27)

From equation (25),(26) & (27), we get

$$R_{a} = \frac{1}{a^{2}} \left[ \left\{ \frac{1}{k_{1}} \left( 1 - \mathrm{nF} \right) - \frac{\gamma}{p_{l}} \right\} J^{2} - \frac{n}{p_{r}\varepsilon} J + \frac{1}{p_{l}} \right] (J+n) + \frac{\left\{ \frac{1}{\varepsilon} (J+n) + \frac{N_{a}}{L_{e}} J \right\}}{\frac{1}{L_{e}} J+n} R_{a_{n}}$$
(28)

where  $J = \pi^2 + a^2$ 

For neutral stability, the real part of n is zero. Hence, on putting  $n = i \omega$ , ( $\omega$  is the real and dimensionless frequency of oscillation) in eq.(28), we get:

$$R_{a} = \Delta_{1} + i \omega \Delta_{2} \qquad (29) \qquad \text{where}$$

$$\Delta_{1} = \frac{J}{a^{2}} \left[ \left( \frac{1}{p_{l}} + \frac{\omega^{2}}{p_{r}\varepsilon} \right) + \left\{ \left( \frac{1}{k_{1}} - \frac{\gamma}{p_{l}} \right) J^{2} + \frac{\omega^{2}F}{k_{1}} J \right\} \right] + \frac{1}{\left\{ \left( \frac{J}{L_{e}} \right)^{2} + \omega^{2} \right\}} \left[ \frac{J^{2}}{L_{e}} \left( \frac{1}{\varepsilon} + \frac{N_{a}}{L_{e}} \right) + \frac{1}{\varepsilon} \omega^{2} \right] R_{a_{n}} \qquad (30)$$

and imaginary part

$$\Delta_{2} = \frac{1}{a^{2}} \left[ \left\{ \frac{1}{k_{1}} - \frac{\gamma}{p_{l}} - \frac{1}{p_{r}\varepsilon} \right\} J^{2} - \frac{FJ^{3}}{k_{1}} + \frac{1}{p_{l}} \right] + \frac{J \left[ \frac{1}{\varepsilon} \left( \frac{1}{L_{e}} - 1 \right) - \left( \frac{N_{A}}{L_{e}} \right) \right]}{\left\{ \left( \frac{J}{L_{e}} \right)^{2} + \omega^{2} \right\}} R_{a_{n}}$$
(31)

 $R_a$  will be real since it is a physical quantity Hence, it follow from Eq.(36) that either  $\omega = 0$  (exchange of stability, steady state) or  $\Delta_2 = 0$  ( $\omega \neq 0$  overstability or oscillatory onset).

#### V. STATIONARY DEPORTATION

When the stability occurs in as stationary convection, the marginal state will be characterized by  $\omega = 0$ . the Eq.(29) reduces as:

$$(R_a)_s = \frac{(\pi^2 + a^2)}{a^2} \left[ \left( \frac{1}{k_1} - \frac{\gamma}{p_l} \right) (\pi^2 + a^2)^2 + \frac{1}{p_l} \right] + \left( \frac{L_e}{\varepsilon} + N_A \right) R_{a_n}$$
(32)

Here  $R_a$  is independent of both the prandtl numbers and the parameters containing the Brownian effects and the thermophoretic effects and presented in the thermal energy equation and the conversation equation for nano particles.

Take 
$$x = \frac{a^2}{\pi^2}$$
 in Eq. (32), then we have  
 $(R_a)_s = \frac{(1+x)}{x} \left[ \left( \frac{1}{k_1} - \frac{\gamma}{p_l} \right) (1+x)^2 \pi^4 + \frac{1}{p_l} \right] + \left( \frac{L_e}{\varepsilon} + N_A \right) R_{a_n}$ 
(33)

To study the effects of Lewis number  $L_e$ , modified diffusivity ratio  $N_A$ , and nano particles Rayleigh number  $R_n$  and couple-stress on stationary convection. We examine the nature of

$$\frac{\partial R_a}{\partial L_e}$$
,  $\frac{\partial R_a}{\partial N_A}$ ,  $\frac{\partial R_a}{\partial R_{a_n}}$ ,  $\frac{\partial R_a}{\partial \gamma}$  analytically.

