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Signed Edge Unidomination Number of a Path

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Abstract *- Graph theory is an important branch of Mathematics that was developed exponentially. In current years domination in graphs is rapidly emerging area of research in graph theory and it has become the basis of interest of many researchers. The concept Signed dominating function was introduced by Dunbar et al. [4]. Edge domination was introduced by Mitchell and Hedetneimi [7]. Bharathi [3] has introduced by new concept edge unidomination and studied this for complete K-partite graph.*

Signed unidominating function was defined and studied by Aruna [1] in 2019 and results on signed unidomination number and upper signed unidomination number of some corona product graphs are discussed. Signed edge domination on rooted product graph studied by Shobha Rani [9]. In this paper, we present signed edge unidominating function of a path and determine signed edge unidomination number.

Keywords - *Signed edge unidominating function, signed edge unidomination number, path.*

I. INTRODUCTION

Currently, the major development of graph theory has occurred and motivated to a larger degree and has become the source of interest to many researchers due to its applications to various branches of Science & Technology.

An introduction and an extensive summary on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [5, 6].The theory of domination in graphs introduced by Ore [8] and Berge [2] is a fascinated area of research in graph theory in the last three decades.

In this paper a new concept signed edge unidominating function of a graph is introduced and this is studied for a path. Also signed edge unidomination number of a path is found in different cases.

II. SIGNED EDGE UNIDOMINATING FUNCTION AND SIGNED EDGE UNIDOMINATION NUMBER

In this section, signed edge unidominating function and signed edge unidomination number of a graph are defined as follows:

Definition1:

Let $G(V, E)$ be a connected graph. A function $f: E \to \{-1,1\}$ is said to be a signed edge unidominating function, if

$$\sum_{e' \in N[e]} f(e') \geq 1 \quad \forall\, e \in E \text{ and } f(e) = 1,$$

$$\sum_{e' \in N[e]} f(e') = 1 \quad \forall\, e \in E \text{ and } f(e) = -1$$

where $N[e]$ is the closed neighbourhood of the edge e.

Definition 2:

The signed edge unidomination number of a graph $G(V,E)$ is defined

as $min\{f(E)/f \text{ is a signed edge unidominating function}\}$.

It is denoted by $\gamma'_{su}(G)$.

Here $f(E) = \sum_{e' \in E} f(e')$ *is called as the weight of the* signed *edge unidominating function* f.

That is the signed edge unidomination number of a graph $G(V,E)$ is the minimum of the weights of the signed edge unidominating functions of G.

III. SIGNED EDGE UNIDOMINATION NUMBER OF A PATH

In this section we discuss signed edge unidominating function of a path and signed edge unidomination number of this graph is obtained in various cases.

Theorem 3.1: The signed edge unidomination number of a path P_n, $n \geq 6$ is

$$\gamma'_{su}(P_n) = \begin{cases} \frac{n+3}{3} & if\ n \equiv 0(mod\ 3), \\ \frac{n+5}{3} & if\ n \equiv 1(mod\ 3), \\ \frac{n+7}{3} & if\ n \equiv 2(mod\ 3). \end{cases}$$

Proof: Let P_n be a path with n vertices $v_1, v_2, \dots \dots v_n$ and $(n-1)$ edges $e_1, e_2, \dots \dots e_m$,

where $m = n - 1$.

In P_n we have

$N[v_1] = \{v_1, v_2\}, N[v_2] = \{v_1, v_2, v_3\}, \dots., N[v_{n-1}] = \{v_{n-2}, v_{n-1}, v_n\}, N[v_n] = \{v_{n-1}, v_n\}$,

$N[e_1] = \{e_1, e_2\}$, $N[e_2] = \{e_1, e_2, e_3\}, \dots., N[e_{m-1}] = \{e_{m-2}, e_{m-1}, e_m\}$, $N[e_m] = \{e_{m-1}, e_m\}$.

That is every vertex and every edge is adjacent to two vertices and two edges respectively except the first and the last.

The following three cases arise.

Case 1:

Let $n \equiv 0(mod\ 3)$.

Now $n - 1 \equiv 2(mod\ 3) \Longrightarrow m \equiv 2(mod\ 3)$.

Define a function $f: E \rightarrow \{-1,1\}$ by

$$f(e_i) = \begin{cases} -1 & for\ i \equiv 0(mod\ 3), \\ 1 & otherwise. \end{cases}$$

We assign the functional values to the edges of P_n in the sequence $1, 1, -1; 1, 1, -1;$ ... ...upto

we get sets of triple edges e_{i-1}, e_i, e_{i+1} of P_n and the remaining edges the value 1.

