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Interval-Valued Intuitionistic Q-Fuzzy Soft Subgroup Structures

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Abstract - *In this paper, we study the notion of Q-fuzzy implications on interval-valued intuitionistic fuzzy soft groupand its related properties are investigated. Characterizations of interval-valued intuitionistic Q-fuzzy soft subgroupare established and how images or inverse images of interval-valued intuitionistic fuzzy soft subgroupbecome interval-valued intuitionistic fuzzy soft subgroup studied.*

Keywords - *fuzzy set, soft set, Q-fuzzy set, interval-valued fuzzy soft set, interval-valued intuitionistic fuzzy soft group, image, inverse image, supremum and infimum.*

I. INTRODUCTION

The concept of fuzzy sets was first introduced by Zadeh in 1965 [22] and since then there has been a tremendous interest in the subject due to diverse applications ranging from engineering and computer science to social behavior studies.The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in 1971 [12]. Several mathematicians have followed the Rosenfeld approach in investigating the fuzzy subgroup theory. Fuzzy normal subgroups were studied by Wu [20,21]. Dib [2], Kumar et.al [5] and Mukherjee [7]. The concept of fuzzy quotient group was studied by some authors [3,4,5,8,9,13,14,16]. The notion of Q-fuzzy subgroups was introduced by [18,19,20,21]. The idea of interval-valued bi-cubic subgroup studied by [11].In this article,the notion of Q-fuzzy implications on interval-valued intuitionistic fuzzy soft groupis introduced and related properties are investigated. Characterizations of Q-fuzzy implications on interval-valued intuitionistic fuzzy soft groupare established and how images or inverse images of interval-valued intuitionisticfuzzy soft subgroupbecome interval-valued intuitionisticfuzzy soft subgroupstudied.

II. PRELIMINARIES

Let I be a closed unit Interval, (ie) I = [0, 1]. By an interval number we mean a closed sub interval $\tilde{a} = [\bar{a}, \overset{+}{a}]$ of I, where $0 \le \bar{a} \le \overset{+}{a} \le 1$. Denote by D[0, 1] the set of all interval numbers. Let us define what is known as refined minimum (briefly, r min) of two elements in D[0,1]. We also define the symbols "≤", "≥", "=" in case of two elements in D[0,1]. Consider two interval numbers $\tilde{a}_1 = [\bar{a}_1, \overset{+}{a}_1]$ and $\tilde{a}_2 = [\bar{a}_2, \overset{+}{a}_2]$.

Then, $r\min\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}]$, $\tilde{a}_1 \ge \tilde{a}_2$ if and only if $a_1^- \ge a_2^-$ and $a_1^+ \ge a_2^+$, and similarly we may have $\tilde{a}_1 \le \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 > \tilde{a}_2$ (respectively $\tilde{a}_1 < \tilde{a}_2$) we mean $\tilde{a}_1 \ge \tilde{a}_2$ and $\tilde{a}_1 \ne \tilde{a}_2$ (respectively $\tilde{a}_1 \le \tilde{a}_2$ *and* $\tilde{a}_1 \ne \tilde{a}_2$). Let $\tilde{a}_i \in D[0,1]$ we have $i \in \wedge$.

We define $r \inf_{i\in\wedge} \tilde{a}_i = [\inf_{i\in\wedge} a_i^-, \inf_{i\in\wedge} a_i^+]$ *and*

$r \sup_{i\in\wedge} \tilde{a}_i = [\sup_{i\in\wedge} a_i^-, \sup_{i\in\wedge} a_i^+]$.

An interval-valued fuzzy set (briefly IVF set) $\tilde{M}_A$ defined on a non-empty set X is given by $\tilde{M}_A = \{(x, [M^-{}_A(x), M^+{}_A(x)]/x \in X\}$, which is briefly denoted by $\tilde{M}_A = [M^-{}_A, M^+{}_A]$ where $M^-{}_A$ *and* $M^+{}_A$ are two Fuzzy sets in X such that $M^-{}_A(x) \leq M^+{}_A(x)$ for all $x \in X$. For any IVF set $\tilde{M}_A$ on X and $x \in X$, $\tilde{M}_A(x) = [M^-{}_A(x), M^+{}_A(x)]$ is called the degree of membership of an element x to $\tilde{M}_A$, in which $M^-{}_A(x)$ *and* $M^+{}_A(x)$ are refixed to as the lower and upper degrees, respectively of membership of x to $\tilde{M}_A$.