From eq. (33)

$$\frac{\partial R_a}{\partial L_e} > 0$$
,  $\frac{\partial R_a}{\partial N_A} > 0$ ,  $\frac{\partial R_a}{\partial R_n} > 0$ ,  $\frac{\partial R_a}{\partial \gamma} < 0$ 

It implies that for stationary convection Lewis number, modified diffusivity ratio and nano particle Rayleigh number have stabilizing effect whenever couple-stress has destabilizing effect on the fluid layer.

## VI. RESULTS AND DISCUSSION

The influence of couple-stress on thermal instability of visco-elastic Walter's (modal B') nanofluid layer heated from below is discussed under realistic boundary conditions.

Figure 1 shows the variation of stationary Rayleigh number with Lewis number  $L_e$  for different values of  $p_l$ . The stationary Rayleigh number  $R_a$  is plotted against Lewis number for fixed values of  $N_A = 5$ ,  $\varepsilon = .1$ ,  $L_e = 30$ ,  $\gamma = 5$  and different values of  $R_{a_n} = 5$ , 10, 15,  $k_1 = 1$ , 2, 3 The Rayleigh number increases with increases in Lewis number, which shows that Lewis number has stabilizing effect on the stationary convection.

Figure 2 shows the variation of stationary Rayleigh number with Lewis number  $L_e$  for different values of  $L_e$ . The stationary Rayleigh number  $R_a$  is plotted against Lewis number for fixed values of  $N_A = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $\gamma = 5$  and different values of  $R_{a_n} = 5$ , 10, 15,  $L_e = 1$ , 2, 3 The Rayleigh number increases with increases in Lewis number, which shows that Lewis number has stabilizing effect on the stationary convection.

Figure 3 shows the variation of stationary Rayleigh number with modified diffusivity number  $N_A$  for different values of  $L_e$ . . The stationary Rayleigh number  $R_a$  is plotted against  $N_A$  for fixed values of  $N_A = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $\gamma = 5$  and different values of  $R_{a_n} = 50, 100, 150, L_e = 1, 2, 3$  The Rayleigh number increases with increases in  $N_A$ , which shows modified diffusivity number  $N_A$  has stabilizing effect on the stationary convection.

Figure 4 shows the variation of stationary Rayleigh number with modified diffusivity number  $N_A$  for different values of  $\gamma$ . The stationary Rayleigh number  $R_a$  is plotted against  $N_A$  for fixed values of  $N_A = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $L_e = 5$  and different values of  $R_{a_n} = 50, 100, 150, \gamma = 1, 2, 3$  The Rayleigh number increases with increases in  $N_A$ , which shows modified diffusivity number  $N_A$  has stabilizing effect on the stationary convection.

Figure 5 shows the variation of stationary Rayleigh number with nanoparticle Rayleigh number  $R_{a_n}$  for different values of  $L_e$ . The stationary Rayleigh number  $R_a$  is plotted against  $R_{a_n}$  for fixed values of  $p_l = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $R_{a_n} = 50$ ,  $\gamma = 1$  and different values of  $N_A = 5$ , 10, 15,  $L_e = 5$ , 10, 15 The Rayleigh number increases with increases in  $R_{a_n}$ , which shows nanoparticle Rayleigh number  $R_{a_n}$  has stabilizing effect on the stationary convection.

Figure 6 shows the variation of stationary Rayleigh number with couple-stress parameter  $\gamma$  for different values of  $L_e$ . The stationary Rayleigh number  $R_a$  is plotted against  $\gamma$  for fixed values of  $p_l = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $R_{a_n} = 50$ ,  $\gamma = 1$  and different values of  $N_A = 5, 10, 15$ ,  $L_e = 5, 10, 15$  The Rayleigh number decreases with increases in  $\gamma$ , which shows couple-stress has destabilizing effect on the stationary convection.