Now we check the condition for signed edge unidominating function at every edge.

We classify the edges in the following way.

$e_i\ with\ i \equiv 0(mod\ 3),$

$e_i\ with\ i \equiv 1(mod\ 3),$

$e_i\ with\ i \equiv 2(mod\ 3).$

As e_1 and e_m are pendent edges, we deal these edges separately, in each case.

Sub case 1:

Let $i \equiv 0(mod\ 3)$. Then $f(e_i) = -1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + (-1) + 1 = 1.$$

Sub case 2:

Let $i \equiv 1(mod\ 3)$ and $i \neq 1$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = (-1) + 1 + 1 = 1.$$

If $i = 1$, we have

$$\sum_{e' \in N[e_1]} f(e') = f(e_1) + f(e_2) = 1 + 1 = 2.$$

Sub case 3:

Let $i \equiv 2(mod\ 3)$. $i \neq m$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + 1 + (-1) = 1.$$

For $i = m,$ we have

$$\sum_{e'\in N[e_m]} f(e') = f(e_{m-1}) + f(e_m) = 1 + 1 = 2.$$

Since $\sum_{e'\in N[e_i]} f(e') \geq 1$ for $f(e_i) = 1$ and $\sum_{e'\in N[e_i]} f(e') = 1$ for $f(e_i) = -1$,

it follows that f is a signed edge unidominating function.

Now $f(E) = \sum_{e'\in P_n} f(e') = \underbrace{(1 + 1 + (-1) + \cdots + (1 + 1 + (-1))}_{\frac{n-3}{3} - times} + 1 + 1$

$$= \frac{n-3}{3} + 2 = \frac{n+3}{3}.$$

(We take three edges as one group whose functional values are $1, 1, -1$ and the sum is 1 and

there are $\frac{n-3}{3}$ such groups).

Thus $f(E) = \frac{n+3}{3}$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n, we can show that

the resulting functions are not signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n) = \frac{n+3}{3}, when\, n \equiv 0(mod 3)$.

Case 2:

Let $n \equiv 1(mod\ 3)$.

Then $n - 1 \equiv 0(mod\ 3) \Longrightarrow m \equiv 0(mod\ 3)$.

Define a function $f: E \to \{-1,1\}\ by$

$$f(e_i) = \begin{cases} -1 & for\ i\ \equiv 0(mod\ 3)\ and\ i \neq m, \\ 1 & otherwise. \end{cases}$$

We assign the functional values to the edges of P_n in the sequence $1, 1, -1; 1, 1, -1;$ ... ...upto

we get sets of triple edges e_{i-1}, e_i, e_{i+1} of P_n and the remaining edges the value 1.

Now we check the condition for signed edge unidominating function at every edge.

Sub case 1:

Let $i \equiv 0(mod\ 3)$ and $i \neq m$. Then $f(e_i) = -1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + (-1) + 1 = 1.$$

For $i = m$, we have

$$\sum_{e' \in N[e_m]} f(e') = f(e_{m-1}) + f(e_m) = 1 + 1 = 2.$$

Sub case 2:

Let $i \equiv 1(mod\ 3)$ and $i \neq 1$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = (-1) + 1 + 1 = 1.$$

For $i = 1$, we have

$$\sum_{e' \in N[e_1]} f(e') = f(e_1) + f(e_2) = 1 + 1 = 2.$$

Sub case 3:

Let $i \equiv 2(mod\ 3)$ and $i \neq m - 1$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + 1 + (-1) = 1.$$

For $i = m - 1$, we have

$$\sum_{e' \in N[e_{m-1}]} f(e') = f(e_{m-2}) + f(e_{m-1}) + f(e_m) = 1 + 1 + 1 = 3.$$

Since $\sum_{e' \in N[e_i]} f(e') \geq 1$ for $f(e_i) = 1$ and $\sum_{e' \in N[e_i]} f(e') = 1$ for $f(e_i) = -1$,

it follows that f is a signed edge unidominating function.

$$\text{Now} \quad f(E) = \sum_{e' \in P_n} f(e') = \underbrace{(1 + 1 + (-1) + \cdots + (1 + 1 + (-1))}_{\frac{n-4}{3} - times} + 1 + 1 + 1$$

$$= \frac{n-4}{3} + 3 = \frac{n+5}{3}.$$

(Here there are $\frac{n-4}{3}$ groups with functional values sum as 1).