***2.1 Definition*:**Let A and B be fuzzy sets. Then A is a subset of B if $\mu_A(x) \leq \mu_B(x)$ for every x ∈U and it is denoted by A ⊂B or B ⊃ A.

***2.2 Definition*:** Two fuzzy sets A and B are called equal if $\mu_A(x) = \mu_B(x)$for every x ∈U and it is denoted by A= B

***2.3 Definition* :**Let A and B be two fuzzy sets. Then the algebraic product of two fuzzy sets A and B is defined by A•B ={(x, $\mu_{\tilde{A}}$ (x)) / x ∈U, $\mu_{A\bullet B} = \mu_A \bullet \mu_B$}.

2.4 Definition: Let A and B be fuzzy sets. Then the UnionA∪B and IntersectionA∩B are respectively defined by the equations

A∪B={(x,$\mu_{(A\cup B)}$(x))/ x ∈U,$\mu_{(A\cup B)}$(x) = max {(μ_A(x),μ_B(x)} and

A∩B={(x,$\mu_{(A\cap B)}$(x)) / x ∈U,$\mu_{(A\cap B)}$(x) = min {(μ_A(x), μ_B (x)}.

***2.5 remark*:** These definitions can be generated for countable number of fuzzy sets. If $\tilde{A}_1, \tilde{A}_2$,....,are fuzzy sets with membership functions $\mu_{\tilde{A}1}(x), \mu_{\tilde{A}2}(x)$,....,then the membership functions of X =$\cup_{\tilde{A}i}$ and Y= $\cap_{\tilde{A}i}$are defined asμ_X(x)=max{$\mu_{\tilde{A}1(x)}, \mu_{\tilde{A}2(x)}$,..........},x∈U and μ_Y(x)=min{$\mu_{\tilde{A}1}(x), \mu_{\tilde{A}2}(x)$,......},x ∈U respectively.

2.6 Definition: A mapping μ : X→ [0,1], where X is an arbitrary non-empty set and is called a fuzzy set in X.

***2.7 Definition*:** Let G be any group. A mapping μ : G→ [0,1] is a fuzzy groupif

(FG1) μ (xy) ≥ min { μ(x) , μ(y)} (FG2) $\mu(x^{-1}) = \mu(x)$ and (FG3) μ (e) = 1 , for all x,y ∈ G.

2.8 Definition : Let Q and G be a set and a group respectively. A mapping μ : G×Q→ [0,1] is called Q–fuzzy set in G . For any Q-fuzzy set μ in G and t ∈ [0,1] we define the set U(μ; t) = {x ∈ G / μ(x,q) ≥ t, q ∈ Q} which is called an upper cut of 'μ' and can be used to the characterization of μ.

***2.9Definition* :**A soft set f_A over U is defined as f_A: E→ P(U) such that $f_A(x)=\emptyset$ if $x \notin$A.
In other words, a soft set U is a parameterized family of subsets of the universe U. For all $\epsilon \in$ A $f_A(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set f_A. A soft set f_A over U can be presented by the set of ordered pairs:
$f_A = \{(x, f_A(x)) / x \in$ E, $f_A(x)$=P (U)}……….(1)
Clearly, a soft set is not a set. For illustration, Molodtsovconsider several examples in [10].

If f_A is a soft set over U, then the image of f_A is defined by Im(f_A) = $\{f_A(a)/a \in A\}$. The set of all soft sets over U will be denoted by S(U). Some of the operations of soft sets are listed as follows.