Figure 7 shows the variation of stationary Rayleigh number with couple-stress parameter  $\gamma$  for different values of  $\gamma$ . The stationary Rayleigh number  $R_a$  is plotted against  $\gamma$  for fixed values of  $N_A = 5$ ,  $\varepsilon = .1$ ,  $k_1 = 1$ ,  $L_e = 5$  and different values of  $p_l = 5$ , 10, 15,  $R_{a_n} = 1$ , 2, 3 The Rayleigh number decreases with increases in  $\gamma$ , which shows couple-stress has destabilizing effect on the stationary convection.

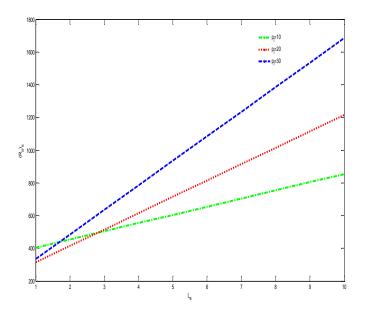


Fig.1: Variations of stationary Rayleigh number with Lewis number

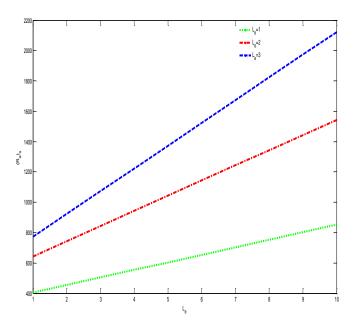


Fig.2: Variations of stationary Rayleigh number with Lewis number

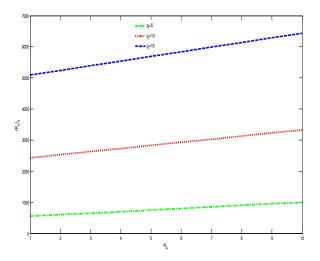


Fig.3: Variations of stationary Rayleigh number with Modified diffusivity ratio number

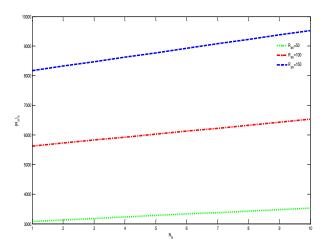


Fig.4: Variations of stationary Rayleigh number with Modified diffusivity ratio number

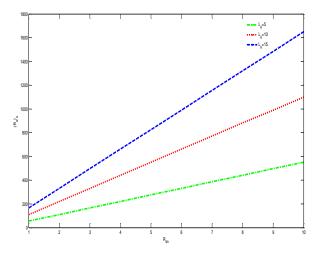


Fig.5: Variations of stationary Rayleigh number with nanoparticle Rayleigh number

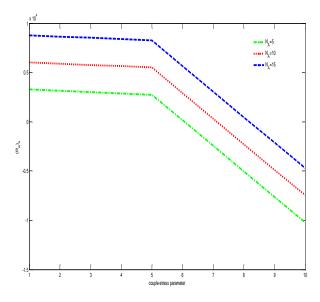


Fig.6: Variations of stationary Rayleigh number with couple-stress parameter

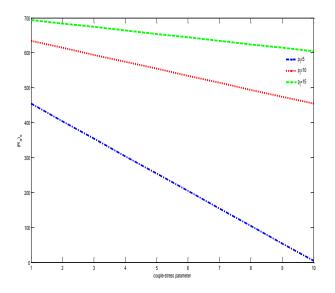


Fig.7: Variations of stationary Rayleigh number with couple-stress parameter

## VII. CONCLUSIONS

The influence of couple-stress on thermal instability of visco-elastic Walter's (modal B') nanofluid layer through a porous medium is observed by using linear instability analysis. The main conclusions from the analysis of this paper are as follows:

(1) For the stationary deportation couple-stress shows destabilizing effect on the system.

(2) Lewis number, modified diffusivity ratio and nano particle Rayleigh number have stabilizing effect on the stationary deportation.

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