Thus $f(E) = \frac{n+5}{3}$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n, we can show that the resulting functions are not signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n) = \frac{n+5}{3}, when\, n \equiv 1(mod 3)$.

Case 3:

Let $n \equiv 2(mod\ 3)$.

Then $n - 1 \equiv 1(mod 3) \Longrightarrow m \equiv 1(mod 3)$.

Define a function $f: E \rightarrow \{-1,1\}$ by

$$f(e_i) = \begin{cases} -1 & \text{for } i \equiv 0(mod\ 3),\ i \neq m-1, \\ 1 & otherwise\ . \end{cases}$$

We assign the functional values to the edges of P_n in the sequence $1, 1, -1; 1, 1, -1; \ldots \ldots \ldots$upto we get sets of triple edges e_{i-1}, e_i, e_{i+1} of P_n and the remaining edges the value 1.

Now we check the condition for signed edge unidominating function at every edge.

Sub case 1:

Let $i \equiv 0(mod\ 3)$ and $i \neq m-1$. Then $f(e_i) = -1$.

$$\text{Now} \sum_{e\prime \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + (-1) + 1 = 1.$$

For $i = m - 1$, we have

$$\sum_{e\prime \in N[e_{m-1}]} f(e') = f(e_{m-2}) + f(e_{m-1}) + f(e_m) = 1 + 1 + 1 = 3.$$

Sub case 2:

Let $i \equiv 1(mod\ 3)$, $i \neq 1$, and $i \neq m$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = (-1) + 1 + 1 = 1.$$

For $i = 1$, we have

$$\sum_{e' \in N[e_1]} f(e') = f(e_1) + f(e_2) = 1 + 1 = 2.$$

For $i = m$, we have

$$\sum_{e' \in N[e_m]} f(e') = f(e_{m-1}) + f(e_m) = 1 + 1 = 2.$$

Sub case 3:

Let $i \equiv 2(mod\ 3)$ and $i \neq m - 2$. Then $f(e_i) = 1$.

$$\text{Now} \sum_{e' \in N[e_i]} f(e') = f(e_{i-1}) + f(e_i) + f(e_{i+1}) = 1 + 1 + (-1) = 1.$$

For $i = m - 2$, we have

$$\text{Now} \sum_{e' \in N[e_{m-2}]} f(e') = f(e_{m-3}) + f(e_{m-2}) + f(e_{m-1}) = 1 + 1 + 1 = 3.$$

Since $\sum_{e' \in N[e_i]} f(e') \geq 1$ for $f(e_i) = 1$ and $\sum_{e' \in N[e_i]} f(e') = 1$ for $f(e_i) = -1$,

it follows that f is a signed edge unidominating function.

$$\text{Now} \quad f(E) = \sum_{e' \in P_n} f\,(e') = \underbrace{(1 + 1 + (-1) + \cdots + (1 + 1 + (-1))}_{\frac{n-5}{3} - times} + 1 + 1 + 1 + 1$$

$$= \frac{n-5}{3} + 4 = \frac{n+7}{3}.$$

(Here there are $\frac{n-5}{3}$ groups with functional values sum as 1).

Thus $f(E) = \frac{n+7}{3}$.

Now for all other possibilities of assigning values 1 and -1 to the edges of P_n, we can show that

the resulting functions are not signed edge unidominating functions.

Hence the function defined above is the only signed edge unidominating function.

Therefore $\gamma'_{su}(P_n) = \frac{n+7}{3}, when\, n \equiv 2(mod3).$

Combining all three cases completes the proof of the theorem.

Thus the signed edge unidomination number of a path $P_n,\ n \geq 6$ is

$$\gamma'_{su}(P_n) = \begin{cases} \dfrac{n+3}{3} & if\ \ n \equiv 0(mod\ 3), \\ \dfrac{n+5}{3} & if\ \ n \equiv 1(mod\ 3), \\ \dfrac{n+7}{3} & if\ \ n \equiv 2(mod\ 3). \end{cases}$$

Theorem 3.2: For $n \equiv 0(mod\ 3), n \equiv 1(mod\ 3), n \equiv 2(mod\ 3)$ the number of signed edge unidominating functions of P_n is 1 with minimum weights $\frac{n+3}{3}, \frac{n+5}{3}, \frac{n+7}{3}$ respectively.

Proof: Follows by Theorem 3.1.

IV. CONCLUSION

In this paper the authors have studied signed edge unidominating function and signed edge unidomination number of a path. This works throws light on further study of some other standard graphs such as complete graph, wheel and etc.

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