***2.10Definition* :**Let$f_A, f_B \in$ S(U). If $f_A(x) \subseteq f_B(x)$, for all $x \in$ E. Then $f_A$is called a soft subset of $f_B$and denoted by $f_A \subseteq f_B$. f_A and $f_B$are called soft equal, denoted by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

***2.11Definition* :** An intuitionistic fuzzy set (IFS) in the universe of discourse X is characterized by two membership functions given by $\mathcal{D}_P$ = { (t_A(x), f_A(x)) / $x \in$X} such that $t_A(x) + f_A(x) \leq 1$, for all $x \in$X.

III. INTERVAL-VALUED INTUITIONISTIC Q-FUZZY SOFT GROUP

In what follows let X denote a group unless otherwise specified.

***3.1Definition*:** Let X be a non-empty set. A Q-fuzzy implications of an interval-valued intuitionistic fuzzy soft set A in a set X is a structure $A = \{((x,q), \tilde{M}_A(x,q), N(x,q)) / x \in X,\ q \in Q\}$which briefly denoted by $A = (\tilde{M}_A, N_A)$ where $\tilde{M}_A = [M^-{}_A,\ M^+{}_A]$ is on IVIF soft set in X and N_A is a fuzzy soft set in X. Denoted by D^N(X) the families of interval-valued intuitionistic Q-fuzzy soft set.

***3.2Definition*:**An interval-valued intuitionistic fuzzy soft set $A = (\tilde{M}_A, N_A)$ in X is called aninterval-valued intuitionistic Q-fuzzy soft subgroupof X if it satisfies: for all $x, y \in X$ *and* $q \in Q$.

$$(QIVIFSG-1): \tilde{M}_A(xy,q) \geq r\ \min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\}$$
$$(QIVIFSG-2): \tilde{M}_A(x^{-1},q) \geq \tilde{M}_A(x,q)$$
$$(QIVIFSG-3): N_A(xy,q) \leq r \max\{N_A(x,q),\ N_A(y,q)\}$$
$$(QIVIFSG-4): N_A(x^{-1},q) \leq N_A(x,q).$$

3.3Example: Let X be the Klein's four group. We have $X = \{e, a, b, ab\}$ where $a^2 = e = b^2$ *and* $ab = ba$. We define $\tilde{M}_A = [M^-{}_A,\ M^+{}_A]$ and N_A by

$$\tilde{M}_A = \begin{pmatrix} e & a & b & ab \\ [-0.7,-0.4] & [-0.8,-0.5] & [-0.1,-0.3] & [-0.1,-0.7] \end{pmatrix} \textit{ and}$$
$$N_A = \begin{pmatrix} e & a & b & ab \\ -0.2 & -0.4 & -0.6 & -0.7 \end{pmatrix}.$$

Then $A = (\tilde{M}_A, N_A)$ is an interval-valued intuitionistic Q- fuzzy soft subgroup.

***3.4Example*:** Let X be a non-trivial group and define an IVF set $\tilde{M}_B = [M^-{}_B,\ M^+{}_B]$ and a $N_A$fuzzy set K by

$\tilde{M}_B(e,q) = [S_c,\ t_c]$ *and* $\tilde{M}_B(x,q) = [S,\ t]$ *for all* $x \neq e$ *where* $[S_c,\ t_c] < [S,\ t]$ *in* $D[-1,0]$,
$K(e) = t_c$ *and* $K(x) = t$ *for all* $x \neq e$ *where* $r_c > r$ *in* $[-1,0]$and'e' is the identity element of X. Then $B = (\tilde{M}_B, k)$ is aninterval-valued intuitionistic Q-fuzzy soft subgroup of X.

***3.5Proposition*:** Let $A = (\tilde{M}_A, N_A)$ be aninterval-valued intuitionistic Q-fuzzy soft subgroupof X, then $\tilde{M}_A(x^{-1},q) = \tilde{M}_A(x,q)$ *and* $N_A(x^{-1},q) = N_A(x,q)$ *for all* $x \in X,\ q \in Q$.

***Proof*:**For any $x \in X$, we have $\tilde{M}_A(x,q) = \tilde{M}_A((x^{-1})^{-1},q) \geq \tilde{M}_A(x^{-1},q) \geq \tilde{M}_A(x,q)$ *and*
$N_A(x,q) = N_A((x^{-1})^{-1},q) \leq N_A(x^{-1},q) \leq N_A(x,q).$

***3.6Proposition*:** Let $A = (\tilde{M}_A, N_A)$ be Q-fuzzy implications of interval-valued intuitionistic fuzzy soft sub groupof X. Then $\tilde{M}_A(e, q) \geq \tilde{M}_A(x,q)$ *and* $N_A(e, q) \leq N_A(x,q)$ *for all* $x \in X$, $q \in Q$, where 'e' is the identity element of X.

Proof: For any $x \in X$, using Proposition 3.5,

we have, $\tilde{M}_A(e,q) = \tilde{M}_A(xx^{-1},q) \geq r\min\{\tilde{M}_A(x,q), \tilde{M}_A(x^{-1},q)\} = \tilde{M}_A(x,q)$ *and*

$N_A(e,q) = N_A(xx^{-1},q) \leq r\max\{N_A(x,q), N_A(x^{-1},q)\} = N_A(x,q).$

This completes the proof.

***3.7Proposition*:** Let $A = (\tilde{M}_A, N_A)$ be Q-fuzzy implications of interval-valued intuitionistic fuzzy soft subgroupof X.Forany $x, y \in X$, if $\tilde{M}_A(xy^{-1},q) = \tilde{M}_A(e,q)$ and $N_A(xy^{-1},q) \leq N_A(e,q)$, then $\tilde{M}_A(x,q) = \tilde{M}_A(y,q)$ and $N_A(x,q) = N_A(y,q)$.

Proof: Let $x \in X$ *and* $q \in Q$ be such that $\tilde{M}_A(xy^{-1}, q) = \tilde{M}_A(e,q)$ *and* $N_A(xy^{-1}, q) \leq N_A(e,q)$.

Using Proposition – 3.6,

We get, $\tilde{M}_A(x,q) = \tilde{M}_A((xy^{-1})y,q) \geq r\min\{\tilde{M}_A(e,q), \tilde{M}_A(y,q)\} = \tilde{M}_A(y,q)$ and

$N_A(x,q) = N_A((xy^{-1})y,q) \leq \max\{N_A(e,q), N_A(y,q)\} = N_A(y,q)$, for all $x \in X$ *and* $q \in Q$.

Similarly,

$$\tilde{M}_A(y,q) \geq \tilde{M}_A(x,q) \text{ and } N_A(y,q) \leq N_A(x,q).$$

Hence the proof.

***3.8 Question*:** For any $x \in X$, *if* $\tilde{M}(y,q) > \tilde{M}(x,q)$ *and* $N(y,q) < N(x,q)$, then the inequalities are $\tilde{M}_A(xy,q) = \tilde{M}_A(x,q) = \tilde{M}_A(yx,q)$ *and* $N_A(xy,q) = N_A(x,q) = N_A(yx,q)$ *true*?

The following example provides a negative answer to the question 3.8.

***3.9 Example*:** In the Klein's four group $X = \{e, a, b, ab\}$, we define $\tilde{M}_A = [M^-{}_A, M^+{}_A]$ and N_A by

$$\tilde{M}_A = \begin{pmatrix} e & a & b & ab \\ [-0.4,-0.9] & [-0.2,-0.7] & [-0.3,-0.8] & [-0.1,-0.9] \end{pmatrix} \text{ and}$$

$$N_A = \begin{pmatrix} e & a & b & ab \\ -0.7 & -0.4 & -0.6 & -0.7 \end{pmatrix}.$$

Then $A = (\tilde{M}_A, N_A)$ is an interval-valued intuitionistic Q-fuzzy soft subgroupof X.

Note that,

$\tilde{M}_A(b,q) = [-0.3,-0.8] > [-0.4, 0.9] = \tilde{M}_A(a,q)$ *and* $N_A(b,q) = -0.6 < -0.4 = N_A(a,q)$.

But, $\tilde{M}_A(ab,q) = [-0.1,-0.9] \neq [-0.4,-0.8]$.

IV. Properties of Aninterval-Valued Intuitionisticq-Fuzzy Soft Subgroup

In this section, we provide characterizations of interval-valued intuitionistic Q-fuzzy soft subgroup.

***4.1Theorem*:** An interval-valued intuitionistic fuzzy soft set $A = (\tilde{M}_A, N_A)$ in X is a Q-fuzzy implications of interval-valued intuitionistic fuzzy soft group[QIVIFSG] of X if and only if satisfies

(i) $\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\}$

(ii) $N_A(xy^{-1},q) \leq r\max\{N_A(x,q),\ N_A(y,q)\}$ for all $x \in X$ *and* $q \in Q$.

Proof: Assume $A=(\tilde{M}_A, N_A)$ is an QIVIFSGof X and let $x \in X$. Then,

$$\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\}$$
$$= r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} \text{ and}$$
$$N_A(xy^{-1},q) \leq r\max\{N_A(x,q),\ N_A(y^{-1},q)\}$$
$$= r\max\{N_A(x,q),\ N_A(y,q)\}$$

By Proposition 3.5,

Conversely, suppose that (i) and (ii) are valid.

If we take y = x in (i) and (ii), then

$$\tilde{M}_A(e,q) = \tilde{M}_A(xx^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} = \tilde{M}_A(x,q) \text{ and}$$
$$N_A(e,q) = N_A(xx^{-1},q) \leq r\max\{N_A(x,q),\ N_A(x,q)\} = N_A(x,q).$$

It follows from (i) and (ii) that

$$\tilde{M}_A(y^{-1},q) = \tilde{M}_A(ey^{-1},q) \geq r\min\{\tilde{M}_A(e,q),\ \tilde{M}_A(y,q)\} = \tilde{M}_A(y,q) \text{ and}$$
$$N_A(y^{-1},q) = N_A(ey^{-1},q) \leq r\max\{N_A(e,q),\ N_A(y,q)\} = N_A(y,q).$$

So that,

$$\tilde{M}_A(xy,q) = \tilde{M}_A(x(y^{-1})^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y^{-1},q)\}$$
$$\geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} \quad \text{and}$$
$$N_A(xy,q) = N_A(x(y^{-1})^{-1},q) \leq r\max\{N_A(x,q),\ N_A(y^{-1},q)\}$$
$$\leq r\max\{N_A(x,q),\ N_A(y,q)\}$$

Therefore, $A=(\tilde{M}_A, N_A)$ is an QIVIFSG of X.

4.2 Theorem **:** If $A=(\tilde{M}_A, N_A)$ is a Q-fuzzy implications of interval-valued intuitionistic fuzzy soft groupQIVIFSG of X, then the set $S=\{\ x \in X \,/\, \tilde{M}_A(x,q) = \tilde{M}_A(e,q),\ N_A(x,q) = N(e,q)\ \}$ is a subgroup of X.

Proof: Let $x, y \in S$, *then* $\tilde{M}(x,q) = \tilde{M}(e,q) = \tilde{M}_A(y,q)$ *and* $N_A(x,q) = N_A(e,q) = N_A(y,q)$.

It follows from theorem 4.1 that

$$\tilde{M}_A(xy^{-1},q) \geq r\min\{\tilde{M}_A(x,q),\ \tilde{M}_A(y,q)\} = \tilde{M}_A(e,q) \text{ and}$$
$$N_A(xy^{-1},q) \leq r\max\{N_A(x,q),\ N_A(y,q)\} = N_A(e,q).$$

So from Proposition 3.6 that

$$\tilde{M}_A(xy^{-1},q) = \tilde{M}_A(e,q) \quad \text{and} \quad N_A(xy^{-1},q) = N_A(e,q).$$

Hence, $xy^{-1} \in S$ *and* $q \in Q$.

So, S is a subgroup of X.

4.3 Definition**:** Let $A=(\tilde{M}_A, N_A)$ be aninterval-valued intuitionistic fuzzy soft setin a set X, $r \in [0,1]$ *and* $[s,t] \in D[0,1]$. The set $\cup(A:[s,t],r) = \{x \in X \,/\, \tilde{M}_A(x,q) \geq [s,t],\ N(x,q) \leq r\}$ is called the Q-fuzzy implications of interval-valued intuitionistic fuzzy level soft set of A.

4.4 Theorem **:** For a interval-valued intuitionistic fuzzy soft set $A=(\tilde{M}_A, N_A)$ in X, the following are equivalent.

(i) $A=(\widetilde{M}_A, N_A)$ is a Q-fuzzy implications of interval-valued intuitionistic fuzzy soft subgroupQIVIFSG of X.

(ii) The non-emptyQ-fuzzy implications of interval-valued intuitionistic fuzzylevel soft set of A is a subgroup of X.

***Proof*:** Assume that A is a QIVIFSG of X. Let $x, y \in \bigcup(A:[s,t], r)$, for all $r \in [0,1]$ *and* $[s,t] \in D[-1,0]$. Then,

$$\widetilde{M}_A(x,q) \geq [s,t], \quad N_A(x,q) \leq r,$$
$$\widetilde{M}_A(y,q) \geq [s,t], \quad N_A(y,q) \leq r.$$

It follows from theorem-4.1 that

$$\widetilde{M}_A(xy^{-1},q) \geq r \min\left\{\widetilde{M}_A(x,q),\, \widetilde{M}_A(y,q)\right\} \geq [s,t] \text{ and}$$
$$N_A(xy^{-1},q) \leq r \max\left\{N_A(x,q),\, N_A(y,q)\right\} = r.$$

So that, $xy^{-1} \in \cup\ (A:[s,t],r)$.

Therefore, the non-empty interval-valued intuitionistic fuzzy level soft set of A is a subgroup of X.

Conversely, let $r \in [-1,0]$ *and* $[s,t] \in D[-1,0]$ be such that $\bigcup(A:[s,t], r) \neq \phi$ *and* $\bigcup(A:[s,t], r)$ is a subgroup of X.

Suppose that theorem 4.1 (i) is not true and theorem 4.1 (ii) is valid, then there exists $[s_0, t_0] \in D[-1,0]$ and $a, b \in X$ such that

$$\widetilde{M}_A(ab^{-1},q) \leq [s_0,t_0] \leq r \min\left\{\widetilde{M}_A(a,q),\, \widetilde{M}_A(b,q)\right\} \text{ and}$$
$$N_A(ab^{-1},q) \leq r \max\left\{N_A(a,q),\, N_A(b,q)\right\}.$$

It follows that $a, b \in \bigcup(A:[s_0,t_0], r)$, $\max\left\{N_A(a,q),\, N_A(b,q)\right\}$

But $ab^{-1} \notin \cup\ (A:[s_0,t_0], max\{N_A(a,q), N_A(b,q)\})$.

This is a contradiction.

If theorem 4.1 (i) is true and Theorem 4.1 (ii) is not valid, then

$$\widetilde{M}_A(ab^{-1},q) \geq r \min\left\{\widetilde{M}_A(a,q),\, \widetilde{M}_A(b,q)\right\} \text{ and}$$
$$N_A(ab^{-1},q) > r_0 \geq r \max\left\{N_A(a,q),\, N_A(b,q)\right\}.$$

for some $r_0 \in [-1,0]$ *and* $a, b \in X$.

Thus $a, b \in \bigcup(A:\ r \min\left\{\widetilde{M}_A(a,q),\, \widetilde{M}_A(b,q), r_0\right\}$

But, $ab^{-1} \notin \cup\ (A: r\ min\{\widetilde{M}_A(a,q), \widetilde{M}_A(b,q), r_0\})$.

This is a contradiction.

Assume that there exist $[s_0,t_0] \in D[-1,0]$, $r_0 \in [-1,0]$ *and* $a, b \in X$ *such that*

$$\widetilde{M}_A(ab^{-1},q) \leq [s_0,t_0] \leq r \min\left\{\widetilde{M}_A(a,q),\, \widetilde{M}_A(b,q)\right\} \text{ and}$$
$$N_A(ab^{-1},q) > r_0 \geq r \max\left\{N_A(a),\, N_A(b)\right\}.$$

Then $a, b \in \bigcup(A:[s_0,t_0], r_0)$ *but* $ab^{-1} \notin \bigcup(A:[s_0,t_0], r_0)$

This is contradiction. Hence (i) and (ii) of theorem 4.1 are valid.

Therefore 'A' is QIVIFSG of X.

***4.5 Definition*:** Let X and Y be given classical set. A mapping $\chi: X \rightarrow Y$ induces two mappings $R_\chi: R(X) \rightarrow R(Y), A \mapsto R_\chi(A)$ and $R_\chi^{-1}: R(Y) \rightarrow R(X), B \mapsto R_{\chi^{-1}}(B)$, where $R_\chi(A)$ is given by

$$R_\chi(\widetilde{M}_A)(y,q) = \begin{cases} r\sup\limits_{y=\chi(x)} \widetilde{M}_A(x,q) & if\ \ \chi^{-1}(y,q) \neq \phi \\ [0,0] & otherwise \end{cases}$$

$$R_\chi(N_A)(y,q) = \begin{cases} r\inf\limits_{y=\chi(x)} N_A(x,q) & if\ \ \chi^{-1}(y,q) \neq \phi \\ -1 & otherwise \end{cases}$$

For all $y \in Y$ and $R_\chi^{-1}(B)$ is defined by $R_\chi^{-1}(\widetilde{M}_B)(x,q) = \widetilde{M}_B(\chi(x))$ and $R_\chi^{-1}(\rho(x)) = \rho(\chi(x))$ for all $x \in X$, then the mapping R_χ (*respectively* R_χ^{-1}) is called an interval-valued intuitionistic fuzzy soft transformation induced by χ.

***4.6Theorem*:** For a homomorphism $\chi: X \to Y$ of groups, let $R_\chi: R(X) \to R(Y)$ and $R_\chi^{-1}: R(Y) \to R(X)$ be the interval-valued intuitionistic fuzzy soft transformation and inverse cubic transformation, respectively induced by χ.

(i) If $A = (\widetilde{M}_A, N_A) \in R(X)$ is QIVIFSG of X, which has the interval-valued intuitionistic fuzzy soft property, then $R_\chi(A)$ is QIVIFSG of Y.

(ii) If $B = (\widetilde{M}_B, \rho) \in R(Y)$ is QIVIFSG of Y, which has the interval-valued intuitionistic fuzzy soft property, then $R_\chi^{-1}(B)$ is QIVIFSG of X.

***Proof*:** (i) Given $\chi(x), \chi(y) \in \chi(X)$, *let* $x_0 \in \chi^{-1}(\chi(x))$ *and* $y_0 \in \chi^{-1}(\chi(y))$ be such that

$\widetilde{M}_A(x_0,q) = r\sup\limits_{a\in\chi^{-1}(\chi(x))} \widetilde{M}_A(a,q), \qquad N_A(x_0,q) = \inf\limits_{a\in\chi^{-1}(\chi(x))} N_A(a,q)$ and

$\widetilde{M}_A(y_0,q) = r\sup\limits_{b\in\chi^{-1}(\chi(y))} \widetilde{M}_A(b,q), \qquad N_A(y_0,q) = \inf\limits_{b\in\chi^{-1}(\chi(y))} N_A(b,q)$, respectively.

Then,

$$\begin{aligned} R_\chi(\widetilde{M}_A)\big(\chi(x)\chi(y),q\big) &= r\sup_{(z,q)\in\chi^{-1}(\chi(x)\chi(y))} \widetilde{M}_A(z,q) \\ &\geq \widetilde{M}_A(x_0y_0,q) \\ &\geq r\min\{\widetilde{M}_A(x_0,q),\ \widetilde{M}_A(y_0,q)\} \\ &= r\min\left\{ r\sup_{a\in\chi^{-1}(\chi(x))} \widetilde{M}_A(a,q),\ r\sup_{b\in\chi^{-1}(\chi(y))} \widetilde{M}_A(b,q) \right\} \\ &= r\min\{R_\chi(\widetilde{M}_A)(\chi(x)),\ R_\chi(\widetilde{M}_A)(\chi(y))\} \end{aligned}$$

$$\begin{aligned} R_\chi(\widetilde{M}_A)\big(\chi^{-1}(x),q\big) &= r\sup_{z\in\chi^{-1}(\chi^{-1}(x))} \widetilde{M}_A(z,q) \\ &\geq \widetilde{M}_A(x_0^{-1},q) \\ &\geq \widetilde{M}_A(x_0,q) \\ &= R_\chi(\widetilde{M}_A)\big(\chi(x)\big). \end{aligned}$$

$$\begin{aligned} R_\chi(N_A)\big(\chi(x)\chi(y),q\big) &= \inf_{(z,q)\in\chi^{-1}(\chi(x)\chi(y))} N_A(z,q) \\ &\leq N_A(x_0y_0,q) \\ &\leq r\max\{N_A(x_0,q),\ N_A(y_0,q)\} \end{aligned}$$

$$= r\max\left\{\inf_{a\in\chi^{-1}(\chi(x))} N_A(a,q),\ \inf_{b\in\chi^{-1}(\chi(y))} N_A(b,q)\right\}$$
$$= r\max\left\{R_\chi(\chi(x)),\ R_\chi(\chi(y))\right\}\text{and}$$
$$R_\chi(N_A)\left(\chi^{-1}(x),q\right) = \inf_{(z,q)\in\chi^{-1}\left(\chi^{-1}(x)\right)} N_A(z,q)$$
$$\leq N_A(x_0^{-1},q)$$
$$\leq N_A(x_0,q)$$
$$= R_\chi(N_A)\left(\chi(x)\right).$$

Therefore $R_\chi(A)$ is a QIVIFSG of Y.

(ii) For any $x, y \in X$, *we have*

$$R_\chi^{-1}(\tilde{M}_B)\left(xy,q\right) = \tilde{M}_B(\chi(xy,q))$$
$$= \tilde{M}_B(\chi(x)\chi(y),q))$$
$$\geq r\ \min\left\{\tilde{M}_B(\chi(x),q),\ \tilde{M}_B(\chi(y),q)\right\}$$
$$\geq r\ \min\left\{R_\chi^{-1}(\tilde{M}_B)(x,q),\ R_\chi^{-1}(\tilde{M}_B)(y,q)\right\}$$
$$R_\chi^{-1}(\tilde{M}_B)\left(x^{-1},q\right) = \tilde{M}_B(\chi(x^{-1},q))$$
$$\geq \tilde{M}_B(\chi(x,q))$$
$$= R_\chi^{-1}(\tilde{M}_B)(x,q)$$
$$R_\chi^{-1}(\rho)\left(xy,q\right) = \rho(\chi(xy),q)$$
$$= \rho(\chi(x)\chi(y),q)$$
$$\leq r\max\left\{\rho(\chi(x),q),\ \rho(\chi(y),q)\right\}$$
$$\leq r\max\left\{R_\chi^{-1}(\rho(x,q),\ R_\chi^{-1}(\rho(y,q)\right\}\text{and}$$
$$R_\chi^{-1}(\rho)\left(x^{-1},q\right) = \rho(\chi(x^{-1}),q)$$
$$\leq \rho(\chi(x),q)$$
$$\leq R_\chi^{-1}(\rho)(x,q).$$ Hence $R_\chi^{-1}(B)$ is a QIVIFSG of X.

4.7 Note**:** Q-Fuzzy implications of interval-valued intuitionistic fuzzy soft set A in X has the interval-valued intuitionistic fuzzy property if for any subset T of X there exists $x_0 \in T$ such that

$$\tilde{M}_A(x_0,q) = \underset{(xo,q)\in T}{rsup}\ \tilde{M}_A(x_0,q),$$
$$N_A(x_0,q) = \inf_{(x_0,q)\in T} N_A(x_0,q).$$

CONCLUSION

Characterizations of interval-valued intuitionisticQ-fuzzy soft subgroupare established and how images or inverse images of interval-valued intuitionisticfuzzy soft subgroupbecome interval-valued intuitionistic fuzzy soft subgroupstudied.

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